13 Gears—General

Chapter Outline

This chapter addresses gear geometry, the kinematic relations, and the forces transmitted by the four principal types of gears: spur, helical, bevel, and worm gears. The forces transmitted between meshing gears supply torsional moments to shafts for motion and power transmission and create forces and moments that affect the shaft and its bearings. The next two chapters will address stress, strength, safety, and reliability of the four types of gears.

13–1 Types of Gears

Spur gears, illustrated in Fig. 13–1, have teeth parallel to the axis of rotation and are used to transmit motion from one shaft to another, parallel, shaft. Of all types, the spur gear is the simplest and, for this reason, will be used to develop the primary kinematic relationships of the tooth form.

Helical gears, shown in Fig. 13–2, have teeth inclined to the axis of rotation. Helical gears can be used for the same applications as spur gears and, when so used, are not as noisy, because of the more gradual engagement of the teeth during meshing. The inclined tooth also develops thrust loads and bending couples, which are not present with spur gearing. Sometimes helical gears are used to transmit motion between nonparallel shafts.

Bevel gears, shown in Fig. 13–3, have teeth formed on conical surfaces and are used mostly for transmitting motion between intersecting shafts. The figure actually illustrates *straight-tooth bevel gears. Spiral bevel gears* are cut so the tooth is no longer straight, but forms a circular arc. *Hypoid gears* are quite similar to spiral bevel gears except that the shafts are offset and nonintersecting.

Figure 13–1

Spur gears are used to transmit rotary motion between parallel shafts.

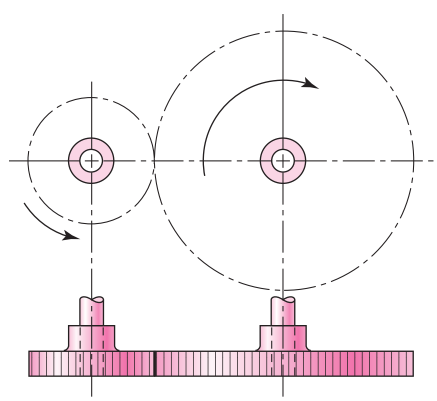
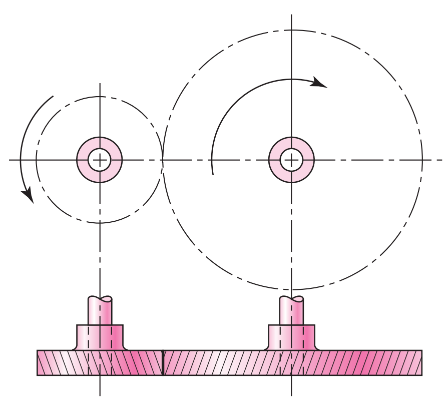


Figure 13–2

Helical gears are used to transmit motion between parallel or nonparallel shafts.



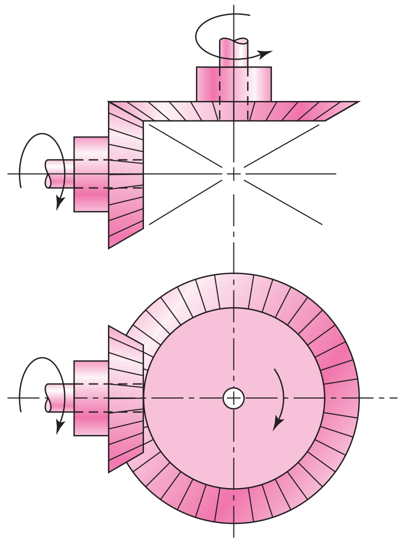


Figure 13–3

Bevel gears are used to transmit rotary motion between intersecting shafts.

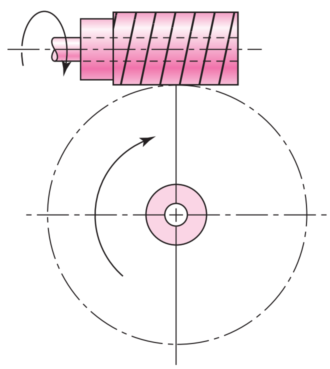


Figure 13–4

Worm gearsets are used to transmit rotary motion between nonparallel and nonintersecting shafts.

Worms and *worm gears,* shown in Fig. 13–4, represent the fourth basic gear type. As shown, the worm resembles a screw. The direction of rotation of the worm gear, also called the worm wheel, depends upon the direction of rotation of the worm and upon whether the worm teeth are cut right-hand or left-hand. Worm-gear sets are also made so that the teeth of one or both wrap partly around the other. Such sets are called *single-enveloping* and *double-enveloping* worm-gear sets. Worm-gear sets are mostly used when the speed ratios of the two shafts are quite high, say, 3 or more.

13–2 Nomenclature

The terminology of spur-gear teeth is illustrated in Fig. 13–5. The *pitch circle* is a theoretical circle upon which all calculations are usually based; its diameter is the *pitch diameter.* The pitch circles of a pair of mating gears are tangent to each other. A *pinion* is the smaller of two mating gears. The larger is often called the *gear.*

The *circular pitch* p is the distance, measured on the pitch circle, from a point on one tooth to a corresponding point on an adjacent tooth. Thus the circular pitch is equal to the sum of the *tooth thickness* and the *width of space.*

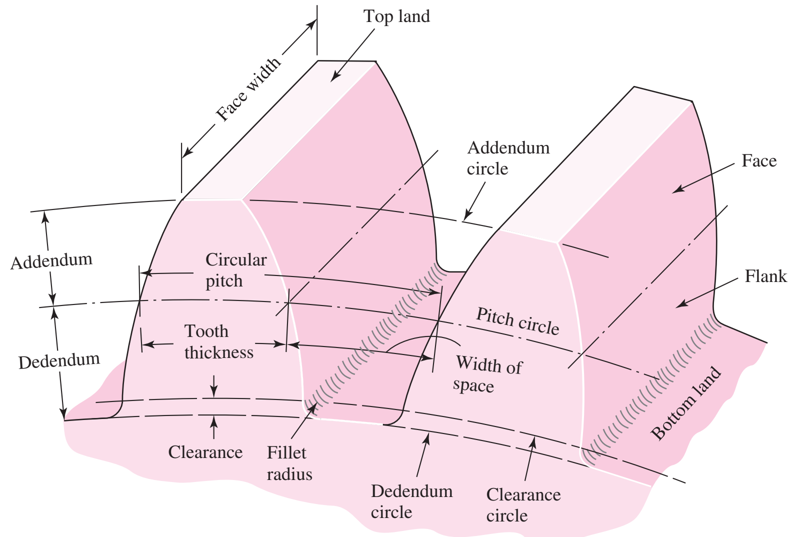


Figure 13–5

Nomenclature of spur-gear teeth.

The *module* m is the ratio of the pitch diameter to the number of teeth. The customary unit of length used is the millimeter. The module is the index of tooth size in SI.

The *diametral pitch* P is the ratio of the number of teeth on the gear to the pitch diameter. Thus, it is the reciprocal of the module. Since diametral pitch is used only with U.S. units, it is expressed as teeth per inch.

The *addendum* a is the radial distance between the *top land* and the pitch circle. The *dedendum* b is the radial distance from the *bottom land* to the pitch circle. The *whole depth* h_t is the sum of the addendum and the dedendum.

The *clearance circle* is a circle that is tangent to the addendum circle of the mating gear. The *clearance* c is the amount by which the dedendum in a given gear exceeds the addendum of its mating gear. The *backlash* is the amount by which the width of a tooth space exceeds the thickness of the engaging tooth measured on the pitch circles.

You should prove for yourself the validity of the following useful relations:

$$P = \frac{N}{d} \tag{13–1}$$

$$m = \frac{d}{N} \tag{13–2}$$

$$p = \frac{\pi d}{N} = \pi m \tag{13–3}$$

$$pP = \pi \tag{13–4}$$

where P = diametral pitch, teeth per inch
N = number of teeth
d = pitch diameter, in
m = module, mm
d = pitch diameter, mm
p = circular pitch

13–3 Conjugate Action

The following discussion assumes the teeth to be perfectly formed, perfectly smooth, and absolutely rigid. Such an assumption is, of course, unrealistic, because the application of forces will cause deflections.

Mating gear teeth acting against each other to produce rotary motion are similar to cams. When the tooth profiles, or cams, are designed so as to produce a constant angular-velocity ratio during meshing, these are said to have *conjugate action*. In theory, at least, it is possible arbitrarily to select any profile for one tooth and then to find a profile for the meshing tooth that will give conjugate action. One of these solutions is the *involute profile,* which, with few exceptions, is in universal use for gear teeth and is the only one with which we should be concerned.

When one curved surface pushes against another (Fig. 13–6), the point of contact occurs where the two surfaces are tangent to each other (point c), and the forces at any instant are directed along the common normal ab to the two curves. The line ab, representing the direction of action of the forces, is called the *line of action*. The line of action will intersect the line of centers O-O at some point P. The angular-velocity ratio between the two arms is inversely proportional to their radii to the point P. Circles drawn through point P from each center are called *pitch circles,* and the radius of each circle is called the *pitch radius*. Point P is called the *pitch point.*

Figure 13–6 is useful in making another observation. A pair of gears is really pairs of cams that act through a small arc and, before running off the involute contour, are replaced by another identical pair of cams. The cams can run in either direction and are configured to transmit a constant angular-velocity ratio. If involute curves are used, the gears tolerate changes in center-to-center distance with *no* variation in constant angular-velocity ratio. Furthermore, the rack profiles are straight-flanked, making primary tooling simpler.

To transmit motion at a constant angular-velocity ratio, the pitch point must remain fixed; that is, all the lines of action for every instantaneous point of contact must pass through the same point P. In the case of the involute profile, it will be shown that all points of contact occur on the same straight line ab, that all normals to the tooth profiles at the point of contact coincide with the line ab, and, thus, that these profiles transmit uniform rotary motion.

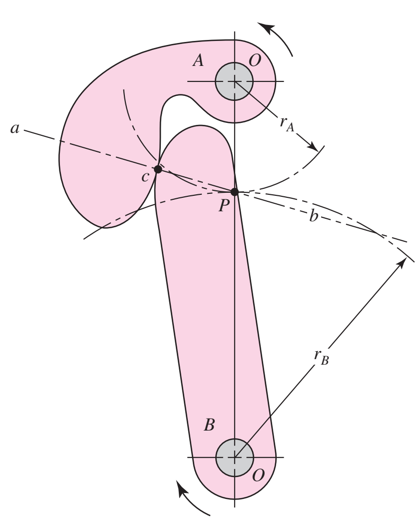
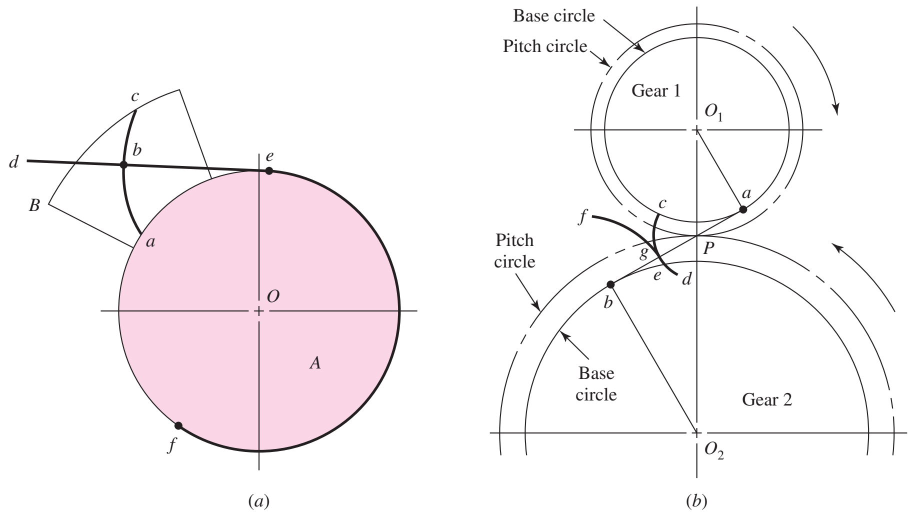


Figure 13–6

Cam *A* and follower *B* in contact. When the contacting surfaces are involute profiles, the ensuing conjugate action produces a constant angular-velocity ratio.

Figure 13–7

(*a*) Generation of an involute; (*b*) involute action.



13–4 Involute Properties

An involute curve may be generated as shown in Fig. 13–7*a*. A partial flange *B* is attached to the cylinder *A*, around which is wrapped a cord *def*, which is held tight. Point *b* on the cord represents the tracing point, and as the cord is wrapped and unwrapped about the cylinder, point *b* will trace out the involute curve *ac*. The radius of the curvature of the involute varies continuously, being zero at point *a* and a maximum at point *c*. At point *b* the radius is equal to the distance *be*, since point *b* is instantaneously rotating about point *e*. Thus the generating line *de* is normal to the involute at all points of intersection and, at the same time, is always tangent to the cylinder *A*. The circle on which the involute is generated is called the *base circle*.

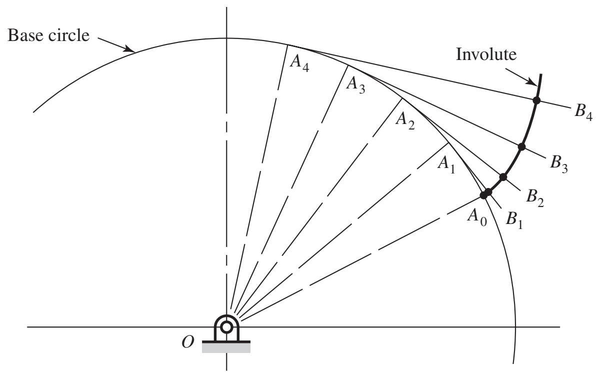
Let us now examine the involute profile to see how it satisfies the requirement for the transmission of uniform motion. In Fig. 13–7*b*, two gear blanks with fixed centers at O_1 and O_2 are shown having base circles whose respective radii are O_1a and O_2b. We now imagine that a cord is wound clockwise around the base circle of gear 1, pulled tight between points *a* and *b*, and wound counterclockwise around the base circle of gear 2. If, now, the base circles are rotated in different directions so as to keep the cord tight, a point *g* on the cord will trace out the involutes *cd* on gear 1 and *ef* on gear 2. The involutes are thus generated simultaneously by the tracing point. The tracing point, therefore, represents the point of contact, while the portion of the cord *ab* is the generating line. The point of contact moves along the generating line; the generating line does not change position, because it is always tangent to the base circles; and since the generating line is always normal to the involutes at the point of contact, the requirement for uniform motion is satisfied.

13–5 Fundamentals

Among other things, it is necessary that you actually be able to draw the teeth on a pair of meshing gears. You should understand, however, that you are not doing this for manufacturing or shop purposes. Rather, we make drawings of gear teeth to obtain an understanding of the problems involved in the meshing of the mating teeth.

Figure 13–8

Construction of an involute curve.



First, it is necessary to learn how to construct an involute curve. As shown in Fig. 13–8, divide the base circle into a number of equal parts, and construct radial lines OA_0, OA_1, OA_2, etc. Beginning at A_1, construct perpendiculars A_1B_1, A_2B_2, A_3B_3, etc. Then along A_1B_1 lay off the distance A_1A_0, along A_2B_2 lay off twice the distance A_1A_0, etc., producing points through which the involute curve can be constructed.

To investigate the fundamentals of tooth action, let us proceed step by step through the process of constructing the teeth on a pair of gears.

When two gears are in mesh, their pitch circles roll on one another without slipping. Designate the pitch radii as r_1 and r_2 and the angular velocities as ω_1 and ω_2, respectively. Then the pitch-line velocity is

$$V = |r_1\omega_1| = |r_2\omega_2|$$

Thus the relation between the radii on the angular velocities is

$$\left|\frac{\omega_1}{\omega_2}\right| = \frac{r_2}{r_1} \tag{13–5}$$

Suppose now we wish to design a speed reducer such that the input speed is 1800 rev/min and the output speed is 1200 rev/min. This is a ratio of 3:2; the gear pitch diameters would be in the same ratio, for example, a 4-in pinion driving a 6-in gear. The various dimensions found in gearing are always based on the pitch circles.

Suppose we specify that an 18-tooth pinion is to mesh with a 30-tooth gear and that the diametral pitch of the gearset is to be 2 teeth per inch. Then, from Eq. (13–1), the pitch diameters of the pinion and gear are, respectively,

$$d_1 = \frac{N_1}{P} = \frac{18}{2} = 9 \text{ in} \qquad d_2 = \frac{N_2}{P} = \frac{30}{2} = 15 \text{ in}$$

The first step in drawing teeth on a pair of mating gears is shown in Fig. 13–9. The center distance is the sum of the pitch radii, in this case 12 in. So locate the pinion and gear centers O_1 and O_2, 12 in apart. Then construct the pitch circles of radii r_1 and r_2. These are tangent at P, the *pitch point*. Next draw line ab, the common tangent, through the pitch point. We now designate gear 1 as the driver, and since it is rotating counterclockwise, we draw a line cd through point P at an angle ϕ to the common tangent ab. The line cd has three names, all of which are in general use. It is called the *pressure line,* the *generating line,* and the *line of action*. It represents the direction in which the resultant force acts between the gears. The angle ϕ is called the *pressure angle,* and it usually has values of 20 or 25°, though $14\frac{1}{2}^\circ$ was once used.

Figure 13–9

Circles of a gear layout.

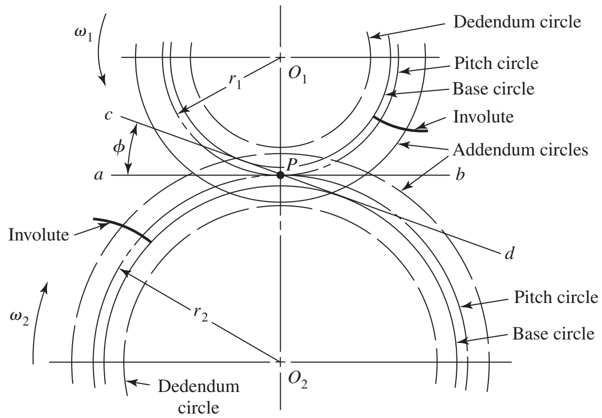
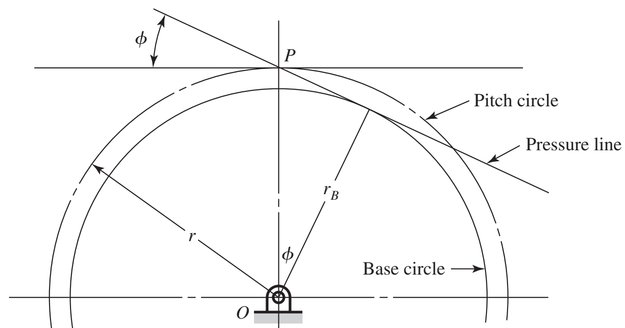


Figure 13–10

Base circle radius can be related to the pressure angle ϕ and the pitch circle radius by $r_b = r\cos\phi$.



Next, on each gear draw a circle tangent to the pressure line. These circles are the *base circles*. Since they are tangent to the pressure line, the pressure angle determines their size. As shown in Fig. 13–10, the radius of the base circle is

$$r_b = r\cos\phi \tag{13–6}$$

where r is the pitch radius.

Now generate an involute on each base circle as previously described and as shown in Fig. 13–9. This involute is to be used for one side of a gear tooth. It is not necessary to draw another curve in the reverse direction for the other side of the tooth, because we are going to use a template which can be turned over to obtain the other side.

The addendum and dedendum distances for standard interchangeable teeth are, as we shall learn later, $1/P$ and $1.25/P$, respectively. Therefore, for the pair of gears we are constructing,

$$a = \frac{1}{P} = \frac{1}{2} = 0.500 \text{ in} \qquad b = \frac{1.25}{P} = \frac{1.25}{2} = 0.625 \text{ in}$$

Using these distances, draw the addendum and dedendum circles on the pinion and on the gear as shown in Fig. 13–9.

Next, using heavy drawing paper, or preferably, a sheet of 0.015- to 0.020-in clear plastic, cut a template for each involute, being careful to locate the gear centers properly with respect to each involute. Figure 13–11 is a reproduction of the template used to create some of the illustrations for this book. Note that only one side of the tooth profile is formed on the template. To get the other side, turn the template over. For some problems you might wish to construct a template for the entire tooth.

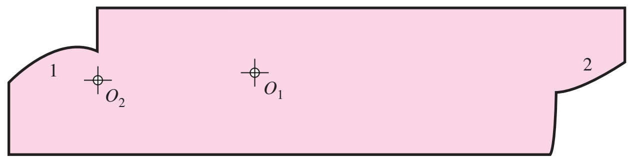


Figure 13–11

A template for drawing gear teeth.

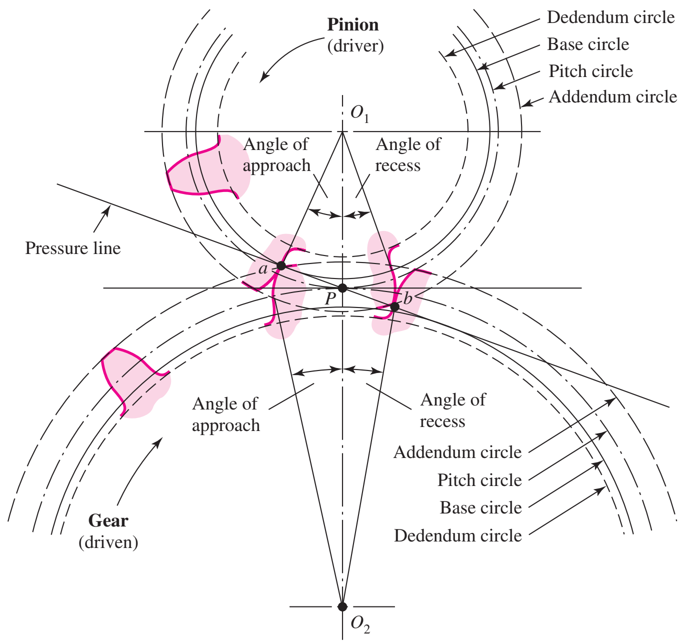


Figure 13–12

Tooth action.

To draw a tooth, we must know the tooth thickness. From Eq. (13–4), the circular pitch is

$$p = \frac{\pi}{P} = \frac{\pi}{2} = 1.57 \text{ in}$$

Therefore, the tooth thickness is

$$t = \frac{p}{2} = \frac{1.57}{2} = 0.785 \text{ in}$$

measured on the pitch circle. Using this distance for the tooth thickness as well as the tooth space, draw as many teeth as desired, using the template, after the points have been marked on the pitch circle. In Fig. 13–12 only one tooth has been drawn on each gear. You may run into trouble in drawing these teeth if one of the base circles happens to be larger than the dedendum circle. The reason for this is that the involute begins at the base circle and is undefined below this circle. So, in drawing gear teeth, we usually draw a radial line for the profile below the base circle. The actual shape, however, will depend upon the kind of machine tool used to form the teeth in manufacture, that is, how the profile is generated.

The portion of the tooth between the clearance circle and the dedendum circle includes the fillet. In this instance the clearance is

$$c = b - a = 0.625 - 0.500 = 0.125 \text{ in}$$

The construction is finished when these fillets have been drawn.

Referring again to Fig. 13–12, the pinion with center at O_1 is the driver and turns counterclockwise. The pressure, or generating, line is the same as the cord used in Fig. 13–7*a* to generate the involute, and contact occurs along this line. The initial contact will take place when the flank of the driver comes into contact with the tip of the driven tooth. This occurs at point *a* in Fig. 13–12, where the addendum circle of the driven gear crosses the pressure line. If we now construct tooth profiles through point *a* and draw radial lines from the intersections of these profiles with the pitch circles to the gear centers, we obtain the *angle of approach* for each gear.

As the teeth go into mesh, the point of contact will slide up the side of the driving tooth so that the tip of the driver will be in contact just before contact ends. The final point of contact will therefore be where the addendum circle of the driver crosses the pressure line. This is point *b* in Fig. 13–12. By drawing another set of tooth profiles through *b*, we obtain the *angle of recess* for each gear in a manner similar to that of finding the angles of approach. The sum of the angle of approach and the angle of recess for either gear is called the *angle of action.* The line *ab* is called *the line of action.*

We may imagine a *rack* as a spur gear having an infinitely large pitch diameter. Therefore, the rack has an infinite number of teeth and a base circle which is an infinite distance from the pitch point. The sides of involute teeth on a rack are straight lines making an angle to the line of centers equal to the pressure angle. Figure 13–13 shows an involute rack in mesh with a pinion. Corresponding sides on involute teeth are parallel curves; the *base pitch* is the constant and fundamental distance between them along a common normal as shown in Fig. 13–13. The base pitch is related to the circular pitch by the equation

$$p_b = p_c \cos\phi \tag{13–7}$$

where p_b is the base pitch.

Figure 13–14 shows a pinion in mesh with an *internal,* or *ring, gear.* Note that both of the gears now have their centers of rotation on the same side of the pitch point. Thus the positions of the addendum and dedendum circles with respect to the pitch circle are reversed; the addendum circle of the internal gear lies *inside* the pitch circle. Note, too, from Fig. 13–14, that the base circle of the internal gear lies inside the pitch circle near the addendum circle.

Another interesting observation concerns the fact that the operating diameters of the pitch circles of a pair of meshing gears need not be the same as the respective design pitch diameters of the gears, though this is the way they have been constructed in Fig. 13–12. If we increase the center distance, we create two new operating pitch circles having larger diameters because they must be tangent to each other at the pitch point.

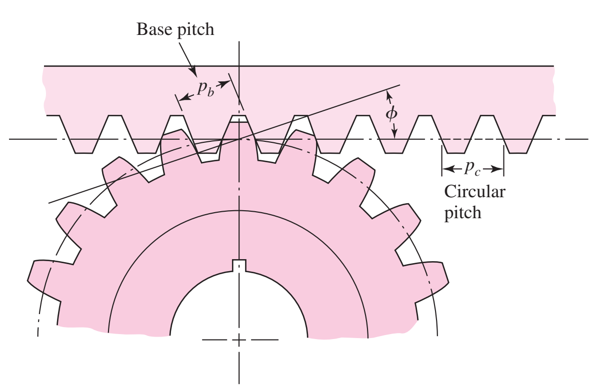


Figure 13–13

Involute-toothed pinion and rack.

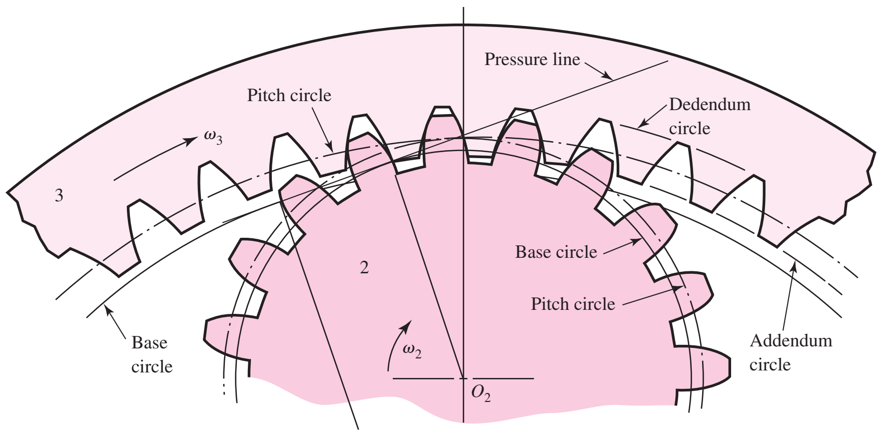


Figure 13–14

Internal gear and pinion.

Thus the pitch circles of gears really do not come into existence until a pair of gears are brought into mesh.

Changing the center distance has no effect on the base circles, because these were used to generate the tooth profiles. Thus the base circle is basic to a gear. Increasing the center distance increases the pressure angle and decreases the length of the line of action, but the teeth are still conjugate, the requirement for uniform motion transmission is still satisfied, and the angular-velocity ratio has not changed.

EXAMPLE 13–1

A gearset consists of a 16-tooth pinion driving a 40-tooth gear. The diametral pitch is 2, and the addendum and dedendum are $1/P$ and $1.25/P$, respectively. The gears are cut using a pressure angle of 20°.

(*a*) Compute the circular pitch, the center distance, and the radii of the base circles.

(*b*) In mounting these gears, the center distance was incorrectly made $\frac{1}{4}$ in larger. Compute the new values of the pressure angle and the pitch-circle diameters.

Solution

Answer (*a*)

$$p = \frac{\pi}{P} = \frac{\pi}{2} = 1.57 \text{ in}$$

The pitch diameters of the pinion and gear are, respectively,

$$d_P = \frac{16}{2} = 8 \text{ in} \qquad d_G = \frac{40}{2} = 20 \text{ in}$$

Therefore the center distance is

Answer

$$\frac{d_P + d_G}{2} = \frac{8 + 20}{2} = 14 \text{ in}$$

Since the teeth were cut on the 20° pressure angle, the base-circle radii are found to be, using $r_b = r \cos \phi$,

Answer

$$r_b \text{ (pinion)} = \frac{8}{2} \cos 20° = 3.76 \text{ in}$$

Answer

$$r_b \text{ (gear)} = \frac{20}{2} \cos 20° = 9.40 \text{ in}$$

(*b*) Designating d'_P and d'_G as the new pitch-circle diameters, the $\frac{1}{4}$-in increase in the center distance requires that

$$\frac{d'_P + d'_G}{2} = 14.250 \tag{1}$$

Also, the velocity ratio does not change, and hence

$$\frac{d'_P}{d'_G} = \frac{16}{40} \tag{2}$$

Solving Eqs. (1) and (2) simultaneously yields

Answer $$d'_P = 8.143 \text{ in} \qquad d'_G = 20.357 \text{ in}$$

Since $r_b = r\cos\phi$, the new pressure angle is

Answer $$\phi' = \cos^{-1}\frac{r_b\ (\text{pinion})}{d'_P/2} = \cos^{-1}\frac{3.76}{8.143/2} = 22.56^\circ$$

13–6 Contact Ratio

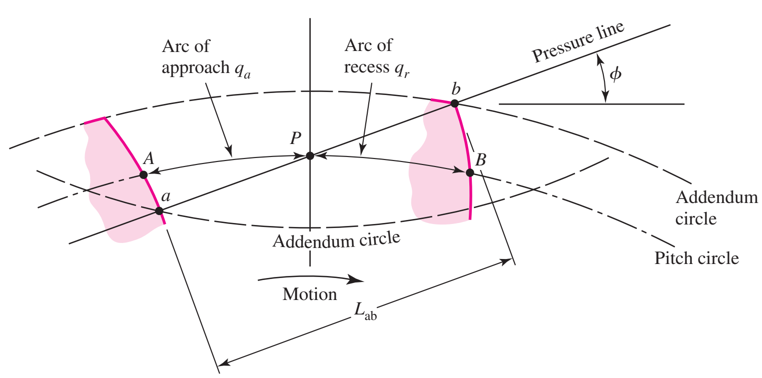
The zone of action of meshing gear teeth is shown in Fig. 13–15. We recall that tooth contact begins and ends at the intersections of the two addendum circles with the pressure line. In Fig. 13–15 initial contact occurs at a and final contact at b. Tooth profiles drawn through these points intersect the pitch circle at A and B, respectively. As shown, the distance AP is called the *arc of approach* q_a, and the distance PB, the *arc of recess* q_r. The sum of these is the *arc of action* q_t.

Now, consider a situation in which the arc of action is exactly equal to the circular pitch, that is, $q_t = p$. This means that one tooth and its space will occupy the entire arc AB. In other words, when a tooth is just beginning contact at a, the previous tooth is simultaneously ending its contact at b. Therefore, during the tooth action from a to b, there will be exactly one pair of teeth in contact.

Next, consider a situation in which the arc of action is greater than the circular pitch, but not very much greater, say, $q_t \doteq 1.2p$. This means that when one pair of teeth is just entering contact at a, another pair, already in contact, will not yet have reached b.

Figure 13–15

Definition of contact ratio.



Thus, for a short period of time, there will be two teeth in contact, one in the vicinity of A and another near B. As the meshing proceeds, the pair near B must cease contact, leaving only a single pair of contacting teeth, until the procedure repeats itself.

Because of the nature of this tooth action, either one or two pairs of teeth in contact, it is convenient to define the term *contact ratio* m_c as

$$m_c = \frac{q_t}{p} \tag{13–8}$$

a number that indicates the average number of pairs of teeth in contact. Note that this ratio is also equal to the length of the path of contact divided by the base pitch. Gears should not generally be designed having contact ratios less than about 1.20, because inaccuracies in mounting might reduce the contact ratio even more, increasing the possibility of impact between the teeth as well as an increase in the noise level.

An easier way to obtain the contact ratio is to measure the line of action ab instead of the arc distance AB. Since ab in Fig. 13–15 is tangent to the base circle when extended, the base pitch p_b must be used to calculate m_c instead of the circular pitch as in Eq. (13–8). If the length of the line of action is L_{ab}, the contact ratio is

$$m_c = \frac{L_{ab}}{p\cos\phi} \tag{13–9}$$

in which Eq. (13–7) was used for the base pitch.

13–7 Interference

The contact of portions of tooth profiles that are not conjugate is called *interference*. Consider Fig. 13–16. Illustrated are two 16-tooth gears that have been cut to the now obsolete $14\frac{1}{2}^\circ$ pressure angle. The driver, gear 2, turns clockwise. The initial and final points of contact are designated A and B, respectively, and are located on the pressure line. Now notice that the points of tangency of the pressure line with the base circles C and D are located *inside* of points A and B. Interference is present.

The interference is explained as follows. Contact begins when the tip of the driven tooth contacts the flank of the driving tooth. In this case the flank of the driving tooth first makes contact with the driven tooth at point A, and this occurs *before* the involute portion of the driving tooth comes within range. In other words, contact is occurring below the base circle of gear 2 on the *noninvolute* portion of the flank. The actual effect is that the involute tip or face of the driven gear tends to dig out the noninvolute flank of the driver.

In this example the same effect occurs again as the teeth leave contact. Contact should end at point D or before. Since it does not end until point B, the effect is for the tip of the driving tooth to dig out, or interfere with, the flank of the driven tooth.

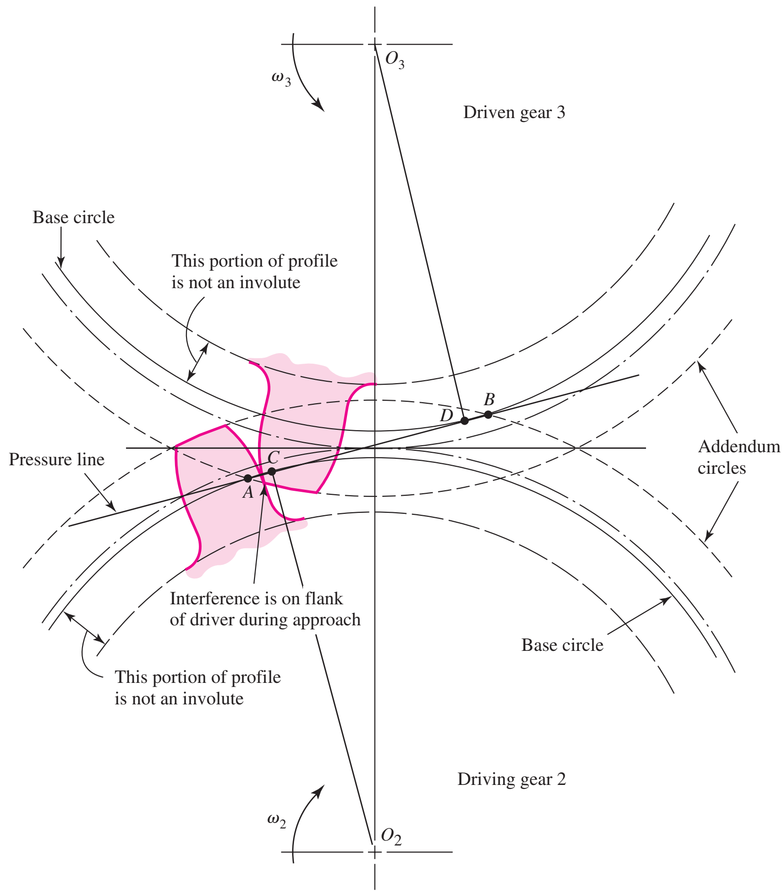
When gear teeth are produced by a generation process, interference is automatically eliminated because the cutting tool removes the interfering portion of the flank. This effect is called *undercutting;* if undercutting is at all pronounced, the undercut tooth is considerably weakened. Thus the effect of eliminating interference by a generation process is merely to substitute another problem for the original one.

The smallest number of teeth on a spur pinion and gear,[1] one-to-one gear ratio, which can exist without interference is N_P. This number of teeth for spur gears is

[1]Robert Lipp, "Avoiding Tooth Interference in Gears," *Machine Design*, Vol. 54, No. 1, 1982, pp. 122–124.

Figure 13–16

Interference in the action of gear teeth.



given by

$$N_P = \frac{2k}{3\sin^2\phi}\left(1 + \sqrt{1 + 3\sin^2\phi}\right) \tag{13–10}$$

where $k = 1$ for full-depth teeth, 0.8 for stub teeth and ϕ = pressure angle.

For a 20° pressure angle, with $k = 1$,

$$N_P = \frac{2(1)}{3\sin^2 20^\circ}\left(1 + \sqrt{1 + 3\sin^2 20^\circ}\right) = 12.3 = 13 \text{ teeth}$$

Thus 13 teeth on pinion and gear are interference-free. Realize that 12.3 teeth is possible in meshing arcs, but for fully rotating gears, 13 teeth represents the least number. For a $14\frac{1}{2}^\circ$ pressure angle, $N_P = 23$ teeth, so one can appreciate why few $14\frac{1}{2}^\circ$-tooth systems are used, as the higher pressure angles can produce a smaller pinion with accompanying smaller center-to-center distances.

If the mating gear has more teeth than the pinion, that is, $m_G = N_G/N_P = m$ is more than one, then the smallest number of teeth on the pinion without interference is given by

$$N_P = \frac{2k}{(1 + 2m)\sin^2\phi}\left(m + \sqrt{m^2 + (1 + 2m)\sin^2\phi}\right) \tag{13–11}$$

For example, if $m = 4$, $\phi = 20°$,

$$N_P = \frac{2(1)}{[1 + 2(4)]\sin^2 20°}\left[4 + \sqrt{4^2 + [1 + 2(4)]\sin^2 20°}\right] = 15.4 = 16 \text{ teeth}$$

Thus a 16-tooth pinion will mesh with a 64-tooth gear without interference.

The largest gear with a specified pinion that is interference-free is

$$N_G = \frac{N_P^2 \sin^2\phi - 4k^2}{4k - 2N_P \sin^2\phi} \tag{13–12}$$

For example, for a 13-tooth pinion with a pressure angle ϕ of 20°,

$$N_G = \frac{13^2 \sin^2 20° - 4(1)^2}{4(1) - 2(13)\sin^2 20°} = 16.45 = 16 \text{ teeth}$$

For a 13-tooth spur pinion, the maximum number of gear teeth possible without interference is 16.

The smallest spur pinion that will operate with a rack without interference is

$$N_P = \frac{2(k)}{\sin^2\phi} \tag{13–13}$$

For a 20° pressure angle full-depth tooth the smallest number of pinion teeth to mesh with a rack is

$$N_P = \frac{2(1)}{\sin^2 20°} = 17.1 = 18 \text{ teeth}$$

Since gear-shaping tools amount to contact with a rack, and the gear-hobbing process is similar, the minimum number of teeth to prevent interference to prevent undercutting by the hobbing process is equal to the value of N_P when N_G is infinite.

The importance of the problem of teeth that have been weakened by undercutting cannot be overemphasized. Of course, interference can be eliminated by using more teeth on the pinion. However, if the pinion is to transmit a given amount of power, more teeth can be used only by increasing the pitch diameter.

Interference can also be reduced by using a larger pressure angle. This results in a smaller base circle, so that more of the tooth profile becomes involute. The demand for smaller pinions with fewer teeth thus favors the use of a 25° pressure angle even though the frictional forces and bearing loads are increased and the contact ratio decreased.

13–8 The Forming of Gear Teeth

There are a large number of ways of forming the teeth of gears, such as *sand casting, shell molding, investment casting, permanent-mold casting, die casting,* and *centrifugal casting*. Teeth can also be formed by using the *powder-metallurgy process;* or, by using *extrusion,* a single bar of aluminum may be formed and then sliced into gears. Gears that carry large loads in comparison with their size are usually made of steel and are cut with either *form cutters* or *generating cutters.* In form cutting, the tooth space takes the exact form of the cutter. In generating, a tool having a shape different from the tooth profile is moved relative to the gear blank so as to obtain the proper tooth shape. One of the newest and most promising of the methods of forming teeth is called *cold forming,* or *cold rolling,* in which dies are rolled against steel blanks to form the teeth. The mechanical properties of the metal are greatly improved by the rolling process, and a high-quality generated profile is obtained at the same time.

Gear teeth may be machined by milling, shaping, or hobbing. They may be finished by shaving, burnishing, grinding, or lapping.

Gears made of thermoplastics such as nylon, polycarbonate, acetal are quite popular and are easily manufactured by *injection molding*. These gears are of low to moderate precision, low in cost for high production quantities, and capable of light loads, and can run without lubrication.

Milling

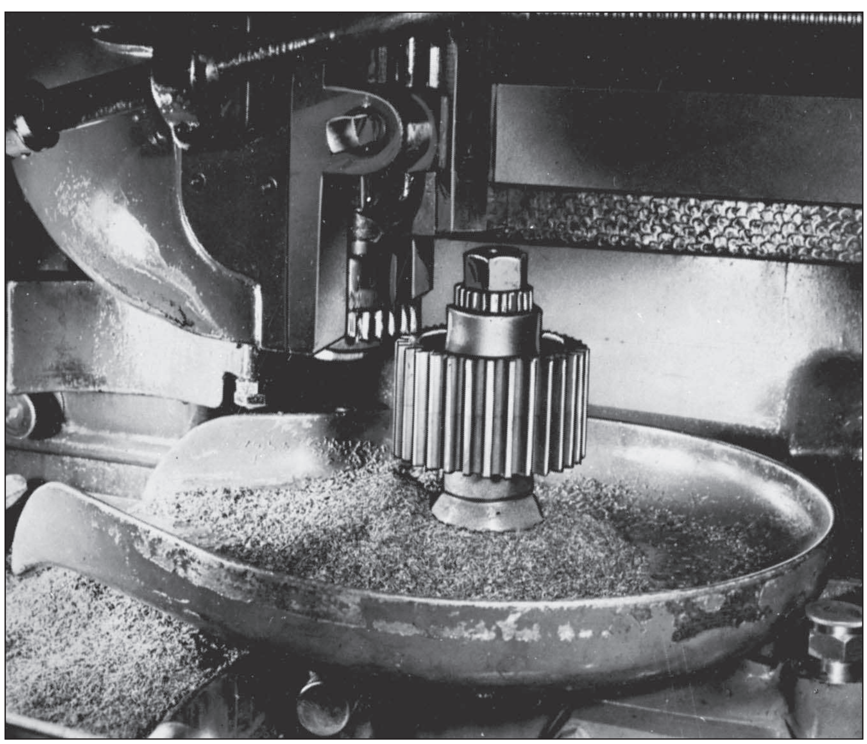
Gear teeth may be cut with a form milling cutter shaped to conform to the tooth space. With this method it is theoretically necessary to use a different cutter for each gear, because a gear having 25 teeth, for example, will have a different-shaped tooth space from one having, say, 24 teeth. Actually, the change in space is not too great, and it has been found that eight cutters may be used to cut with reasonable accuracy any gear in the range of 12 teeth to a rack. A separate set of cutters is, of course, required for each pitch.

Shaping

Teeth may be generated with either a pinion cutter or a rack cutter. The pinion cutter (Fig. 13–17) reciprocates along the vertical axis and is slowly fed into the gear blank to the required depth. When the pitch circles are tangent, both the cutter and the blank rotate slightly after each cutting stroke. Since each tooth of the cutter is a cutting tool, the teeth are all cut after the blank has completed one rotation. The sides of an involute rack tooth are straight. For this reason, a rack-generating tool provides an accurate method of cutting gear teeth. This is also a shaping operation and is illustrated by the drawing of Fig. 13–18. In operation, the cutter reciprocates and is first fed into the gear blank until the pitch circles are tangent. Then, after each cutting stroke, the gear blank

Figure 13–17

Generating a spur gear with a pinion cutter. *(Courtesy of Boston Gear Works, Inc.)*



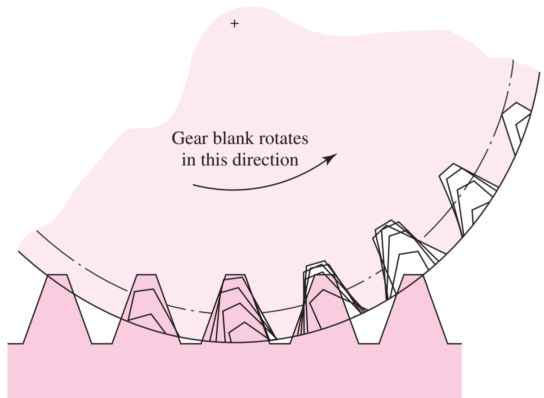


Figure 13–18

Shaping teeth with a rack. (This is a drawing-board figure that J. E. Shigley executed over 35 years ago in response to a question from a student at the University of Michigan.)

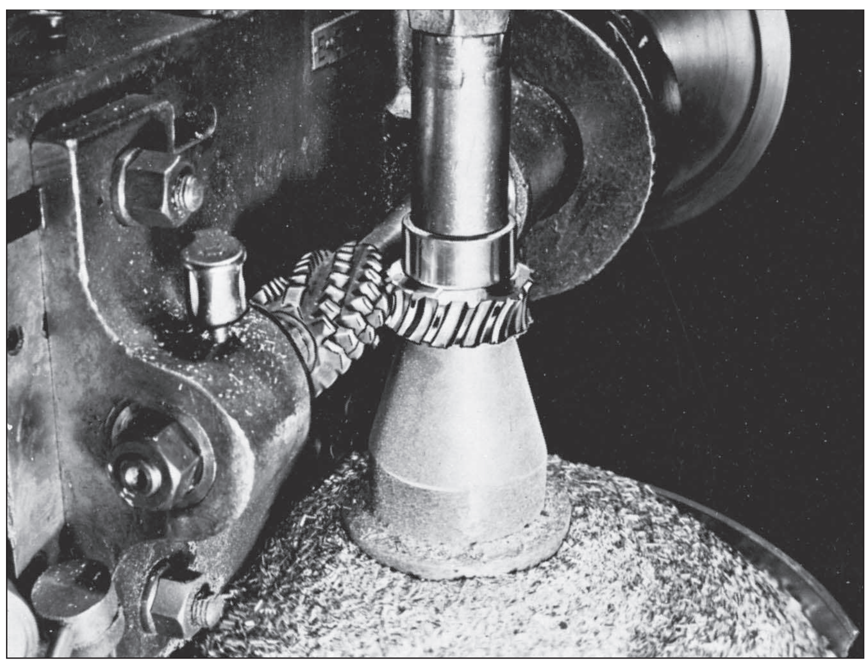


Figure 13–19

Hobbing a worm gear. *(Courtesy of Boston Gear Works, Inc.)*

and cutter roll slightly on their pitch circles. When the blank and cutter have rolled a distance equal to the circular pitch, the cutter is returned to the starting point, and the process is continued until all the teeth have been cut.

Hobbing

The hobbing process is illustrated in Fig. 13–19. The hob is simply a cutting tool that is shaped like a worm. The teeth have straight sides, as in a rack, but the hob axis must be turned through the lead angle in order to cut spur-gear teeth. For this reason, the teeth generated by a hob have a slightly different shape from those generated by a rack cutter. Both the hob and the blank must be rotated at the proper angular-velocity ratio. The hob is then fed slowly across the face of the blank until all the teeth have been cut.

Finishing

Gears that run at high speeds and transmit large forces may be subjected to additional dynamic forces if there are errors in tooth profiles. Errors may be diminished somewhat by finishing the tooth profiles. The teeth may be finished, after cutting, by either shaving or burnishing. Several shaving machines are available that cut off a minute amount of metal, bringing the accuracy of the tooth profile within the limits of 250 μin.

Burnishing, like shaving, is used with gears that have been cut but not heat-treated. In burnishing, hardened gears with slightly oversize teeth are run in mesh with the gear until the surfaces become smooth.

Grinding and lapping are used for hardened gear teeth after heat treatment. The grinding operation employs the generating principle and produces very accurate teeth. In lapping, the teeth of the gear and lap slide axially so that the whole surface of the teeth is abraded equally.

13–9 Straight Bevel Gears

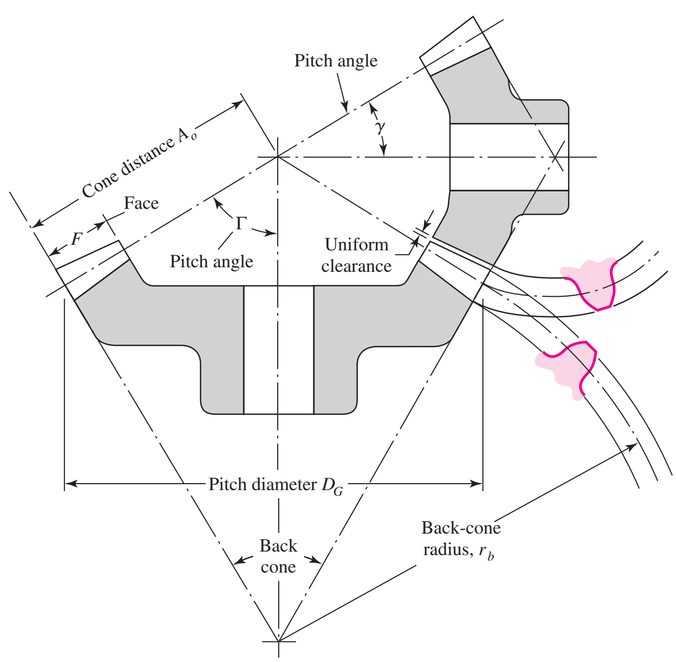
When gears are used to transmit motion between intersecting shafts, some form of bevel gear is required. A bevel gearset is shown in Fig. 13–20. Although bevel gears are usually made for a shaft angle of 90°, they may be produced for almost any angle. The teeth may be cast, milled, or generated. Only the generated teeth may be classed as accurate.

The terminology of bevel gears is illustrated in Fig. 13–20. The pitch of bevel gears is measured at the large end of the tooth, and both the circular pitch and the pitch diameter are calculated in the same manner as for spur gears. It should be noted that the clearance is uniform. The pitch angles are defined by the pitch cones meeting at the apex, as shown in the figure. They are related to the tooth numbers as follows:

$$\tan\gamma = \frac{N_P}{N_G} \qquad \tan\Gamma = \frac{N_G}{N_P} \tag{13–14}$$

Figure 13–20

Terminology of bevel gears.



where the subscripts P and G refer to the pinion and gear, respectively, and where γ and Γ are, respectively, the pitch angles of the pinion and gear.

Figure 13–20 shows that the shape of the teeth, when projected on the back cone, is the same as in a spur gear having a radius equal to the back-cone distance r_b. This is called Tredgold's approximation. The number of teeth in this imaginary gear is

$$N' = \frac{2\pi r_b}{p} \tag{13–15}$$

where N' is the *virtual number of teeth* and p is the circular pitch measured at the large end of the teeth. Standard straight-tooth bevel gears are cut by using a 20° pressure angle, unequal addenda and dedenda, and full-depth teeth. This increases the contact ratio, avoids undercut, and increases the strength of the pinion.

13–10 Parallel Helical Gears

Helical gears, used to transmit motion between parallel shafts, are shown in Fig. 13–2. The helix angle is the same on each gear, but one gear must have a right-hand helix and the other a left-hand helix. The shape of the tooth is an involute helicoid and is illustrated in Fig. 13–21. If a piece of paper cut in the shape of a parallelogram is wrapped around a cylinder, the angular edge of the paper becomes a helix. If we unwind this paper, each point on the angular edge generates an involute curve. This surface obtained when every point on the edge generates an involute is called an *involute helicoid.*

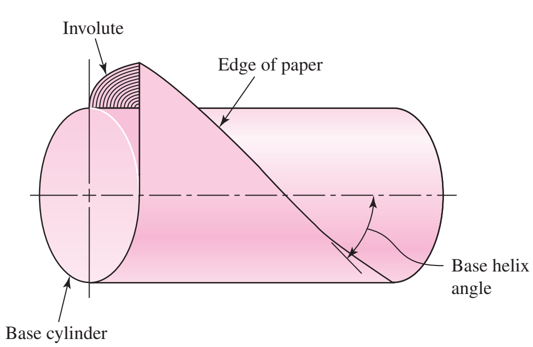
The initial contact of spur-gear teeth is a line extending all the way across the face of the tooth. The initial contact of helical-gear teeth is a point that extends into a line as the teeth come into more engagement. In spur gears the line of contact is parallel to the axis of rotation; in helical gears the line is diagonal across the face of the tooth. It is this gradual engagement of the teeth and the smooth transfer of load from one tooth to another that gives helical gears the ability to transmit heavy loads at high speeds. Because of the nature of contact between helical gears, the contact ratio is of only minor importance, and it is the contact area, which is proportional to the face width of the gear, that becomes significant.

Helical gears subject the shaft bearings to both radial and thrust loads. When the thrust loads become high or are objectionable for other reasons, it may be desirable to use double helical gears. A double helical gear (herringbone) is equivalent to two helical gears of opposite hand, mounted side by side on the same shaft. They develop opposite thrust reactions and thus cancel out the thrust load.

When two or more single helical gears are mounted on the same shaft, the hand of the gears should be selected so as to produce the minimum thrust load.

Figure 13–21

An involute helicoid.



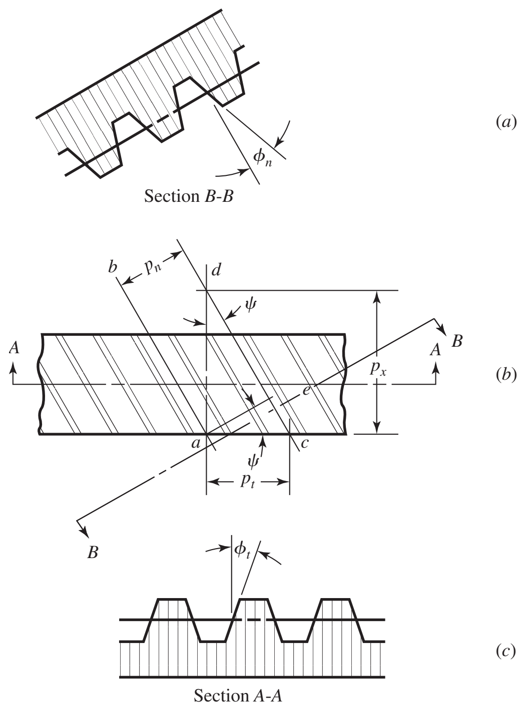


Figure 13–22

Nomenclature of helical gears.

Figure 13–22 represents a portion of the top view of a helical rack. Lines *ab* and *cd* are the centerlines of two adjacent helical teeth taken on the same pitch plane. The angle ψ is the *helix angle*. The distance *ac* is the *transverse circular pitch* p_t in the plane of rotation (usually called the *circular pitch*). The distance *ae* is the *normal circular pitch* p_n and is related to the transverse circular pitch as follows:

$$p_n = p_t \cos\psi \tag{13–16}$$

The distance *ad* is called the *axial pitch* p_x and is related by the expression

$$p_x = \frac{p_t}{\tan\psi} \tag{13–17}$$

Since $p_n P_n = \pi$, the *normal diametral pitch* is

$$P_n = \frac{P_t}{\cos\psi} \tag{13–18}$$

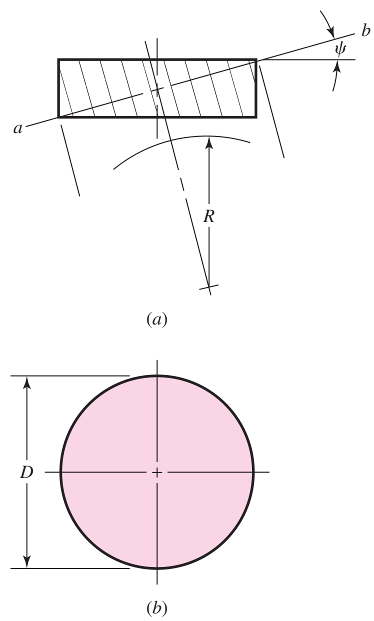
The pressure angle ϕ_n in the normal direction is different from the pressure angle ϕ_t in the direction of rotation, because of the angularity of the teeth. These angles are related by the equation

$$\cos\psi = \frac{\tan\phi_n}{\tan\phi_t} \tag{13–19}$$

Figure 13–23 illustrates a cylinder cut by an oblique plane *ab* at an angle ψ to a right section. The oblique plane cuts out an arc having a radius of curvature of *R*. For the condition that $\psi = 0$, the radius of curvature is $R = D/2$. If we imagine the angle ψ to be slowly increased from zero to 90°, we see that *R* begins at a value of $D/2$ and

Figure 13–23

A cylinder cut by an oblique plane.



increases until, when $\psi = 90^\circ$, $R = \infty$. The radius R is the apparent pitch radius of a helical-gear tooth when viewed in the direction of the tooth elements. A gear of the same pitch and with the radius R will have a greater number of teeth, because of the increased radius. In helical-gear terminology this is called the *virtual number of teeth.* It can be shown by analytical geometry that the virtual number of teeth is related to the actual number by the equation

$$N' = \frac{N}{\cos^3 \psi} \tag{13–20}$$

where N' is the virtual number of teeth and N is the actual number of teeth. It is necessary to know the virtual number of teeth in design for strength and also, sometimes, in cutting helical teeth. This apparently larger radius of curvature means that few teeth may be used on helical gears, because there will be less undercutting.

EXAMPLE 13–2 A stock helical gear has a normal pressure angle of 20°, a helix angle of 25°, and a transverse diametral pitch of 6 teeth/in, and has 18 teeth. Find:
(*a*) The pitch diameter
(*b*) The transverse, the normal, and the axial pitches
(*c*) The normal diametral pitch
(*d*) The transverse pressure angle

Solution

Answer (*a*)

$$d = \frac{N}{P_t} = \frac{18}{6} = 3 \text{ in}$$

Answer (*b*)

$$p_t = \frac{\pi}{P_t} = \frac{\pi}{6} = 0.5236 \text{ in}$$

Answer
$$p_n = p_t \cos\psi = 0.5236 \cos 25^\circ = 0.4745 \text{ in}$$

Answer
$$p_x = \frac{p_t}{\tan\psi} = \frac{0.5236}{\tan 45^\circ} = 1.123 \text{ in}$$

Answer (*c*)
$$P_n = \frac{P_t}{\cos\psi} = \frac{6}{\cos 25^\circ} = 6.620 \text{ teeth/in}$$

Answer (*d*)
$$\phi_t = \tan^{-1}\left(\frac{\tan\phi_n}{\cos\psi}\right) = \tan^{-1}\left(\frac{\tan 20^\circ}{\cos 25^\circ}\right) = 21.88^\circ$$

Just like teeth on spur gears, helical-gear teeth can interfere. Equation (13–19) can be solved for the pressure angle ϕ_t in the tangential (rotation) direction to give

$$\phi_t = \tan^{-1}\left(\frac{\tan\phi_n}{\cos\psi}\right)$$

The smallest tooth number N_P of a helical-spur pinion that will run without interference[2] with a gear with the same number of teeth is

$$N_P = \frac{2k\cos\psi}{3\sin^2\phi_t}\left(1 + \sqrt{1 + 3\sin^2\phi_t}\right) \tag{13–21}$$

For example, if the normal pressure angle ϕ_n is 20°, the helix angle ψ is 30°, then ϕ_t is

$$\phi_t = \tan^{-1}\left(\frac{\tan 20^\circ}{\cos 30^\circ}\right) = 22.80^\circ$$

$$N_P = \frac{2(1)\cos 30^\circ}{3\sin^2 22.80^\circ}\left(1 + \sqrt{1 + 3\sin^2 22.80^\circ}\right) = 8.48 = 9 \text{ teeth}$$

For a given gear ratio $m_G = N_G/N_P = m$, the smallest pinion tooth count is

$$N_P = \frac{2k\cos\psi}{(1+2m)\sin^2\phi_t}\left[m + \sqrt{m^2 + (1+2m)\sin^2\phi_t}\right] \tag{13–22}$$

The largest gear with a specified pinion is given by

$$N_G = \frac{N_P^2\sin^2\phi_t - 4k^2\cos^2\psi}{4k\cos\psi - 2N_P\sin^2\phi_t} \tag{13–23}$$

For example, for a nine-tooth pinion with a pressure angle ϕ_n of 20°, a helix angle ψ of 30°, and recalling that the tangential pressure angle ϕ_t is 22.80°,

$$N_G = \frac{9^2\sin^2 22.80^\circ - 4(1)^2\cos^2 30^\circ}{4(1)\cos 30^\circ - 2(9)\sin^2 22.80^\circ} = 12.02 = 12$$

The smallest pinion that can be run with a rack is

$$N_P = \frac{2k\cos\psi}{\sin^2\phi_t} \tag{13–24}$$

[2]Op. cit., Robert Lipp, *Machine Design*, pp. 122–124.

For a normal pressure angle ϕ_n of 20° and a helix angle ψ of 30°, and $\phi_t = 22.80°$,

$$N_P = \frac{2(1)\cos 30°}{\sin^2 22.80°} = 11.5 = 12 \text{ teeth}$$

For helical-gear teeth the number of teeth in mesh across the width of the gear will be greater than unity and a term called *face-contact ratio* is used to describe it. This increase of contact ratio, and the gradual sliding engagement of each tooth, results in quieter gears.

13–11 Worm Gears

The nomenclature of a worm gear is shown in Fig. 13–24. The worm and worm gear of a set have the same hand of helix as for crossed helical gears, but the helix angles are usually quite different. The helix angle on the worm is generally quite large, and that on the gear very small. Because of this, it is usual to specify the lead angle λ on the worm and helix angle ψ_G on the gear; the two angles are equal for a 90° shaft angle. The worm lead angle is the complement of the worm helix angle, as shown in Fig. 13–24.

In specifying the pitch of worm gearsets, it is customary to state the *axial pitch* p_x of the worm and the *transverse circular pitch* p_t, often simply called the circular pitch, of the mating gear. These are equal if the shaft angle is 90°. The pitch diameter of the gear is the diameter measured on a plane containing the worm axis, as shown in Fig. 13–24; it is the same as for spur gears and is

$$d_G = \frac{N_G p_t}{\pi} \tag{13–25}$$

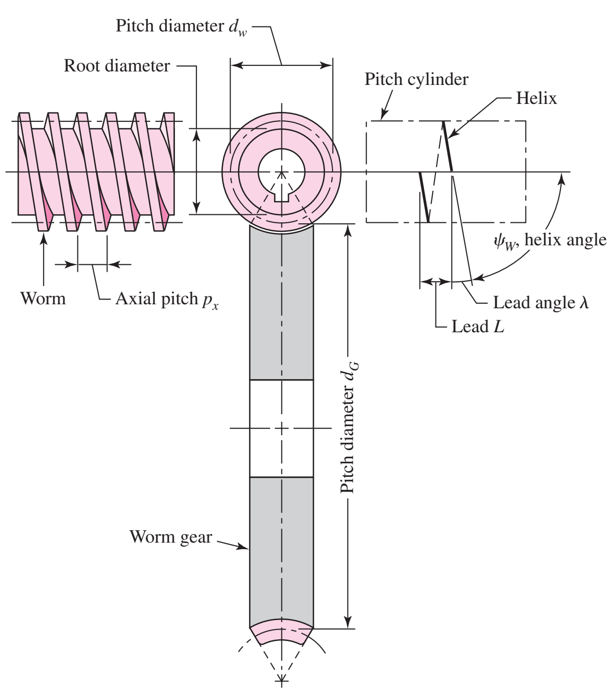


Figure 13–24

Nomenclature of a single-enveloping worm gearset.

Since it is not related to the number of teeth, the worm may have any pitch diameter; this diameter should, however, be the same as the pitch diameter of the hob used to cut the worm-gear teeth. Generally, the pitch diameter of the worm should be selected so as to fall into the range

$$\frac{C^{0.875}}{3.0} \le d_W \le \frac{C^{0.875}}{1.7} \tag{13–26}$$

where C is the center distance. These proportions appear to result in optimum horsepower capacity of the gearset.

The *lead* L and the *lead angle* λ of the worm have the following relations:

$$L = p_x N_W \tag{13–27}$$

$$\tan\lambda = \frac{L}{\pi d_W} \tag{13–28}$$

13–12 Tooth Systems[3]

A *tooth system* is a standard that specifies the relationships involving addendum, dedendum, working depth, tooth thickness, and pressure angle. The standards were originally planned to attain interchangeability of gears of all tooth numbers, but of the same pressure angle and pitch.

Table 13–1 contains the standards most used for spur gears. A $14\frac{1}{2}^\circ$ pressure angle was once used for these but is now obsolete; the resulting gears had to be comparatively larger to avoid interference problems.

Table 13–2 is particularly useful in selecting the pitch or module of a gear. Cutters are generally available for the sizes shown in this table.

Table 13–3 lists the standard tooth proportions for straight bevel gears. These sizes apply to the large end of the teeth. The nomenclature is defined in Fig. 13–20.

Standard tooth proportions for helical gears are listed in Table 13–4. Tooth proportions are based on the normal pressure angle; these angles are standardized the same

Table 13–1

Standard and Commonly Used Tooth Systems for Spur Gears

Tooth System	Pressure Angle ϕ, deg	Addendum a	Dedendum b
Full depth	20	$1/P_d$ or $1m$	$1.25/P_d$ or $1.25m$
			$1.35/P_d$ or $1.35m$
	$22\frac{1}{2}$	$1/P_d$ or $1m$	$1.25/P_d$ or $1.25m$
			$1.35/P_d$ or $1.35m$
	25	$1/P_d$ or $1m$	$1.25/P_d$ or $1.25m$
			$1.35/P_d$ or $1.35m$
Stub	20	$0.8/P_d$ or $0.8m$	$1/P_d$ or $1m$

[3]Standardized by the American Gear Manufacturers Association (AGMA). Write AGMA for a complete list of standards, because changes are made from time to time. The address is: 1500 King Street, Suite 201, Alexandria, VA 22314; or, www.agma.org.

Table 13–2

Tooth Sizes in General Uses

Diametral Pitch	
Coarse	$2, 2\frac{1}{4}, 2\frac{1}{2}, 3, 4, 6, 8, 10, 12, 16$
Fine	20, 24, 32, 40, 48, 64, 80, 96, 120, 150, 200

Modules	
Preferred	1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, 20, 25, 32, 40, 50
Next Choice	1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5, 5.5, 7, 9, 11, 14, 18, 22, 28, 36, 45

Table 13–3

Tooth Proportions for 20° Straight Bevel-Gear Teeth

Item	Formula
Working depth	$h_k = 2.0/P$
Clearance	$c = (0.188/P) + 0.002$ in
Addendum of gear	$a_G = \dfrac{0.54}{P} + \dfrac{0.460}{P(m_{90})^2}$
Gear ratio	$m_G = N_G/N_P$
Equivalent 90° ratio	$m_{90} = m_G$ when $\Gamma = 90°$
	$m_{90} = \sqrt{m_G \dfrac{\cos\gamma}{\cos\Gamma}}$ when $\Gamma \neq 90°$
Face width	$F = 0.3A_0$ or $F = \dfrac{10}{P}$, whichever is smaller
Minimum number of teeth	Pinion: 16, 15, 14, 13; Gear: 16, 17, 20, 30

Table 13–4

Standard Tooth Proportions for Helical Gears

Quantity*	Formula	Quantity*	Formula
Addendum	$\dfrac{1.00}{P_n}$	External gears:	
Dedendum	$\dfrac{1.25}{P_n}$	Standard center distance	$\dfrac{D+d}{2}$
Pinion pitch diameter	$\dfrac{N_P}{P_n \cos\psi}$	Gear outside diameter	$D + 2a$
Gear pitch diameter	$\dfrac{N_G}{P_n \cos\psi}$	Pinion outside diameter	$d + 2a$
Normal arc tooth thickness†	$\dfrac{\pi}{P_n} - \dfrac{B_n}{2}$	Gear root diameter	$D - 2b$
Pinion base diameter	$d \cos\phi_t$	Pinion root diameter	$d - 2b$
		Internal gears:	
Gear base diameter	$D \cos\phi_t$	Center distance	$\dfrac{D-d}{2}$
Base helix angle	$\tan^{-1}(\tan\psi \cos\phi_t)$	Inside diameter	$D - 2a$
		Root diameter	$D + 2b$

*All dimensions are in inches, and angles are in degrees.

†B_n is the normal backlash.

Table 13–5

Recommended Pressure Angles and Tooth Depths for Worm Gearing

Lead Angle λ, deg	Pressure Angle ϕ_n, deg	Addendum a	Dedendum b_G
0–15	$14\frac{1}{2}$	$0.3683p_x$	$0.3683p_x$
15–30	20	$0.3683p_x$	$0.3683p_x$
30–35	25	$0.2865p_x$	$0.3314p_x$
35–40	25	$0.2546p_x$	$0.2947p_x$
40–45	30	$0.2228p_x$	$0.2578p_x$

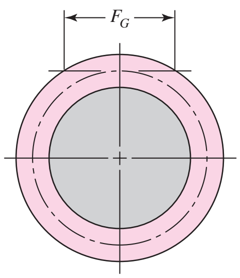


Figure 13–25

A graphical depiction of the face width of the worm of a worm gearset.

as for spur gears. Though there will be exceptions, the face width of helical gears should be at least 2 times the axial pitch to obtain good helical-gear action.

Tooth forms for worm gearing have not been highly standardized, perhaps because there has been less need for it. The pressure angles used depend upon the lead angles and must be large enough to avoid undercutting of the worm-gear tooth on the side at which contact ends. A satisfactory tooth depth, which remains in about the right proportion to the lead angle, may be obtained by making the depth a proportion of the axial circular pitch. Table 13–5 summarizes what may be regarded as good practice for pressure angle and tooth depth.

The *face width* F_G of the worm gear should be made equal to the length of a tangent to the worm pitch circle between its points of intersection with the addendum circle, as shown in Fig. 13–25.

13–13 Gear Trains

Consider a pinion 2 driving a gear 3. The speed of the driven gear is

$$n_3 = \left|\frac{N_2}{N_3}n_2\right| = \left|\frac{d_2}{d_3}n_2\right| \tag{13–29}$$

where n = revolutions or rev/min

N = number of teeth

d = pitch diameter

Equation (13–29) applies to any gearset no matter whether the gears are spur, helical, bevel, or worm. The absolute-value signs are used to permit complete freedom in choosing positive and negative directions. In the case of spur and parallel helical gears, the directions ordinarily correspond to the right-hand rule and are positive for counterclockwise rotation.

Rotational directions are somewhat more difficult to deduce for worm and crossed helical gearsets. Figure 13–26 will be of help in these situations.

The gear train shown in Fig. 13–27 is made up of five gears. The speed of gear 6 is

$$n_6 = -\frac{N_2}{N_3}\frac{N_3}{N_4}\frac{N_5}{N_6}n_2 \tag{a}$$

Hence we notice that gear 3 is an idler, that its tooth numbers cancel in Eq. (*a*), and hence that it affects only the direction of rotation of gear 6. We notice, furthermore, that

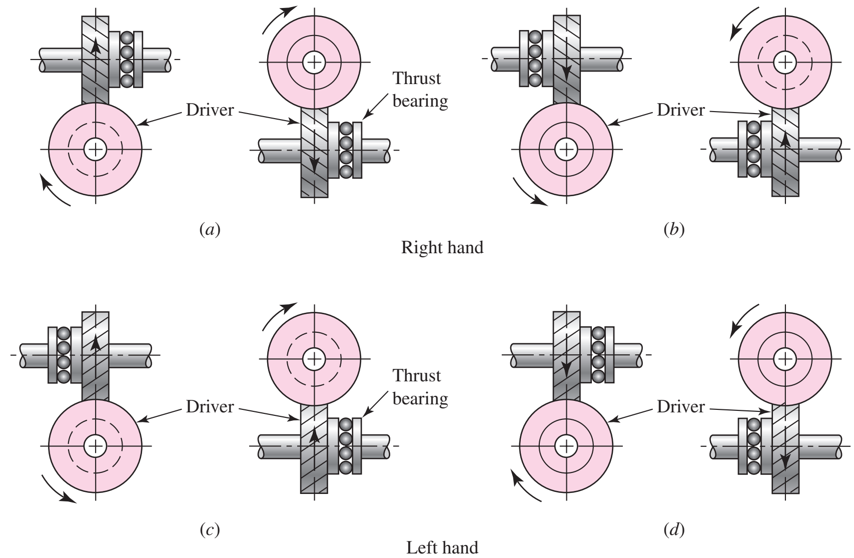


Figure 13–26

Thrust, rotation, and hand relations for crossed helical gears. Note that each pair of drawings refers to a single gearset. These relations also apply to worm gearsets. *(Reproduced by permission, Boston Gear Division, Colfax Corp.)*

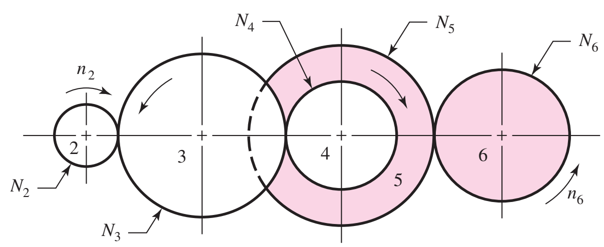


Figure 13–27

A gear train.

gears 2, 3, and 5 are drivers, while 3, 4, and 6 are driven members. We define the *train value e* as

$$e = \frac{\text{product of driving tooth numbers}}{\text{product of driven tooth numbers}} \tag{13–30}$$

Note that pitch diameters can be used in Eq. (13–30) as well. When Eq. (13–30) is used for spur gears, e is positive if the last gear rotates in the same sense as the first, and negative if the last rotates in the opposite sense.

Now we can write

$$n_L = e n_F \tag{13–31}$$

where n_L is the speed of the last gear in the train and n_F is the speed of the first.

As a rough guideline, a train value of up to 10 to 1 can be obtained with one pair of gears. Greater ratios can be obtained in less space and with fewer dynamic problems by compounding additional pairs of gears. A two-stage compound gear train, such as shown in Fig. 13–28, can obtain a train value of up to 100 to 1.

The design of gear trains to accomplish a specific train value is straightforward. Since numbers of teeth on gears must be integers, it is better to determine them first, and then obtain pitch diameters second. Determine the number of stages necessary to obtain the overall ratio, then divide the overall ratio into portions to be accomplished in each

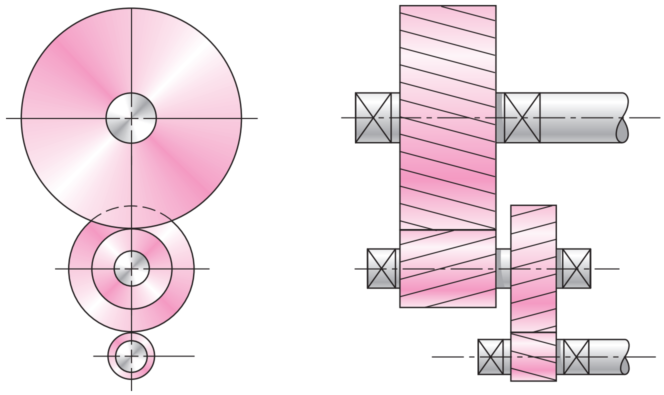


Figure 13–28

A two stage compound gear train.

stage. To minimize package size, keep the portions as evenly divided between the stages as possible. In cases where the overall train value need only be approximated, each stage can be identical. For example, in a two-stage compound gear train, assign the square root of the overall train value to each stage. If an exact train value is needed, attempt to factor the overall train value into integer components for each stage. Then assign the smallest gear(s) to the minimum number of teeth allowed for the specific ratio of each stage, in order to avoid interference (see Sec. 13–7). Finally, applying the ratio for each stage, determine the necessary number of teeth for the mating gears. Round to the nearest integer and check that the resulting overall ratio is within acceptable tolerance.

EXAMPLE 13–3

A gearbox is needed to provide a 30:1 ($\pm$ 1 percent) increase in speed, while minimizing the overall gearbox size. Specify appropriate teeth numbers.

Solution

Since the ratio is greater than 10:1, but less than 100:1, a two-stage compound gear train, such as in Figure 13–28, is needed. The portion to be accomplished in each stage is $\sqrt{30} = 5.4772$. For this ratio, assuming a typical 20° pressure angle, the minimum number of teeth to avoid interference is 16, according to Eq. (13–11). The number of teeth necessary for the mating gears is

Answer

$$16\sqrt{30} = 87.64 \doteq 88$$

From Eq. (13–30), the overall train value is

$$e = (88/16)(88/16) = 30.25$$

This is within the 1 percent tolerance. If a closer tolerance is desired, then increase the pinion size to the next integer and try again.

EXAMPLE 13–4 A gearbox is needed to provide an *exact* 30:1 increase in speed, while minimizing the overall gearbox size. Specify appropriate teeth numbers.

Solution The previous example demonstrated the difficulty with finding integer numbers of teeth to provide an exact ratio. In order to obtain integers, factor the overall ratio into two integer stages.

$$e = 30 = (6)(5)$$

$$N_2/N_3 = 6 \quad \text{and} \quad N_4/N_5 = 5$$

With two equations and four unknown numbers of teeth, two free choices are available. Choose N_3 and N_5 to be as small as possible without interference. Assuming a 20° pressure angle, Eq. (13–11) gives the minimum as 16.

Then

$$N_2 = 6\,N_3 = 6\,(16) = 96$$

$$N_4 = 5\,N_5 = 5\,(16) = 80$$

The overall train value is then exact.

$$e = (96/16)(80/16) = (6)(5) = 30$$

It is sometimes desirable for the input shaft and the output shaft of a two-stage compound geartrain to be in-line, as shown in Fig. 13–29. This configuration is called a *compound reverted geartrain*. This requires the distances between the shafts to be the same for both stages of the train, which adds to the complexity of the design task. The distance constraint is

$$d_2/2 + d_3/2 = d_4/2 + d_5/2$$

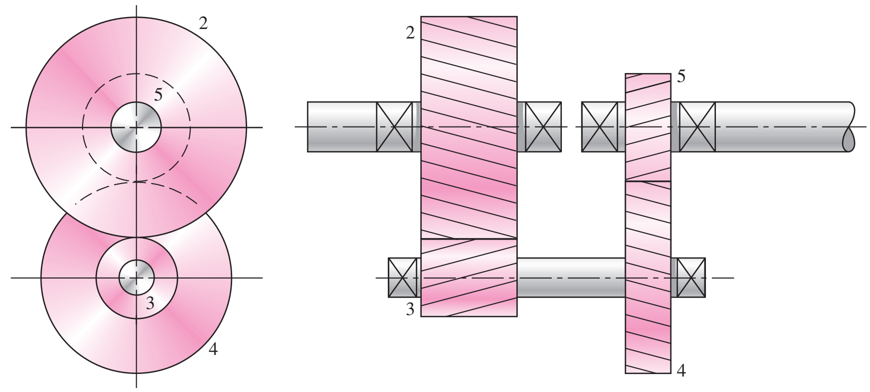


Figure 13–29

A compound reverted gear train.

The diametral pitch relates the diameters and the numbers of teeth, $P = N/d$. Replacing all the diameters gives

$$N_2/(2P) + N_3/(2P) = N_4/(2P) + N_5/(2P)$$

Assuming a constant diametral pitch in both stages, we have the geometry condition stated in terms of numbers of teeth:

$$N_2 + N_3 = N_4 + N_5$$

This condition must be exactly satisfied, in addition to the previous ratio equations, to provide for the in-line condition on the input and output shafts.

EXAMPLE 13–5

A gearbox is needed to provide an exact 30:1 increase in speed, while minimizing the overall gearbox size. The input and output shafts should be in-line. Specify appropriate teeth numbers.

Solution

The governing equations are

$$N_2/N_3 = 6$$

$$N_4/N_5 = 5$$

$$N_2 + N_3 = N_4 + N_5$$

With three equations and four unknown numbers of teeth, only one free choice is available. Of the two smaller gears, N_3 and N_5, the free choice should be used to minimize N_3 since a greater gear ratio is to be achieved in this stage. To avoid interference, the minimum for N_3 is 16.

Applying the governing equations yields

$$N_2 = 6N_3 = 6(16) = 96$$

$$N_2 + N_3 = 96 + 16 = 112 = N_4 + N_5$$

Substituting $N_4 = 5N_5$ gives

$$112 = 5N_5 + N_5 = 6N_5$$

$$N_5 = 112/6 = 18.67$$

If the train value need only be approximated, then this can be rounded to the nearest integer. But for an exact solution, it is necessary to choose the initial free choice for N_3 such that solution of the rest of the teeth numbers results exactly in integers. This can be done by trial and error, letting $N_3 = 17$, then 18, etc., until it works. Or, the problem can be normalized to quickly determine the minimum free choice. Beginning again, let the free choice be $N_3 = 1$. Applying the governing equations gives

$$N_2 = 6N_3 = 6(1) = 6$$

$$N_2 + N_3 = 6 + 1 = 7 = N_4 + N_5$$

Substituting $N_4 = 5N_5$, we find

$$7 = 5N_5 + N_5 = 6N_5$$
$$N_5 = 7/6$$

This fraction could be eliminated if it were multiplied by a multiple of 6. The free choice for the smallest gear N_3 should be selected as a multiple of 6 that is greater than the minimum allowed to avoid interference. This would indicate that $N_3 = 18$. Repeating the application of the governing equations for the final time yields

$$N_2 = 6N_3 = 6(18) = 108$$
$$N_2 + N_3 = 108 + 18 = 126 = N_4 + N_5$$
$$126 = 5N_5 + N_5 = 6N_5$$
$$N_5 = 126/6 = 21$$
$$N_4 = 5N_5 = 5(21) = 105$$

Thus,

Answer

$$N_2 = 108$$
$$N_3 = 18$$
$$N_4 = 105$$
$$N_5 = 21$$

Checking, we calculate $e = (108/18)(105/21) = (6)(5) = 30$.

And checking the geometry constraint for the in-line requirement, we calculate

$$N_2 + N_3 = N_4 + N_5$$
$$108 + 18 = 105 + 21$$
$$126 = 126$$

Unusual effects can be obtained in a gear train by permitting some of the gear axes to rotate about others. Such trains are called *planetary,* or *epicyclic, gear trains.* Planetary trains always include a *sun gear,* a *planet carrier* or *arm,* and one or more *planet gears,* as shown in Fig. 13–30. Planetary gear trains are unusual mechanisms because they have two degrees of freedom; that is, for constrained motion, a planetary train must have two inputs. For example, in Fig. 13–30 these two inputs could be the motion of any two of the elements of the train. We might, say, specify that the sun gear rotates at 100 rev/min clockwise and that the ring gear rotates at 50 rev/min counterclockwise; these are the inputs. The output would be the motion of the arm. In most planetary trains one of the elements is attached to the frame and has no motion.

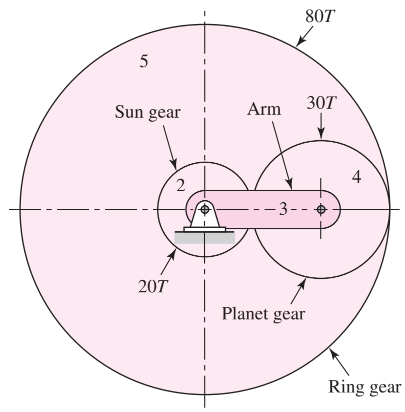


Figure 13–30

A planetary gear train.

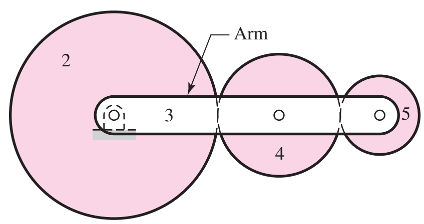


Figure 13–31

A gear train on the arm of a planetary gear train.

Figure 13–31 shows a planetary train composed of a sun gear 2, an arm or carrier 3, and planet gears 4 and 5. The angular velocity of gear 2 relative to the arm in rev/min is

$$n_{23} = n_2 - n_3 \tag{b}$$

Also, the velocity of gear 5 relative to the arm is

$$n_{53} = n_5 - n_3 \tag{c}$$

Dividing Eq. (*c*) by Eq. (*b*) gives

$$\frac{n_{53}}{n_{23}} = \frac{n_5 - n_3}{n_2 - n_3} \tag{d}$$

Equation (*d*) expresses the ratio of gear 5 to that of gear 2, and both velocities are taken relative to the arm. Now this ratio is the same and is proportional to the tooth numbers, whether the arm is rotating or not. It is the train value. Therefore, we may write

$$e = \frac{n_5 - n_3}{n_2 - n_3} \tag{e}$$

This equation can be used to solve for the output motion of any planetary train. It is more conveniently written in the form

$$e = \frac{n_L - n_A}{n_F - n_A} \tag{13–32}$$

where n_F = rev/min of first gear in planetary train
n_L = rev/min of last gear in planetary train
n_A = rev/min of arm

EXAMPLE 13–6 In Fig. 13–30 the sun gear is the input, and it is driven clockwise at 100 rev/min. The ring gear is held stationary by being fastened to the frame. Find the rev/min and direction of rotation of the arm and gear 4.

Solution Designate $n_F = n_2 = -100$ rev/min, and $n_L = n_5 = 0$. Unlocking gear 5 and holding the arm stationary, in our imagination, we find

$$e = -\left(\frac{20}{30}\right)\left(\frac{30}{80}\right) = -0.25$$

Substituting this value in Eq. (13–32) gives

$$-0.25 = \frac{0 - n_A}{(-100) - n_A}$$

or

Answer

$$n_A = -20 \text{ rev/min}$$

To obtain the speed of gear 4, we follow the procedure outlined by Eqs. (*b*), (*c*), and (*d*). Thus

$$n_{43} = n_4 - n_3 \qquad n_{23} = n_2 - n_3$$

and so

$$\frac{n_{43}}{n_{23}} = \frac{n_4 - n_3}{n_2 - n_3} \tag{1}$$

But

$$\frac{n_{43}}{n_{23}} = -\frac{20}{30} = -\frac{2}{3} \tag{2}$$

Substituting the known values in Eq. (1) gives

$$-\frac{2}{3} = \frac{n_4 - (-20)}{(-100) - (-20)}$$

Solving gives

Answer

$$n_4 = 33\tfrac{1}{3} \text{ rev/min}$$

13–14 Force Analysis—Spur Gearing

Before beginning the force analysis of gear trains, let us agree on the notation to be used. Beginning with the numeral 1 for the frame of the machine, we shall designate the input gear as gear 2, and then number the gears successively 3, 4, etc., until we arrive at the last gear in the train. Next, there may be several shafts involved, and usually one or two gears are mounted on each shaft as well as other elements. We shall designate the shafts, using lowercase letters of the alphabet, a, b, c, etc.

With this notation we can now speak of the force exerted by gear 2 against gear 3 as F_{23}. The force of gear 2 against a shaft a is F_{2a}. We can also write F_{a2} to mean the force of a shaft a against gear 2. Unfortunately, it is also necessary to use superscripts to indicate directions. The coordinate directions will usually be indicated by the x, y, and z coordinates, and the radial and tangential directions by superscripts r and t. With this notation, F^t_{43} is the tangential component of the force of gear 4 acting against gear 3.

Figure 13–32*a* shows a pinion mounted on shaft a rotating clockwise at n_2 rev/min and driving a gear on shaft b at n_3 rev/min. The reactions between the mating teeth occur along the pressure line. In Fig. 13–32*b* the pinion has been separated from the gear and the shaft, and their effects have been replaced by forces. F_{a2} and T_{a2} are the force and torque, respectively, exerted by shaft a against pinion 2. F_{32} is the force exerted by gear 3 against the pinion. Using a similar approach, we obtain the free-body diagram of the gear shown in Fig. 13–32*c*.

In Fig. 13–33, the free-body diagram of the pinion has been redrawn and the forces have been resolved into tangential and radial components. We now define

$$W_t = F^t_{32} \tag{a}$$

as the *transmitted load*. This tangential load is really the useful component, because the radial component F^r_{32} serves no useful purpose. It does not transmit power. The applied torque and the transmitted load are seen to be related by the equation

$$T = \frac{d}{2} W_t \tag{b}$$

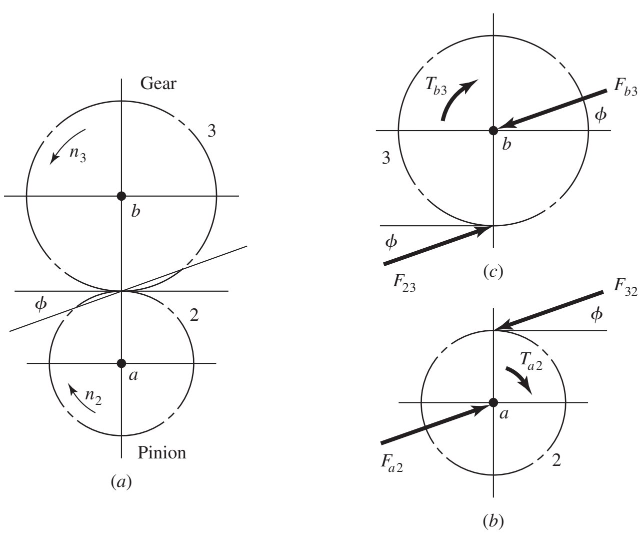
where we have used $T = T_{a2}$ and $d = d_2$ to obtain a general relation.

The power H transmitted through a rotating gear can be obtained from the standard relationship of the product of torque T and angular velocity ω.

$$H = T\omega = (W_t d/2)\,\omega \tag{13–33}$$

Figure 13–32

Free-body diagrams of the forces and moments acting upon two gears of a simple gear train.



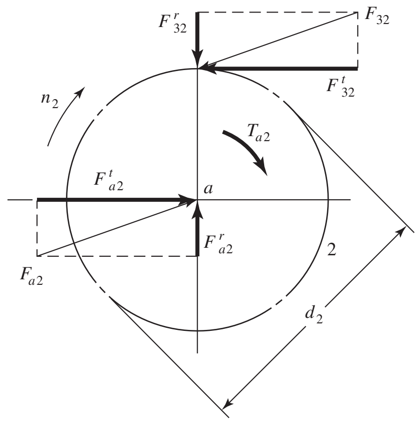


Figure 13–33

Resolution of gear forces.

While any units can be used in this equation, the units of the resulting power will obviously be dependent on the units of the other parameters. It will often be desirable to work with the power in either horsepower or kilowatts, and appropriate conversion factors should be used.

Since meshed gears are reasonably efficient, with losses of less than 2 percent, the power is generally treated as constant through the mesh. Consequently, with a pair of meshed gears, Eq. (13–33) will give the same power regardless of which gear is used for d and ω.

Gear data is often tabulated using *pitch-line velocity*, which is the linear velocity of a point on the gear at the radius of the pitch circle; thus $V = (d/2)\,\omega$. Converting this to customary units gives

$$V = \pi dn/12 \tag{13–34}$$

where V = pitch-line velocity, ft/min
d = gear diameter, in
n = gear speed, rev/min

Many gear design problems will specify the power and speed, so it is convenient to solve Eq. (13–33) for W_t. With the pitch-line velocity and appropriate conversion factors incorporated, Eq. (13–33) can be rearranged and expressed in customary units as

$$W_t = 33\,000\frac{H}{V} \tag{13–35}$$

where W_t = transmitted load, lbf
H = power, hp
V = pitch-line velocity, ft/min

The corresponding equation in SI is

$$W_t = \frac{60\,000H}{\pi dn} \tag{13–36}$$

where W_t = transmitted load, kN
H = power, kW
d = gear diameter, mm
n = speed, rev/min

EXAMPLE 13–7

Pinion 2 in Fig. 13–34*a* runs at 1750 rev/min and transmits 2.5 kW to idler gear 3. The teeth are cut on the 20° full-depth system and have a module of $m = 2.5$ mm. Draw a free-body diagram of gear 3 and show all the forces that act upon it.

Solution

The pitch diameters of gears 2 and 3 are

$$d_2 = N_2 m = 20(2.5) = 50 \text{ mm}$$

$$d_3 = N_3 m = 50(2.5) = 125 \text{ mm}$$

From Eq. (13–36) we find the transmitted load to be

$$W_t = \frac{60\,000H}{\pi d_2 n} = \frac{60\,000(2.5)}{\pi(50)(1750)} = 0.546 \text{ kN}$$

Thus, the tangential force of gear 2 on gear 3 is $F^t_{23} = 0.546$ kN, as shown in Fig. 13–34*b*. Therefore

$$F^r_{23} = F^t_{23} \tan 20° = (0.546) \tan 20° = 0.199 \text{ kN}$$

and so

$$F_{23} = \frac{F^t_{23}}{\cos 20°} = \frac{0.546}{\cos 20°} = 0.581 \text{ kN}$$

Since gear 3 is an idler, it transmits no power (torque) to its shaft, and so the tangential reaction of gear 4 on gear 3 is also equal to W_t. Therefore

$$F^t_{43} = 0.546 \text{ kN} \qquad F^r_{43} = 0.199 \text{ kN} \qquad F_{43} = 0.581 \text{ kN}$$

and the directions are shown in Fig. 13–34*b*.

The shaft reactions in the x and y directions are

$$F^x_{b3} = -\left(F^t_{23} + F^r_{43}\right) = -(-0.546 + 0.199) = 0.347 \text{ kN}$$

$$F^y_{b3} = -\left(F^r_{23} + F^t_{43}\right) = -(0.199 - 0.546) = 0.347 \text{ kN}$$

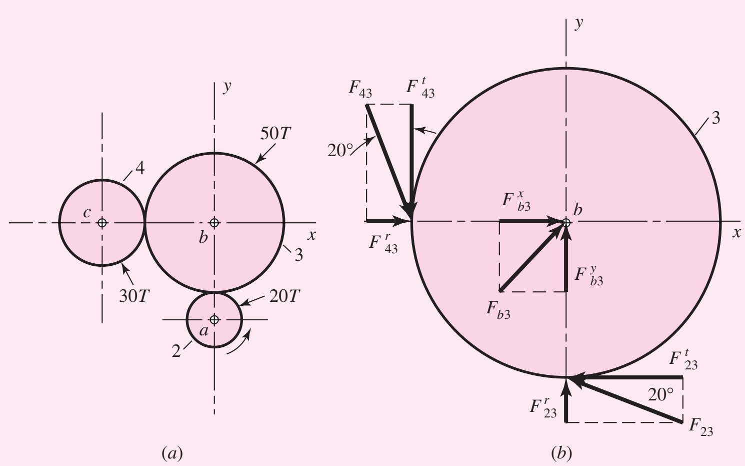


Figure 13–34

A gear train containing an idler gear. (*a*) The gear train. (*b*) Free-body of the idler gear.

The resultant shaft reaction is

$$F_{b3} = \sqrt{(0.347)^2 + (0.347)^2} = 0.491 \text{ kN}$$

These are shown on the figure.

13–15 Force Analysis—Bevel Gearing

In determining shaft and bearing loads for bevel-gear applications, the usual practice is to use the tangential or transmitted load that would occur if all the forces were concentrated at the midpoint of the tooth. While the actual resultant occurs somewhere between the midpoint and the large end of the tooth, there is only a small error in making this assumption. For the transmitted load, this gives

$$W_t = \frac{T}{r_{\text{av}}} \tag{13–37}$$

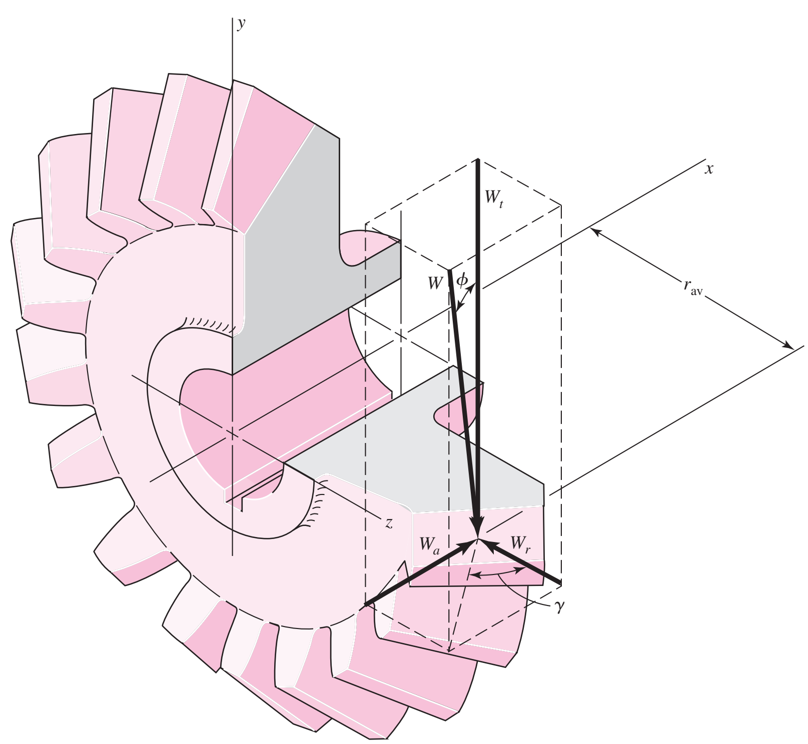
where T is the torque and r_{av} is the pitch radius at the midpoint of the tooth for the gear under consideration.

The forces acting at the center of the tooth are shown in Fig. 13–35. The resultant force W has three components: a tangential force W_t, a radial force W_r, and an axial force W_a. From the trigonometry of the figure,

$$\begin{aligned} W_r &= W_t \tan\phi \cos\gamma \\ W_a &= W_t \tan\phi \sin\gamma \end{aligned} \tag{13–38}$$

Figure 13–35

Bevel-gear tooth forces.



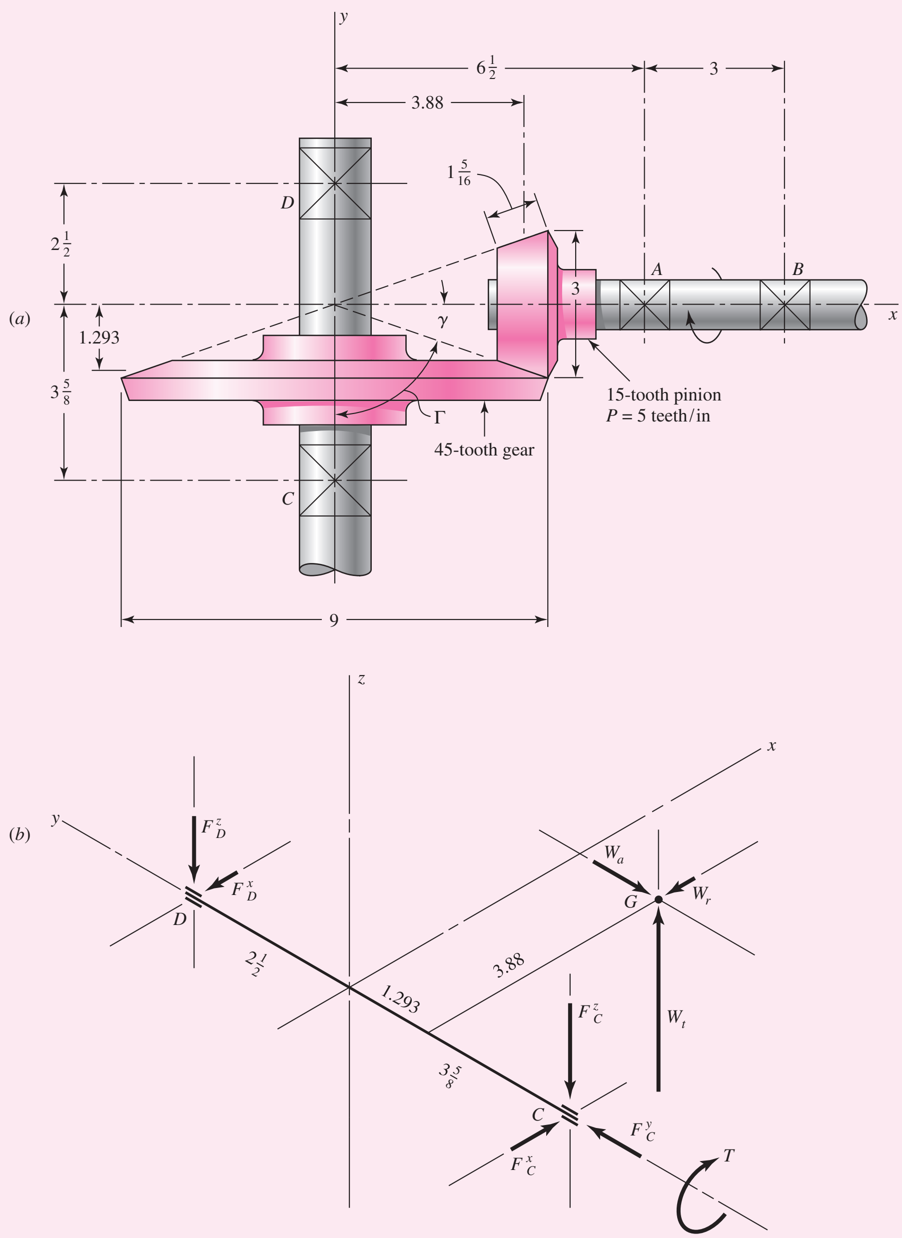
The three forces W_t, W_r, and W_a are at right angles to each other and can be used to determine the bearing loads by using the methods of statics.

EXAMPLE 13–8

The bevel pinion in Fig. 13–36*a* rotatès at 600 rev/min in the direction shown and transmits 5 hp to the gear. The mounting distances, the location of all bearings, and the average pitch radii of the pinion and gear are shown in the figure. For simplicity, the teeth have been replaced by pitch cones. Bearings *A* and *C* should take the thrust loads. Find the bearing forces on the gearshaft.

Figure 13–36

(*a*) Bevel-gear set of Ex. 13–8. (*b*) Free body diagram of shaft *CD*. Dimensions in inches.



Solution The pitch angles are

$$\gamma = \tan^{-1}\left(\frac{3}{9}\right) = 18.4^\circ \qquad \Gamma = \tan^{-1}\left(\frac{9}{3}\right) = 71.6^\circ$$

The pitch-line velocity corresponding to the average pitch radius is

$$V = \frac{2\pi r_P n}{12} = \frac{2\pi(1.293)(600)}{12} = 406\ \text{ft/min}$$

Therefore the transmitted load is

$$W_t = \frac{33\,000H}{V} = \frac{(33\,000)(5)}{406} = 406\ \text{lbf}$$

which acts in the positive z direction, as shown in Fig. 13–36*b*. We next have

$$W_r = W_t \tan\phi \cos\Gamma = 406 \tan 20^\circ \cos 71.6^\circ = 46.6\ \text{lbf}$$
$$W_a = W_t \tan\phi \sin\Gamma = 406 \tan 20^\circ \sin 71.6^\circ = 140\ \text{lbf}$$

where W_r is in the $-x$ direction and W_a is in the $-y$ direction, as illustrated in the isometric sketch of Fig. 13–36*b*.

In preparing to take a sum of the moments about bearing D, define the position vector from D to G as

$$\mathbf{R}_G = 3.88\mathbf{i} - (2.5 + 1.293)\mathbf{j} = 3.88\mathbf{i} - 3.793\mathbf{j}$$

We shall also require a vector from D to C:

$$\mathbf{R}_C = -(2.5 + 3.625)\mathbf{j} = -6.125\mathbf{j}$$

Then, summing moments about D gives

$$\mathbf{R}_G \times \mathbf{W} + \mathbf{R}_C \times \mathbf{F}_C + \mathbf{T} = \mathbf{0} \tag{1}$$

When we place the details in Eq. (1), we get

$$\begin{aligned}&(3.88\mathbf{i} - 3.793\mathbf{j}) \times (-46.6\mathbf{i} - 140\mathbf{j} + 406\mathbf{k})\\ &\quad + (-6.125\mathbf{j}) \times \left(F_C^x\mathbf{i} + F_C^y\mathbf{j} + F_C^z\mathbf{k}\right) + T\mathbf{j} = \mathbf{0}\end{aligned} \tag{2}$$

After the two cross products are taken, the equation becomes

$$(-1540\mathbf{i} - 1575\mathbf{j} - 720\mathbf{k}) + \left(-6.125F_C^z\mathbf{i} + 6.125F_C^x\mathbf{k}\right) + T\mathbf{j} = \mathbf{0}$$

from which

$$\mathbf{T} = 1575\mathbf{j}\ \text{lbf}\cdot\text{in} \qquad F_C^x = 118\ \text{lbf} \qquad F_C^z = -251\ \text{lbf} \tag{3}$$

Now sum the forces to zero. Thus

$$\mathbf{F}_D + \mathbf{F}_C + \mathbf{W} = \mathbf{0} \tag{4}$$

When the details are inserted, Eq. (4) becomes

$$\left(F_D^x\mathbf{i} + F_D^z\mathbf{k}\right) + \left(118\mathbf{i} + F_C^y\mathbf{j} - 251\mathbf{k}\right) + (-46.6\mathbf{i} - 140\mathbf{j} + 406\mathbf{k}) = \mathbf{0} \tag{5}$$

First we see that $F_C^y = 140$ lbf, and so

Answer
$$\mathbf{F}_C = 118\mathbf{i} + 140\mathbf{j} - 251\mathbf{k}\ \text{lbf}$$

Then, from Eq. (5),

Answer
$$\mathbf{F}_D = -71.4\mathbf{i} - 155\mathbf{k}\ \text{lbf}$$

These are all shown in Fig. 13–36*b* in the proper directions.The analysis for the pinion shaft is quite similar.

13–16 Force Analysis—Helical Gearing

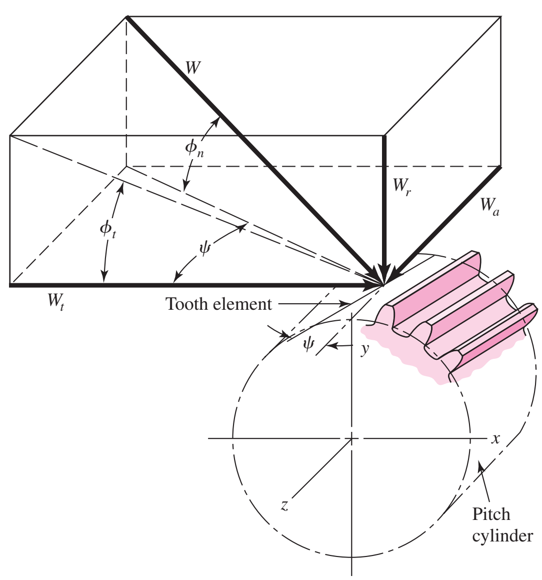
Figure 13–37 is a three-dimensional view of the forces acting against a helical-gear tooth. The point of application of the forces is in the pitch plane and in the center of the gear face. From the geometry of the figure, the three components of the total (normal) tooth force W are

$$\begin{aligned} W_r &= W \sin\phi_n \\ W_t &= W \cos\phi_n \cos\psi \\ W_a &= W \cos\phi_n \sin\psi \end{aligned} \tag{13–39}$$

where W = total force
W_r = radial component
W_t = tangential component, also called transmitted load
W_a = axial component, also called thrust load

Figure 13–37

Tooth forces acting on a right-hand helical gear.



Usually W_t is given and the other forces are desired. In this case, it is not difficult to discover that

$$W_r = W_t \tan\phi_t$$
$$W_a = W_t \tan\psi \tag{13–40}$$
$$W = \frac{W_t}{\cos\phi_n \cos\psi}$$

EXAMPLE 13–9 In Fig. 13–38 a 1-hp electric motor runs at 1800 rev/min in the clockwise direction, as viewed from the positive x axis. Keyed to the motor shaft is an 18-tooth helical pinion having a normal pressure angle of 20°, a helix angle of 30°, and a normal diametral pitch of 12 teeth/in. The hand of the helix is shown in the figure. Make a three-dimensional sketch of the motor shaft and pinion, and show the forces acting on the pinion and the bearing reactions at A and B. The thrust should be taken out at A.

Solution From Eq. (13–19) we find

$$\phi_t = \tan^{-1}\frac{\tan\phi_n}{\cos\psi} = \tan^{-1}\frac{\tan 20^\circ}{\cos 30^\circ} = 22.8^\circ$$

Also, $P_t = P_n \cos\psi = 12\cos 30^\circ = 10.39$ teeth/in. Therefore the pitch diameter of the pinion is $d_p = 18/10.39 = 1.732$ in. The pitch-line velocity is

$$V = \frac{\pi d n}{12} = \frac{\pi(1.732)(1800)}{12} = 816\text{ ft/min}$$

The transmitted load is

$$W_t = \frac{33\,000H}{V} = \frac{(33\,000)(1)}{816} = 40.4\text{ lbf}$$

From Eq. (13–40) we find

$$W_r = W_t \tan\phi_t = (40.4)\tan 22.8^\circ = 17.0\text{ lbf}$$
$$W_a = W_t \tan\psi = (40.4)\tan 30^\circ = 23.3\text{ lbf}$$
$$W = \frac{W_t}{\cos\phi_n\cos\psi} = \frac{40.4}{\cos 20^\circ \cos 30^\circ} = 49.6\text{ lbf}$$

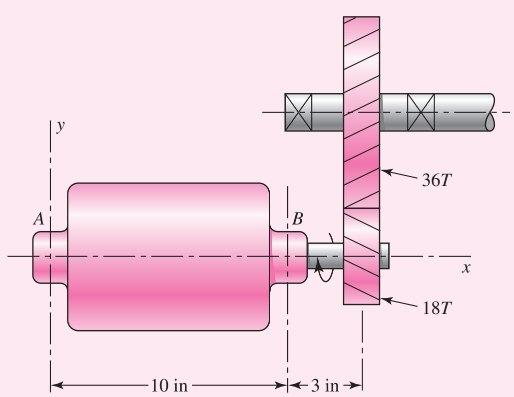
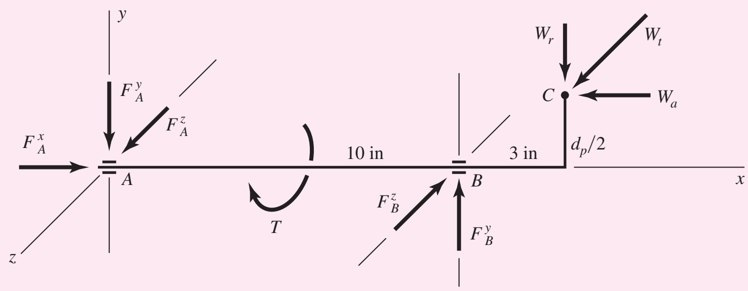


Figure 13–38
The motor and gear train of Ex. 13–9.

Figure 13–39

Free body diagram of motor shaft of Ex. 13–9.



These three forces, W_r in the $-y$ direction, W_a in the $-x$ direction, and W_t in the $+z$ direction, are shown acting at point C in Fig. 13–39. We assume bearing reactions at A and B as shown. Then $F_A^x = W_a = 23.3$ lbf. Taking moments about the z axis,

$$-(17.0)(13) + (23.3)\left(\frac{1.732}{2}\right) + 10F_B^y = 0$$

or $F_B^y = 20.1$ lbf. Summing forces in the y direction then gives $F_A^y = 3.1$ lbf. Taking moments about the y axis, next

$$10F_B^z - (40.4)(13) = 0$$

or $F_B^z = 52.5$ lbf. Summing forces in the z direction and solving gives $F_A^z = 12.1$ lbf. Also, the torque is $T = W_t d_p/2 = (40.4)(1.732/2) = 35\ \text{lbf}\cdot\text{in}$.

For comparison, solve the problem again using vectors. The force at C is

$$\mathbf{W} = -23.3\mathbf{i} - 17.0\mathbf{j} + 40.4\mathbf{k}\ \text{lbf}$$

Position vectors to B and C from origin A are

$$\mathbf{R}_B = 10\,\mathbf{i} \qquad \mathbf{R}_C = 13\mathbf{i} + 0.866\mathbf{j}$$

Taking moments about A, we have

$$\mathbf{R}_B \times \mathbf{F}_B + \mathbf{T} + \mathbf{R}_C \times \mathbf{W} = \mathbf{0}$$

Using the directions assumed in Fig. 13–39 and substituting values gives

$$10\,\mathbf{i} \times \left(F_B^y\mathbf{j} - F_B^z\mathbf{k}\right) - T\,\mathbf{i} + (13\mathbf{i} + 0.866\mathbf{j}) \times (-23.3\mathbf{i} - 17.0\mathbf{j} + 40.4\mathbf{k}) = \mathbf{0}$$

When the cross products are formed, we get

$$\left(10F_B^y\mathbf{k} + 10F_B^z\mathbf{j}\right) - T\,\mathbf{i} + (35\mathbf{i} - 525\mathbf{j} - 201\mathbf{k}) = \mathbf{0}$$

whence $T = 35\ \text{lbf}\cdot\text{in}$, $F_B^y = 20.1$ lbf, and $F_B^z = 52.5$ lbf.

Next,

$$\mathbf{F}_A = -\mathbf{F}_B - \mathbf{W}, \text{ and so } \mathbf{F}_A = 23.3\mathbf{i} - 3.1\mathbf{j} + 12.1\mathbf{k}\ \text{lbf}.$$

13–17 Force Analysis—Worm Gearing

If friction is neglected, then the only force exerted by the gear will be the force W, shown in Fig. 13–40, having the three orthogonal components W^x, W^y, and W^z. From the geometry of the figure, we see that

$$
\begin{aligned}
W^x &= W \cos\phi_n \sin\lambda \\
W^y &= W \sin\phi_n \\
W^z &= W \cos\phi_n \cos\lambda
\end{aligned}
\tag{13–41}
$$

We now use the subscripts W and G to indicate forces acting against the worm and gear, respectively. We note that W^y is the separating, or radial, force for both the worm and the gear. The tangential force on the worm is W^x and is W^z on the gear, assuming a 90° shaft angle. The axial force on the worm is W^z, and on the gear, W^x. Since the gear forces are opposite to the worm forces, we can summarize these relations by writing

$$
\begin{aligned}
W_{Wt} &= -W_{Ga} = W^x \\
W_{Wr} &= -W_{Gr} = W^y \\
W_{Wa} &= -W_{Gt} = W^z
\end{aligned}
\tag{13–42}
$$

It is helpful in using Eq. (13–41) and also Eq. (13–42) to observe that *the gear axis is parallel to the x direction and the worm axis is parallel to the z direction* and that we are employing a right-handed coordinate system.

In our study of spur-gear teeth we have learned that the motion of one tooth relative to the mating tooth is primarily a rolling motion; in fact, when contact occurs at the pitch point, the motion is pure rolling. In contrast, the relative motion between worm and worm-gear teeth is pure sliding, and so we must expect that friction plays an important role in the performance of worm gearing. By introducing a coefficient of friction f, we can develop another set of relations similar to those of Eq. (13–41). In Fig. 13–40 we see that the force W acting normal to the worm-tooth profile produces a frictional force $W_f = fW$, having a component $fW \cos\lambda$ in the negative x direction and another component $fW \sin\lambda$ in the positive z direction. Equation (13–41) therefore becomes

$$
\begin{aligned}
W^x &= W(\cos\phi_n \sin\lambda + f \cos\lambda) \\
W^y &= W \sin\phi_n \\
W^z &= W(\cos\phi_n \cos\lambda - f \sin\lambda)
\end{aligned}
\tag{13–43}
$$

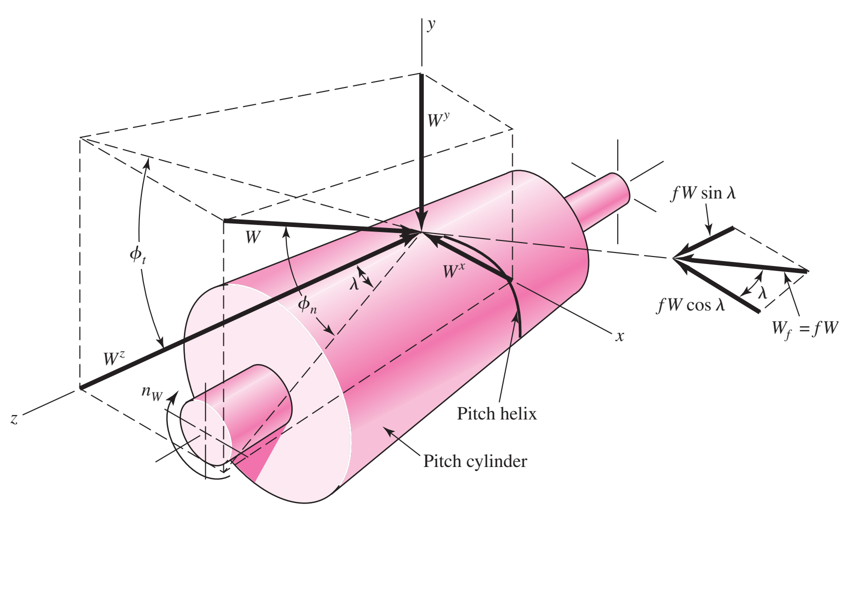


Figure 13–40

Drawing of the pitch cylinder of a worm, showing the forces exerted upon it by the worm gear.

Equation (13–42), of course, still applies.

Inserting $-W_{Gt}$ from Eq. (13–42) for W^z in Eq. (13–43) and multiplying both sides by f, we find the frictional force W_f to be

$$W_f = fW = \frac{fW_{Gt}}{f\sin\lambda - \cos\phi_n \cos\lambda} \tag{13–44}$$

A useful relation between the two tangential forces, W_{Wt} and W_{Gt}, can be obtained by equating the first and third parts of Eqs. (13–42) and (13–43) and eliminating W. The result is

$$W_{Wt} = W_{Gt}\frac{\cos\phi_n \sin\lambda + f\cos\lambda}{f\sin\lambda - \cos\phi_n \cos\lambda} \tag{13–45}$$

Efficiency η can be defined by using the equation

$$\eta = \frac{W_{Wt}(\text{without friction})}{W_{Wt}(\text{with friction})} \tag{a}$$

Substitute Eq. (13–45) with $f = 0$ in the numerator of Eq. (a) and the same equation in the denominator. After some rearranging, you will find the efficiency to be

$$\eta = \frac{\cos\phi_n - f\tan\lambda}{\cos\phi_n + f\cot\lambda} \tag{13–46}$$

Selecting a typical value of the coefficient of friction, say $f = 0.05$, and the pressure angles shown in Table 13–6, we can use Eq. (13–46) to get some useful design information. Solving this equation for helix angles from 1 to 30° gives the interesting results shown in Table 13–6.

Many experiments have shown that the coefficient of friction is dependent on the relative or sliding velocity. In Fig. 13–41, V_G is the pitch-line velocity of the gear and V_W the pitch-line velocity of the worm. Vectorially, $\mathbf{V}_W = \mathbf{V}_G + \mathbf{V}_S$; consequently, the sliding velocity is

$$V_S = \frac{V_W}{\cos\lambda} \tag{13–47}$$

Table 13–6

Efficiency of Worm Gearsets for $f = 0.05$

Helix Angle ψ, deg	Efficiency η, %
1.0	25.2
2.5	45.7
5.0	62.0
7.5	71.3
10.0	76.6
15.0	82.7
20.0	85.9
30.0	89.1

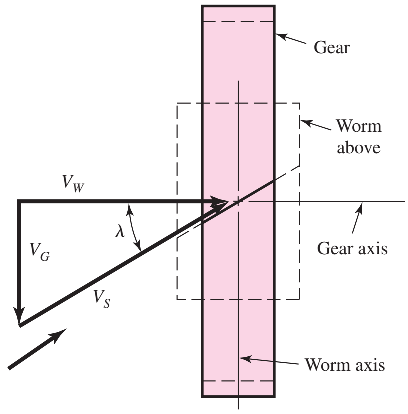


Figure 13–41

Velocity components in worm gearing.

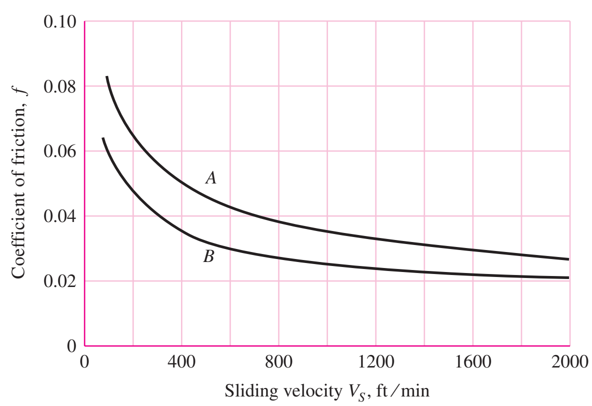


Figure 13–42

Representative values of the coefficient of friction for worm gearing. These values are based on good lubrication. Use curve *B* for high-quality materials, such as a case-hardened steel worm mating with a phosphor-bronze gear. Use curve *A* when more friction is expected, as with a cast-iron worm mating with a cast-iron worm gear.

Published values of the coefficient of friction vary as much as 20 percent, undoubtedly because of the differences in surface finish, materials, and lubrication. The values on the chart of Fig. 13–42 are representative and indicate the general trend.

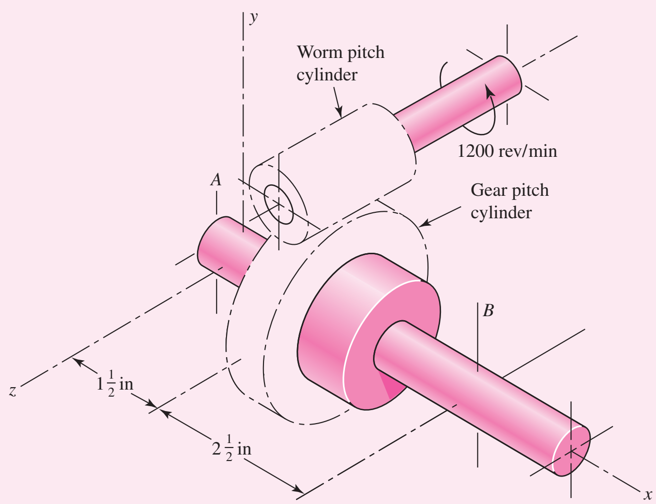
EXAMPLE 13–10 A 2-tooth right-hand worm transmits 1 hp at 1200 rev/min to a 30-tooth worm gear. The gear has a transverse diametral pitch of 6 teeth/in and a face width of 1 in. The worm has a pitch diameter of 2 in and a face width of $2\frac{1}{2}$ in. The normal pressure angle is $14\frac{1}{2}^\circ$. The materials and quality of work needed are such that curve *B* of Fig. 13–42 should be used to obtain the coefficient of friction.

(*a*) Find the axial pitch, the center distance, the lead, and the lead angle.

(*b*) Figure 13–43 is a drawing of the worm gear oriented with respect to the coordinate system described earlier in this section; the gear is supported by bearings *A* and *B*. Find the forces exerted by the bearings against the worm-gear shaft, and the output torque.

Figure 13–43

The pitch cylinders of the worm gear train of Ex. 13–10.



Solution (*a*) The axial pitch is the same as the transverse circular pitch of the gear, which is

Answer

$$p_x = p_t = \frac{\pi}{P} = \frac{\pi}{6} = 0.5236 \text{ in}$$

The pitch diameter of the gear is $d_G = N_G/P = 30/6 = 5$ in. Therefore, the center distance is

Answer

$$C = \frac{d_W + d_G}{2} = \frac{2+5}{2} = 3.5 \text{ in}$$

From Eq. (13–27), the lead is

$$L = p_x N_W = (0.5236)(2) = 1.0472 \text{ in}$$

Answer Also using Eq. (13–28), find

Answer

$$\lambda = \tan^{-1}\frac{L}{\pi d_W} = \tan^{-1}\frac{1.0472}{\pi(2)} = 9.46^\circ$$

(*b*) Using the right-hand rule for the rotation of the worm, you will see that your thumb points in the positive z direction. Now use the bolt-and-nut analogy (the worm is right-handed, as is the screw thread of a bolt), and turn the bolt clockwise with the right hand while preventing nut rotation with the left. The nut will move axially along the bolt toward your right hand. Therefore the surface of the gear (Fig. 13–43) in contact with the worm will move in the negative z direction. Thus, the gear rotates clockwise about x, with your right thumb pointing in the negative x direction.

The pitch-line velocity of the worm is

$$V_W = \frac{\pi d_W n_W}{12} = \frac{\pi(2)(1200)}{12} = 628 \text{ ft/min}$$

The speed of the gear is $n_G = (\frac{2}{30})(1200) = 80$ rev/min. Therefore the pitch-line velocity of the gear is

$$V_G = \frac{\pi d_G n_G}{12} = \frac{\pi(5)(80)}{12} = 105 \text{ ft/min}$$

Then, from Eq. (13–47), the sliding velocity V_S is found to be

$$V_S = \frac{V_W}{\cos\lambda} = \frac{628}{\cos 9.46^\circ} = 637 \text{ ft/min}$$

Getting to the forces now, we begin with the horsepower formula

$$W_{Wt} = \frac{33\,000H}{V_W} = \frac{(33\,000)(1)}{628} = 52.5 \text{ lbf}$$

This force acts in the negative x direction, the same as in Fig. 13–40. Using Fig. 13–42, we find $f = 0.03$. Then, the first equation of group (13–42) and (13–43) gives

$$\begin{aligned} W &= \frac{W^x}{\cos\phi_n \sin\lambda + f\cos\lambda} \\ &= \frac{52.5}{\cos 14.5^\circ \sin 9.46^\circ + 0.03\cos 9.46^\circ} = 278 \text{ lbf} \end{aligned}$$

Also, from Eq. (13–43),

$$\begin{aligned} W^y &= W\sin\phi_n = 278\sin 14.5^\circ = 69.6 \text{ lbf} \\ W^z &= W(\cos\phi_n\cos\lambda - f\sin\lambda) \\ &= 278(\cos 14.5^\circ\cos 9.46^\circ - 0.03\sin 9.46^\circ) = 264 \text{ lbf} \end{aligned}$$

We now identify the components acting on the gear as

$$\begin{aligned} W_{Ga} &= -W^x = 52.5 \text{ lbf} \\ W_{Gr} &= -W^y = -69.6 \text{ lbf} \\ W_{Gt} &= -W^z = -264 \text{ lbf} \end{aligned}$$

At this point a three-dimensional line drawing should be made in order to simplify the work to follow. An isometric sketch, such as the one of Fig. 13–44, is easy to make and will help you to avoid errors.

We shall make B a thrust bearing in order to place the gearshaft in compression. Thus, summing forces in the x direction gives

Answer
$$F_B^x = -52.5 \text{ lbf}$$

Taking moments about the z axis, we have

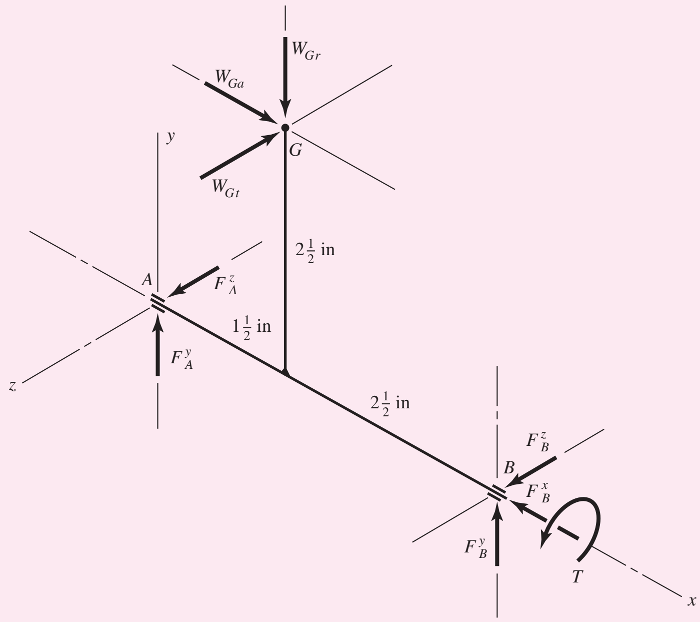
Answer
$$-(52.5)(2.5) - (69.6)(1.5) + 4F_B^y = 0 \qquad F_B^y = 58.9 \text{ lbf}$$

Taking moments about the y axis,

Answer
$$(264)(1.5) - 4F_B^z = 0 \qquad F_B^z = 99 \text{ lbf}$$

Figure 13–44

An isometric sketch used in Ex. 13–10.



These three components are now inserted on the sketch as shown at B in Fig. 13–44. Summing forces in the y direction,

Answer
$$-69.6 + 58.9 + F_A^y = 0 \qquad F_A^y = 10.7 \text{ lbf}$$

Similarly, summing forces in the z direction,

Answer
$$-264 + 99 + F_A^z = 0 \qquad F_A^z = 165 \text{ lbf}$$

These two components can now be placed at A on the sketch. We still have one more equation to write. Summing moments about x,

Answer
$$-(264)(2.5) + T = 0 \qquad T = 660 \text{ lbf} \cdot \text{in}$$

It is because of the frictional loss that this output torque is less than the product of the gear ratio and the input torque.

PROBLEMS

13–1 A 17-tooth spur pinion has a diametral pitch of 8 teeth/in, runs at 1120 rev/min, and drives a gear at a speed of 544 rev/min. Find the number of teeth on the gear and the theoretical center-to-center distance.

13–2 A 15-tooth spur pinion has a module of 3 mm and runs at a speed of 1600 rev/min. The driven gear has 60 teeth. Find the speed of the driven gear, the circular pitch, and the theoretical center-to-center distance.

13–3 A spur gearset has a module of 4 mm and a velocity ratio of 2.80. The pinion has 20 teeth. Find the number of teeth on the driven gear, the pitch diameters, and the theoretical center-to-center distance.

13–4 A 21-tooth spur pinion mates with a 28-tooth gear. The diametral pitch is 3 teeth/in and the pressure angle is 20°. Make a drawing of the gears showing one tooth on each gear. Find and tabulate the following results: the addendum, dedendum, clearance, circular pitch, tooth thickness, and base-circle diameters; the lengths of the arc of approach, recess, and action; and the base pitch and contact ratio.

13–5 A 20° straight-tooth bevel pinion having 14 teeth and a diametral pitch of 6 teeth/in drives a 32-tooth gear. The two shafts are at right angles and in the same plane. Find:
(*a*) The cone distance
(*b*) The pitch angles
(*c*) The pitch diameters
(*d*) The face width

13–6 A parallel helical gearset uses a 17-tooth pinion driving a 34-tooth gear. The pinion has a right-hand helix angle of 30°, a normal pressure angle of 20°, and a normal diametral pitch of 5 teeth/in. Find:
(*a*) The normal, transverse, and axial circular pitches
(*b*) The normal base circular pitch
(*c*) The transverse diametral pitch and the transverse pressure angle
(*d*) The addendum, dedendum, and pitch diameter of each gear

13–7 A parallel helical gearset consists of a 19-tooth pinion driving a 57-tooth gear. The pinion has a left-hand helix angle of 20°, a normal pressure angle of $14\frac{1}{2}^\circ$, and a normal diametral pitch of 10 teeth/in. Find:
(*a*) The normal, transverse, and axial circular pitches
(*b*) The transverse diametral pitch and the transverse pressure angle
(*c*) The addendum, dedendum, and pitch diameter of each gear

13–8 For a spur gearset with $\phi = 20^\circ$, while avoiding interference, find:
(*a*) The smallest pinion tooth count that will run with itself
(*b*) The smallest pinion tooth count at a ratio $m_G = 2.5$, and the largest gear tooth count possible with this pinion
(*c*) The smallest pinion that will run with a rack

13–9 Repeat problem 13-8 for a helical gearset with $\phi_n = 20^\circ$ and $\psi = 30^\circ$.

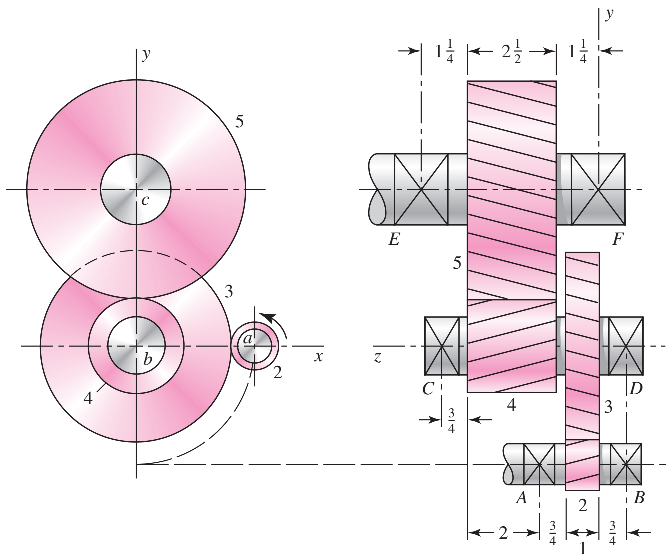
13–10 The decision has been made to use $\phi_n = 20^\circ$, $P_t = 6$ teeth/in, and $\psi = 30^\circ$ for a 2:1 reduction. Choose a suitable pinion and gear tooth count to avoid interference.

13–11 Repeat Problem 13–10 with a 6:1 reduction.

13–12 By employing a pressure angle larger than standard, it is possible to use fewer pinion teeth, and hence obtain smaller gears without undercutting during machining. If the gears are spur gears, what is the smallest possible pressure angle ϕ_t that can be obtained without undercutting for a 9-tooth pinion to mesh with a rack?

13–13 A parallel-shaft gearset consists of an 18-tooth helical pinion driving a 32-tooth gear. The pinion has a left-hand helix angle of 25°, a normal pressure angle of 20°, and a normal module of 3 mm. Find:
(*a*) The normal, transverse, and axial circular pitches
(*b*) The transverse module and the transverse pressure angle
(*c*) The pitch diameters of the two gears

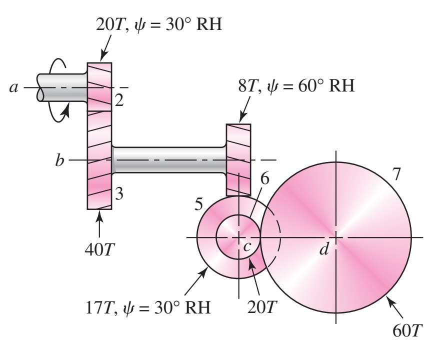
13–14 The double-reduction helical gearset shown in the figure is driven through shaft *a* at a speed of 900 rev/min. Gears 2 and 3 have a normal diametral pitch of 10 teeth/in, a 30° helix angle, and a



Problem 13–14
Dimensions in inches.

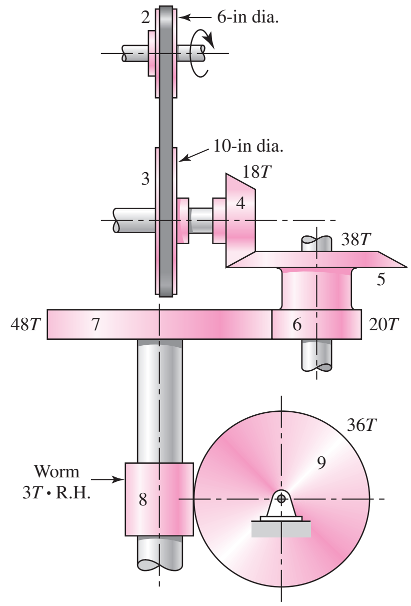
normal pressure angle of 20°. The second pair of gears in the train, gears 4 and 5, have a normal diametral pitch of 6 teeth/in, a 25° helix angle, and a normal pressure angle of 20°. The tooth numbers are: $N_2 = 14$, $N_3 = 54$, $N_4 = 16$, $N_5 = 36$. Find:
(*a*) The directions of the thrust force exerted by each gear upon its shaft
(*b*) The speed and direction of shaft *c*
(*c*) The center distance between shafts

13–15 Shaft *a* in the figure rotates at 600 rev/min in the direction shown. Find the speed and direction of rotation of shaft *d*.



Problem 13–15

13–16 The mechanism train shown consists of an assortment of gears and pulleys to drive gear 9. Pulley 2 rotates at 1200 rev/min in the direction shown. Determine the speed and direction of rotation of gear 9.



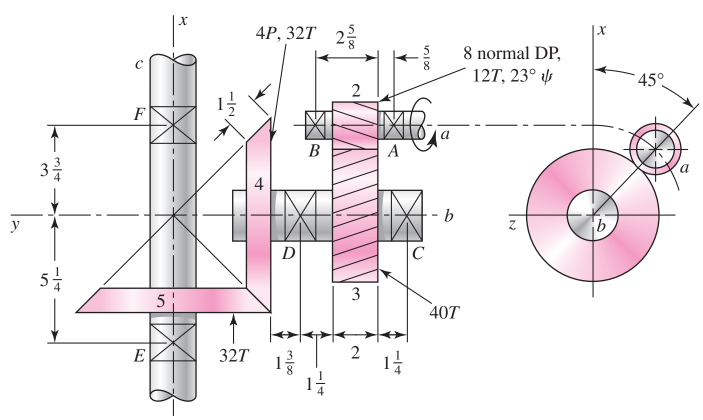
Problem 13–16

13–17 The figure shows a gear train consisting of a pair of helical gears and a pair of miter gears. The helical gears have a $17\frac{1}{2}^\circ$ normal pressure angle and a helix angle as shown. Find:

(*a*) The speed of shaft *c*

(*b*) The distance between shafts *a* and *b*

(*c*) The diameter of the miter gears

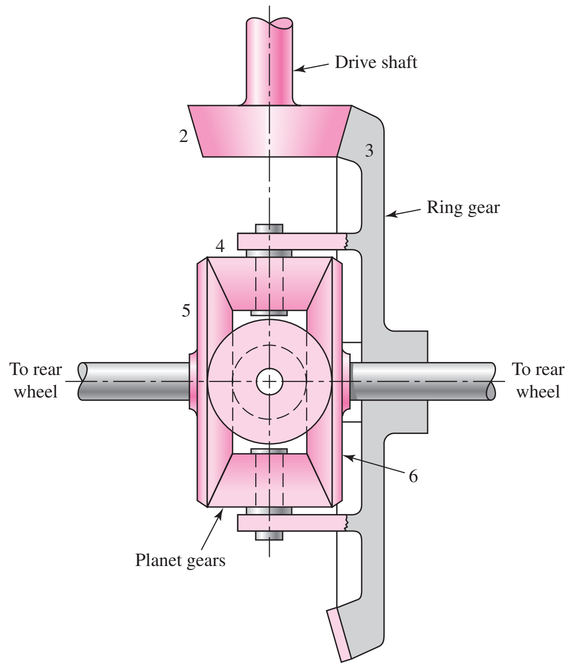


Problem 13–17
Dimensions in inches.

13–18 The tooth numbers for the automotive differential shown in the figure are $N_2 = 17$, $N_3 = 54$, $N_4 = 11$, $N_5 = N_6 = 16$. The drive shaft turns at 1200 rev/min.

(*a*) What are the wheel speeds if the car is traveling in a straight line on a good road surface?

(*b*) Suppose the right wheel is jacked up and the left wheel resting on a good road surface. What is the speed of the right wheel?



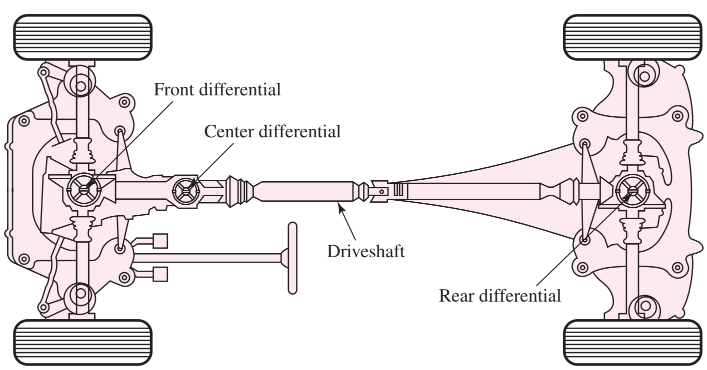
Problem 13–18

(*c*) Suppose, with a rear-wheel drive vehicle, the auto is parked with the right wheel resting on a wet icy surface. Does the answer to part (*b*) give you any hint as to what would happen if you started the car and attempted to drive on?

13–19 The figure illustrates an all-wheel drive concept using three differentials, one for the front axle, another for the rear, and the third connected to the drive shaft.

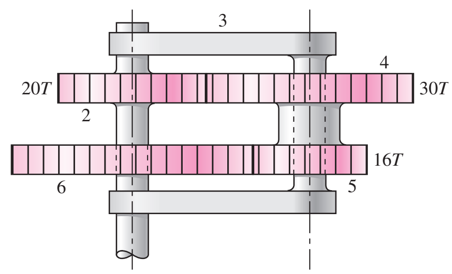
(*a*) Explain why this concept may allow greater acceleration.

(*b*) Suppose either the center of the rear differential, or both, can be locked for certain road conditions. Would either or both of these actions provide greater traction? Why?



Problem 13–19
The Audi "Quattro concept," showing the three differentials that provide permanent all-wheel drive. *(Reprinted by permission of Audi of America, Inc.)*

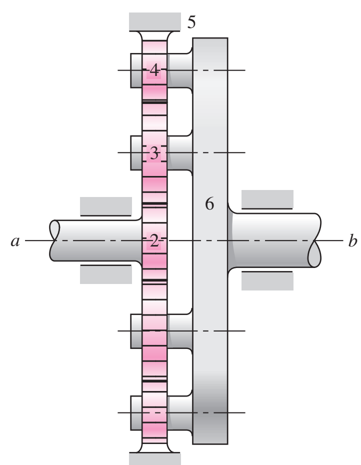
13–20 In the reverted planetary train illustrated, find the speed and direction of rotation of the arm if gear 2 is unable to rotate and gear 6 is driven at 12 rev/min in the clockwise direction.



Problem 13–20

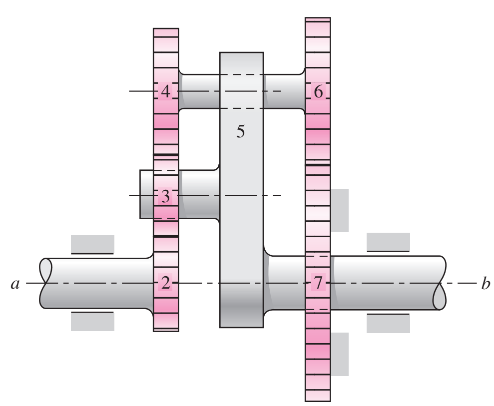
13–21 In the gear train of Prob. 13–20, let gear 2 be driven at 180 rev/min counterclockwise while gear 6 is held stationary. What is the speed and direction of rotation of the arm?

13–22 Tooth numbers for the gear train shown in the figure are $N_2 = 12$, $N_3 = 16$, and $N_4 = 12$. How many teeth must internal gear 5 have? Suppose gear 5 is fixed. What is the speed of the arm if shaft a rotates counterclockwise at 320 rev/min?



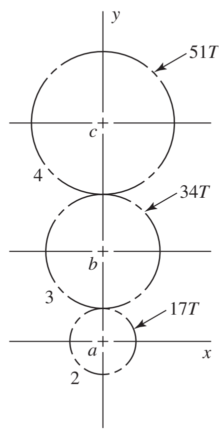
Problem 13–22

13–23 The tooth numbers for the gear train illustrated are $N_2 = 24$, $N_3 = 18$, $N_4 = 30$, $N_6 = 36$, and $N_7 = 54$. Gear 7 is fixed. If shaft b is turned through 5 revolutions, how many turns will shaft a make?



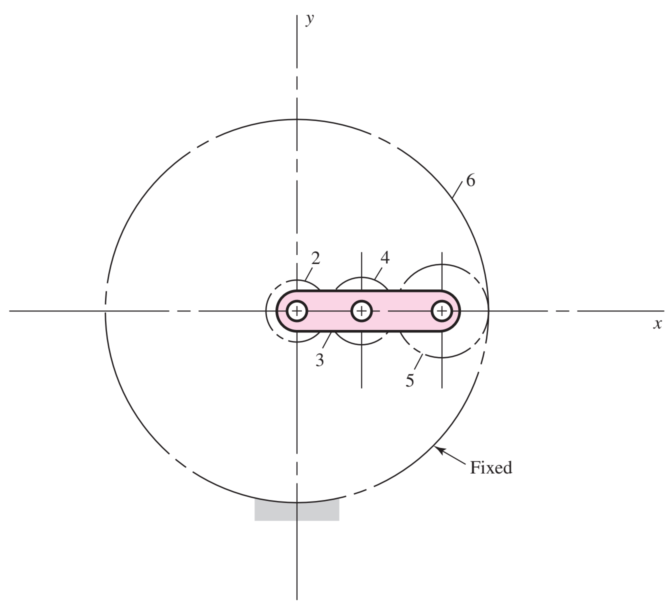
Problem 13–23

13–24 Shaft a in the figure has a power input of 75 kW at a speed of 1000 rev/min in the counterclockwise direction. The gears have a module of 5 mm and a 20° pressure angle. Gear 3 is an idler.
(a) Find the force F_{3b} that gear 3 exerts against shaft b.
(b) Find the torque T_{4c} that gear 4 exerts on shaft c.



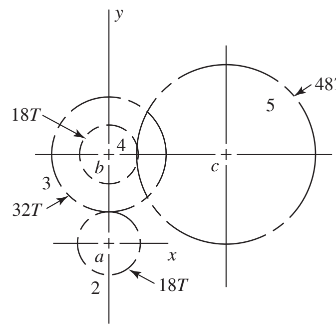
Problem 13–24

13–25 The 24T 6-pitch 20° pinion 2 shown in the figure rotates clockwise at 1000 rev/min and is driven at a power of 25 hp. Gears 4, 5, and 6 have 24, 36, and 144 teeth, respectively. What torque can arm 3 deliver to its output shaft? Draw free-body diagrams of the arm and of each gear and show all forces that act upon them.



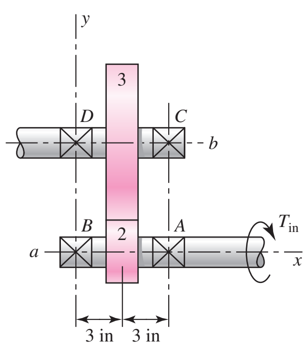
Problem 13–25

13–26 The gears shown in the figure have a diametral pitch of 2 teeth per inch and a 20° pressure angle. The pinion rotates at 1800 rev/min clockwise and transmits 200 hp through the idler pair to gear 5 on shaft c. What forces do gears 3 and 4 transmit to the idler shaft?



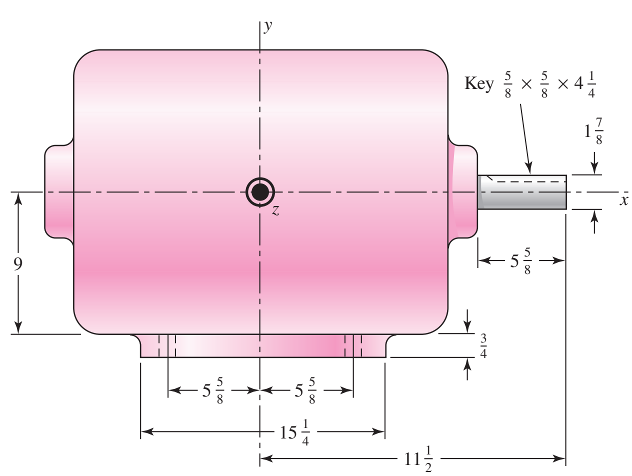
Problem 13–26

13–27 The figure shows a pair of shaft-mounted spur gears having a diametral pitch of 5 teeth/in with an 18-tooth 20° pinion driving a 45-tooth gear. The horsepower input is 32 maximum at 1800 rev/min. Find the direction and magnitude of the maximum forces acting on bearings A, B, C, and D.



Problem 13–27

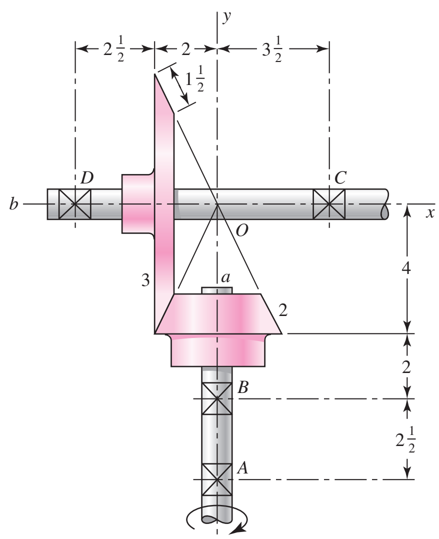
13–28 The figure shows the electric-motor frame dimensions for a 30-hp 900 rev/min motor. The frame is bolted to its support using four $\frac{3}{4}$-in bolts spaced $11\frac{1}{4}$ in apart in the view shown and 14 in apart when viewed from the end of the motor. A 4 diametral pitch 20° spur pinion having 20 teeth and a face width of 2 in is keyed to the motor shaft. This pinion drives another gear whose axis is in the same xz plane. Determine the maximum shear and tensile forces on the mounting bolts based on 200 percent overload torque. Does the direction of rotation matter?



Problem 13–28

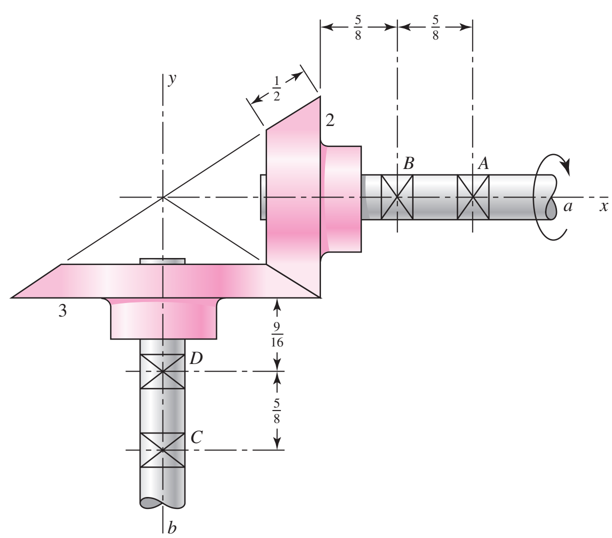
NEMA No. 364 frame; dimensions in inches. The z axis is directed out of the paper.

13–29 The figure shows a 16*T* 20° straight bevel pinion driving a 32*T* gear, and the location of the bearing centerlines. Pinion shaft *a* receives 2.5 hp at 240 rev/min. Determine the bearing reactions at *A* and *B* if *A* is to take both radial and thrust loads.



Problem 13–29
Dimensions in inches.

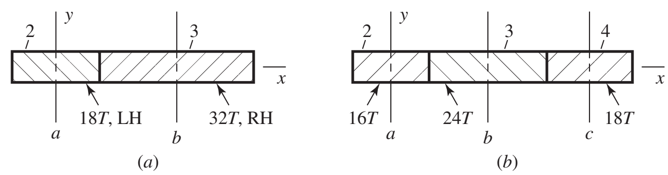
13–30 The figure shows a 10 diametral pitch 15-tooth 20° straight bevel pinion driving a 25-tooth gear. The transmitted load is 30 lbf. Find the bearing reactions at *C* and *D* on the output shaft if *D* is to take both radial and thrust loads.



Problem 13–30
Dimensions in inches.

13–31 The gears in the two trains shown in the figure have a normal diametral pitch of 5 teeth/in, a normal pressure angle of 20°, and a 30° helix angle. For both gear trains the transmitted load is 800 lbf. In part *a* the pinion rotates counterclockwise about the *y* axis. Find the force exerted by each gear in part *a* on its shaft.

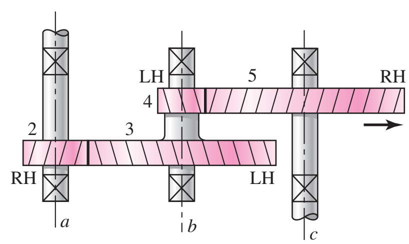
Problem 13–31



13–32 This is a continuation of Prob. 13–31. Here, you are asked to find the forces exerted by gears 2 and 3 on their shafts as shown in part *b*. Gear 2 rotates clockwise about the *y* axis. Gear 3 is an idler.

13–33 A gear train is composed of four helical gears with the three shaft axes in a single plane, as shown in the figure. The gears have a normal pressure angle of 20° and a 30° helix angle. Shaft *b* is an idler and the transmitted load acting on gear 3 is 500 lbf. The gears on shaft *b* both have a normal diametral pitch of 7 teeth/in and have 54 and 14 teeth, respectively. Find the forces exerted by gears 3 and 4 on shaft *b*.

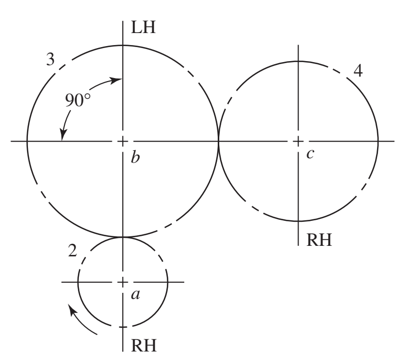
Problem 13–33



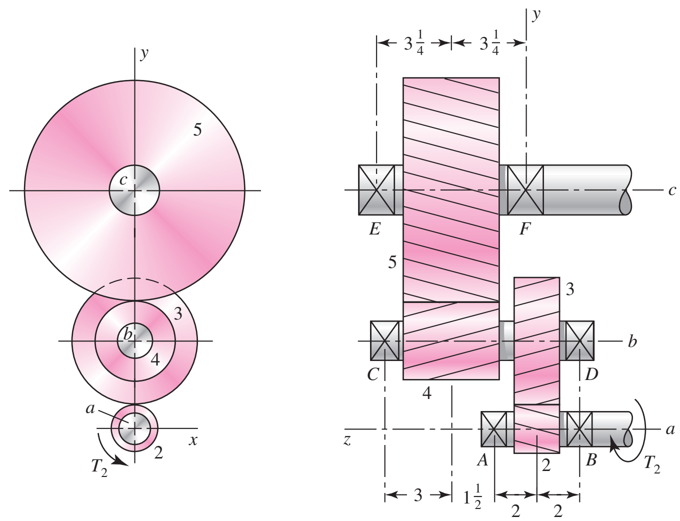
13–34 In the figure for Prob. 13–27, pinion 2 is to be a right-hand helical gear having a helix angle of 30°, a normal pressure angle of 20°, 16 teeth, and a normal diametral pitch of 6 teeth/in. A 25-hp motor drives shaft *a* at a speed of 1720 rev/min clockwise about the *x* axis. Gear 3 has 42 teeth. Find the reaction exerted by bearings *C* and *D* on shaft *b*. One of these bearings is to take both radial and thrust loads. This bearing should be selected so as to place the shaft in compression.

13–35 Gear 2, in the figure, has 16 teeth, a 20° transverse angle, a 15° helix angle, and a normal diametral pitch of 8 teeth/in. Gear 2 drives the idler on shaft *b*, which has 36 teeth. The driven gear on shaft *c* has 28 teeth. If the driver rotates at 1720 rev/min and transmits $7\frac{1}{2}$ hp, find the radial and thrust load on each shaft.

Problem 13–35



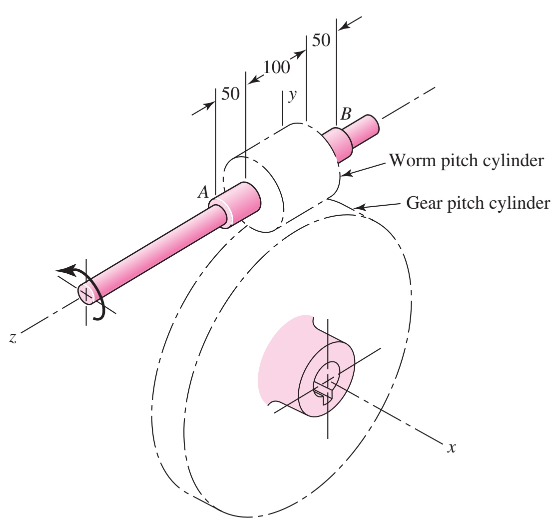
13–36 The figure shows a double-reduction helical gearset. Pinion 2 is the driver, and it receives a torque of 1200 lbf · in from its shaft in the direction shown. Pinion 2 has a normal diametral pitch of 8 teeth/in, 14 teeth, and a normal pressure angle of 20° and is cut right-handed with a helix angle



Problem 13–36
Dimensions in inches.

of 30°. The mating gear 3 on shaft *b* has 36 teeth. Gear 4, which is the driver for the second pair of gears in the train, has a normal diametral pitch of 5 teeth/in, 15 teeth, and a normal pressure angle of 20° and is cut left-handed with a helix angle of 15°. Mating gear 5 has 45 teeth. Find the magnitude and direction of the force exerted by the bearings *C* and *D* on shaft *b* if bearing *C* can take only radial load while bearing *D* is mounted to take both radial and thrust load.

13–37 A right-hand single-tooth hardened-steel (hardness not specified) worm has a catalog rating of 2000 W at 600 rev/min when meshed with a 48-tooth cast-iron gear. The axial pitch of the worm is 25 mm, the normal pressure angle is $14\frac{1}{2}^\circ$, the pitch diameter of the worm is 100 mm, and the face widths of the worm and gear are, respectively, 100 mm and 50 mm. The figure shows bearings *A* and *B* on the worm shaft symmetrically located with respect to the worm and 200 mm apart. Determine which should be the thrust bearing, and find the magnitudes and directions of the forces exerted by both bearings.



Problem 13–37
Dimensions in millimeters.

13–38 The hub diameter and projection for the gear of Prob. 13–37 are 100 and 37.5 mm, respectively. The face width of the gear is 50 mm. Locate bearings C and D on opposite sides, spacing C 10 mm from the gear on the hidden face (see figure) and D 10 mm from the hub face. Find the output torque and the magnitudes and directions of the forces exerted by the bearings on the gearshaft.

13–39 A 2-tooth left-hand worm transmits $\frac{1}{2}$ hp at 900 rev/min to a 36-tooth gear having a transverse diametral pitch of 10 teeth/in. The worm has a normal pressure angle of $14\frac{1}{2}^\circ$, a pitch diameter of $1\frac{1}{2}$ in, and a face width of $1\frac{1}{2}$ in. Use a coefficient of friction of 0.05 and find the force exerted by the gear on the worm and the torque input. For the same geometry as shown for Prob. 13–37, the worm velocity is clockwise about the z axis.

13–40 Write a computer program that will analyze a spur gear or helical-mesh gear, accepting ϕ_n, ψ, P_t, N_P, and N_G; compute m_G, d_P, d_G, p_t, p_n, p_x, and ϕ_t; and give advice as to the smallest tooth count that will allow a pinion to run with itself without interference, run with its gear, and run with a rack. Also have it give the largest tooth count possible with the intended pinion.

14 Spur and Helical Gears

Chapter Outline

This chapter is devoted primarily to analysis and design of spur and helical gears to resist bending failure of the teeth as well as pitting failure of tooth surfaces. Failure by bending will occur when the significant tooth stress equals or exceeds either the yield strength or the bending endurance strength. A surface failure occurs when the significant contact stress equals or exceeds the surface endurance strength. The first two sections present a little of the history of the analyses from which current methodology developed.

The American Gear Manufacturers Association[1] (AGMA) has for many years been the responsible authority for the dissemination of knowledge pertaining to the design and analysis of gearing. The methods this organization presents are in general use in the United States when strength and wear are primary considerations. In view of this fact it is important that the AGMA approach to the subject be presented here.

The general AGMA approach requires a great many charts and graphs—too many for a single chapter in this book. We have omitted many of these here by choosing a single pressure angle and by using only full-depth teeth. This simplification reduces the complexity but does not prevent the development of a basic understanding of the approach. Furthermore, the simplification makes possible a better development of the fundamentals and hence should constitute an ideal introduction to the use of the general AGMA method.[2] Sections 14–1 and 14–2 are elementary and serve as an examination of the foundations of the AGMA method. Table 14–1 is largely AGMA nomenclature.

14–1 The Lewis Bending Equation

Wilfred Lewis introduced an equation for estimating the bending stress in gear teeth in which the tooth form entered into the formulation. The equation, announced in 1892, still remains the basis for most gear design today.

To derive the basic Lewis equation, refer to Fig. 14–1*a*, which shows a cantilever of cross-sectional dimensions F and t, having a length l and a load W^t, uniformly distributed across the face width F. The section modulus I/c is $Ft^2/6$, and therefore the bending stress is

$$\sigma = \frac{M}{I/c} = \frac{6W^t l}{Ft^2} \tag{a}$$

Gear designers denote the components of gear-tooth forces as W_t, W_r, W_a or W^t, W^r, W^a interchangeably. The latter notation leaves room for post-subscripts essential to free-body diagrams. For instance, for gears 2 and 3 in mesh, W^t_{23} is the transmitted force of

[1] 500 Montgomery Street, Suite 350, Alexandria, VA 22314-1560.

[2] The standards ANSI/AGMA 2001-D04 (revised AGMA 2001-C95) and ANSI/AGMA 2101-D04 (metric edition of ANSI/AGMA 2001-D04), *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth,* are used in this chapter. The use of American National Standards is completely voluntary; their existence does not in any respect preclude people, whether they have approved the standards or not, from manufacturing, marketing, purchasing, or using products, processes, or procedures not conforming to the standards.

The American National Standards Institute does not develop standards and will in no circumstances give an interpretation of any American National Standard. Requests for interpretation of these standards should be addressed to the American Gear Manufacturers Association. [Tables or other self-supporting sections may be quoted or extracted in their entirety. Credit line should read: "Extracted from ANSI/AGMA Standard 2001-D04 or 2101-D04 *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*" with the permission of the publisher, American Gear Manufacturers Association, 500 Montgomery Street, Suite 350, Alexandria, Virginia 22314-1560.] The foregoing is adapted in part from the ANSI foreword to these standards.

Table 14–1

Symbols, Their Names, and Locations*

Symbol	Name	Where Found
b	Net width of face of narrowest member	Eq. (14–16)
C_e	Mesh alignment correction factor	Eq. (14–35)
C_f	Surface condition factor	Eq. (14–16)
C_H	Hardness-ratio factor	Eq. (14–18)
C_{ma}	Mesh alignment factor	Eq. (14–34)
C_{mc}	Load correction factor	Eq. (14–31)
C_{mf}	Face load-distribution factor	Eq. (14–30)
C_p	Elastic coefficient	Eq. (14–13)
C_{pf}	Pinion proportion factor	Eq. (14–32)
C_{pm}	Pinion proportion modifier	Eq. (14–33)
d	Operating pitch diameter of pinion	Ex. (14–1)
d_P	Pitch diameter, pinion	Eq. (14–22)
d_G	Pitch diameter, gear	Eq. (14–22)
E	Modulus of elasticity	Eq. (14–10)
F	Net face width of narrowest member	Eq. (14–15)
f_P	Pinion surface finish	Fig. 14–13
H	Power	Fig. 14–17
H_B	Brinell hardness	Ex. 14–3
H_{BG}	Brinell hardness of gear	Sec. 14–12
H_{BP}	Brinell hardness of pinion	Sec. 14–12
hp	Horsepower	Ex. 14–1
h_t	Gear-tooth whole depth	Sec. 14–16
I	Geometry factor of pitting resistance	Eq. (14–16)
J	Geometry factor for bending strength	Eq. (14–15)
K	Contact load factor for pitting resistance	Eq. (6–65)
K_B	Rim-thickness factor	Eq. (14–40)
K_f	Fatigue stress-concentration factor	Eq. (14–9)
K_m	Load-distribution factor	Eq. (14–30)
K_o	Overload factor	Eq. (14–15)
K_R	Reliability factor	Eq. (14–17)
K_s	Size factor	Sec. 14–10
K_T	Temperature factor	Eq. (14–17)
K_v	Dynamic factor	Eq. (14–27)
m	Metric module	Eq. (14–15)
m_B	Backup ratio	Eq. (14–39)
m_G	Gear ratio (never less than 1)	Eq. (14–22)
m_N	Load-sharing ratio	Eq. (14–21)
N	Number of stress cycles	Fig. 14–14
N_G	Number of teeth on gear	Eq. (14–22)
N_P	Number of teeth on pinion	Eq. (14–22)
n	Speed	Ex. 14–1

(Continued)

Table 14–1

Symbols, Their Names, and Locations* (*Continued*)

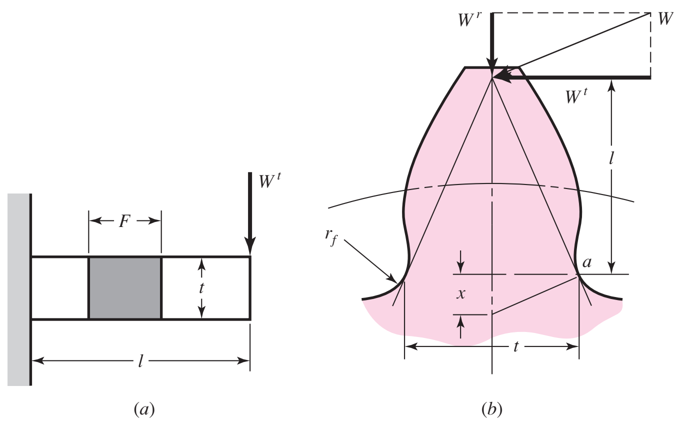
Symbol	Name	Where Found
n_P	Pinion speed	Ex. 14–4
P	Diametral pitch	Eq. (14–2)
P_d	Diametral pitch of pinion	Eq. (14–15)
p_N	Normal base pitch	Eq. (14–24)
p_n	Normal circular pitch	Eq. (14–24)
p_x	Axial pitch	Eq. (14–19)
Q_v	Transmission accuracy level number	Eq. (14–29)
R	Reliability	Eq. (14–38)
R_a	Root-mean-squared roughness	Fig. 14–13
r_f	Tooth fillet radius	Fig. 14–1
r_G	Pitch-circle radius, gear	In standard
r_P	Pitch-circle radius, pinion	In standard
r_{bP}	Pinion base-circle radius	Eq. (14–25)
r_{bG}	Gear base-circle radius	Eq. (14–25)
S_C	Buckingham surface endurance strength	Ex. 14–3
S_c	AGMA surface endurance strength	Eq. (14–18)
S_t	AGMA bending strength	Eq. (14–17)
S	Bearing span	Fig. 14–10
S_1	Pinion offset from center span	Fig. 14–10
S_F	Safety factor—bending	Eq. (14–41)
S_H	Safety factor—pitting	Eq. (14–42)
W^t or $W_t^\dagger$	Transmitted load	Fig. 14–1
Y_N	Stress cycle factor for bending strength	Fig. 14–14
Z_N	Stress cycle factor for pitting resistance	Fig. 14–15
β	Exponent	Eq. (14–44)
σ	Bending stress	Eq. (14–2)
σ_C	Contact stress from Hertzian relationships	Eq. (14–14)
σ_c	Contact stress from AGMA relationships	Eq. (14–16)
σ_{all}	Allowable bending stress	Eq. (14–17)
$\sigma_{c,\text{all}}$	Allowable contact stress, AGMA	Eq. (14–18)
ϕ	Pressure angle	Eq. (14–12)
ϕ_t	Transverse pressure angle	Eq. (14–23)
ψ	Helix angle at standard pitch diameter	Ex. 14–5

*Because ANSI/AGMA 2001-C95 introduced a significant amount of new nomenclature, and continued in ANSI/AGMA 2001-D04, this summary and references are provided for use until the reader's vocabulary has grown.

†See preference rationale following Eq. (*a*), Sec. 14–1.

body 2 on body 3, and W_{32}^t is the transmitted force of body 3 on body 2. When working with double- or triple-reduction speed reducers, this notation is compact and essential to clear thinking. Since gear-force components rarely take exponents, this causes no complication. Pythagorean combinations, if necessary, can be treated with parentheses or avoided by expressing the relations trigonometrically.

Figure 14–1



Referring now to Fig. 14–1*b*, we assume that the maximum stress in a gear tooth occurs at point *a*. By similar triangles, you can write

$$\frac{t/2}{x} = \frac{l}{t/2} \qquad \text{or} \qquad x = \frac{t^2}{4l} \tag{b}$$

By rearranging Eq. (*a*),

$$\sigma = \frac{6W^t l}{Ft^2} = \frac{W^t}{F}\frac{1}{t^2/6l} = \frac{W^t}{F}\frac{1}{t^2/4l}\frac{1}{\frac{4}{6}} \tag{c}$$

If we now substitute the value of x from Eq. (*b*) in Eq. (*c*) and multiply the numerator and denominator by the circular pitch p, we find

$$\sigma = \frac{W^t p}{F\left(\frac{2}{3}\right)xp} \tag{d}$$

Letting $y = 2x/3p$, we have

$$\sigma = \frac{W^t}{Fpy} \tag{14–1}$$

This completes the development of the original Lewis equation. The factor y is called the *Lewis form factor,* and it may be obtained by a graphical layout of the gear tooth or by digital computation.

In using this equation, most engineers prefer to employ the diametral pitch in determining the stresses. This is done by substituting $P = \pi/p$ and $Y = \pi y$ in Eq. (14–1). This gives

$$\sigma = \frac{W^t P}{FY} \tag{14–2}$$

where

$$Y = \frac{2xP}{3} \tag{14–3}$$

The use of this equation for Y means that only the bending of the tooth is considered and that the compression due to the radial component of the force is neglected. Values of Y obtained from this equation are tabulated in Table 14–2.

Table 14–2

Values of the Lewis Form Factor Y (These Values Are for a Normal Pressure Angle of 20°, Full-Depth Teeth, and a Diametral Pitch of Unity in the Plane of Rotation)

Number of Teeth	Y	Number of Teeth	Y
12	0.245	28	0.353
13	0.261	30	0.359
14	0.277	34	0.371
15	0.290	38	0.384
16	0.296	43	0.397
17	0.303	50	0.409
18	0.309	60	0.422
19	0.314	75	0.435
20	0.322	100	0.447
21	0.328	150	0.460
22	0.331	300	0.472
24	0.337	400	0.480
26	0.346	Rack	0.485

The use of Eq. (14–3) also implies that the teeth do not share the load and that the greatest force is exerted at the tip of the tooth. But we have already learned that the contact ratio should be somewhat greater than unity, say about 1.5, to achieve a quality gearset. If, in fact, the gears are cut with sufficient accuracy, the tip-load condition is not the worst, because another pair of teeth will be in contact when this condition occurs. Examination of run-in teeth will show that the heaviest loads occur near the middle of the tooth. Therefore the maximum stress probably occurs while a single pair of teeth is carrying the full load, at a point where another pair of teeth is just on the verge of coming into contact.

Dynamic Effects

When a pair of gears is driven at moderate or high speed and noise is generated, it is certain that dynamic effects are present. One of the earliest efforts to account for an increase in the load due to velocity employed a number of gears of the same size, material, and strength. Several of these gears were tested to destruction by meshing and loading them at zero velocity. The remaining gears were tested to destruction at various pitch-line velocities. For example, if a pair of gears failed at 500 lbf tangential load at zero velocity and at 250 lbf at velocity V_1, then a *velocity factor,* designated K_v, of 2 was specified for the gears at velocity V_1. Then another, identical, pair of gears running at a pitch-line velocity V_1 could be assumed to have a load equal to twice the tangential or transmitted load.

Note that the definition of dynamic factor K_v has been altered. AGMA standards ANSI/AGMA 2001-D04 and 2101-D04 contain this caution:

> Dynamic factor K_v has been redefined as the reciprocal of that used in previous AGMA standards. It is now greater than 1.0. In earlier AGMA standards it was less than 1.0.

Care must be taken in referring to work done prior to this change in the standards.

In the nineteenth century, Carl G. Barth first expressed the velocity factor, and in terms of the current AGMA standards, they are represented as

$$K_v = \frac{600 + V}{600} \qquad \text{(cast iron, cast profile)} \tag{14–4a}$$

$$K_v = \frac{1200 + V}{1200} \qquad \text{(cut or milled profile)} \tag{14–4b}$$

where V is the pitch-line velocity in feet per minute. It is also quite probable, because of the date that the tests were made, that the tests were conducted on teeth having a cycloidal profile instead of an involute profile. Cycloidal teeth were in general use in the nineteenth century because they were easier to cast than involute teeth. Equation (14–4*a*) is called the *Barth equation*. The Barth equation is often modified into Eq. (14–4*b*), for cut or milled teeth. Later AGMA added

$$K_v = \frac{50 + \sqrt{V}}{50} \qquad \text{(hobbed or shaped profile)} \tag{14–5a}$$

$$K_v = \sqrt{\frac{78 + \sqrt{V}}{78}} \qquad \text{(shaved or ground profile)} \tag{14–5b}$$

In SI units, Eqs. (14–4*a*) through (14–5*b*) become

$$K_v = \frac{3.05 + V}{3.05} \qquad \text{(cast iron, cast profile)} \tag{14–6a}$$

$$K_v = \frac{6.1 + V}{6.1} \qquad \text{(cut or milled profile)} \tag{14–6b}$$

$$K_v = \frac{3.56 + \sqrt{V}}{3.56} \qquad \text{(hobbed or shaped profile)} \tag{14–6c}$$

$$K_v = \sqrt{\frac{5.56 + \sqrt{V}}{5.56}} \qquad \text{(shaved or ground profile)} \tag{14–6d}$$

where V is in meters per second (m/s).

Introducing the velocity factor into Eq. (14–2) gives

$$\sigma = \frac{K_v W^t P}{FY} \tag{14–7}$$

The metric version of this equation is

$$\sigma = \frac{K_v W^t}{FmY} \tag{14–8}$$

where the face width F and the module m are both in millimeters (mm). Expressing the tangential component of load W^t in newtons (N) then results in stress units of megapascals (MPa).

As a general rule, spur gears should have a face width F from 3 to 5 times the circular pitch p.

Equations (14–7) and (14–8) are important because they form the basis for the AGMA approach to the bending strength of gear teeth. They are in general use for estimating the capacity of gear drives when life and reliability are not important considerations. The equations can be useful in obtaining a preliminary estimate of gear sizes needed for various applications.

EXAMPLE 14–1

A stock spur gear is available having a diametral pitch of 8 teeth/in, a $1\frac{1}{2}$-in face, 16 teeth, and a pressure angle of 20° with full-depth teeth. The material is AISI 1020 steel in as-rolled condition. Use a design factor of $n_d = 3$ to rate the horsepower output of the gear corresponding to a speed of 1200 rev/m and moderate applications.

Solution

The term *moderate applications* seems to imply that the gear can be rated by using the yield strength as a criterion of failure. From Table A–20, we find $S_{ut} = 55$ kpsi and $S_y = 30$ kpsi. A design factor of 3 means that the allowable bending stress is $30/3 = 10$ kpsi. The pitch diameter is $N/P = 16/8 = 2$ in, so the pitch-line velocity is

$$V = \frac{\pi dn}{12} = \frac{\pi(2)1200}{12} = 628 \text{ ft/min}$$

The velocity factor from Eq. (14–4*b*) is found to be

$$K_v = \frac{1200 + V}{1200} = \frac{1200 + 628}{1200} = 1.52$$

Table 14–2 gives the form factor as $Y = 0.296$ for 16 teeth. We now arrange and substitute in Eq. (14–7) as follows:

$$W^t = \frac{FY\sigma_{\text{all}}}{K_v P} = \frac{1.5(0.296)10\,000}{1.52(8)} = 365 \text{ lbf}$$

The horsepower that can be transmitted is

Answer

$$hp = \frac{W^t V}{33\,000} = \frac{365(628)}{33\,000} = 6.95 \text{ hp}$$

It is important to emphasize that this is a rough estimate, and that this approach must not be used for important applications. The example is intended to help you understand some of the fundamentals that will be involved in the AGMA approach.

EXAMPLE 14–2

Estimate the horsepower rating of the gear in the previous example based on obtaining an infinite life in bending.

Solution

The rotating-beam endurance limit is estimated from Eq. (6–8)

$$S'_e = 0.5S_{ut} = 0.5(55) = 27.5 \text{ kpsi}$$

To obtain the surface finish Marin factor k_a we refer to Table 6–3 for machined surface, finding $a = 2.70$ and $b = -0.265$. Then Eq. (6–19) gives the surface finish Marin factor k_a as

$$k_a = aS_{ut}^b = 2.70(55)^{-0.265} = 0.934$$

The next step is to estimate the size factor k_b. From Table 13–1, the sum of the addendum and dedendum is

$$l = \frac{1}{P} + \frac{1.25}{P} = \frac{1}{8} + \frac{1.25}{8} = 0.281 \text{ in}$$

The tooth thickness t in Fig. 14–1b is given in Sec. 14–1 [Eq. (b)] as $t = (4lx)^{1/2}$ when $x = 3Y/(2P)$ from Eq. (14–3). Therefore, since from Ex. 14–1 $Y = 0.296$ and $P = 8$,

$$x = \frac{3Y}{2P} = \frac{3(0.296)}{2(8)} = 0.0555 \text{ in}$$

then

$$t = (4lx)^{1/2} = [4(0.281)0.0555]^{1/2} = 0.250 \text{ in}$$

We have recognized the tooth as a cantilever beam of rectangular cross section, so the equivalent rotating-beam diameter must be obtained from Eq. (6–25):

$$d_e = 0.808(hb)^{1/2} = 0.808(Ft)^{1/2} = 0.808[1.5(0.250)]^{1/2} = 0.495 \text{ in}$$

Then, Eq. (6–20) gives k_b as

$$k_b = \left(\frac{d_e}{0.30}\right)^{-0.107} = \left(\frac{0.495}{0.30}\right)^{-0.107} = 0.948$$

The load factor k_c from Eq. (6–26) is unity. With no information given concerning temperature and reliability we will set $k_d = k_e = 1$.

Two effects are used to evaluate the miscellaneous-effects Marin factor k_f. The first of these is the effect of one-way bending. In general, a gear tooth is subjected only to one-way bending. Exceptions include idler gears and gears used in reversing mechanisms.

For one-way bending the steady and alternating stress components are $\sigma_a = \sigma_m = \sigma/2$ where σ is the largest repeatedly applied bending stress as given in Eq. (14–7). If a material exhibited a Goodman failure locus,

$$\frac{S_a}{S'_e} + \frac{S_m}{S_{ut}} = 1$$

Since S_a and S_m are equal for one-way bending, we substitute S_a for S_m and solve the preceding equation for S_a, giving

$$S_a = \frac{S'_e S_{ut}}{S'_e + S_{ut}}$$

Now replace S_a with $\sigma/2$, and in the denominator replace S'_e with $0.5S_{ut}$ to obtain

$$\sigma = \frac{2S'_e S_{ut}}{0.5S_{ut} + S_{ut}} = \frac{2S'_e}{0.5 + 1} = 1.33S'_e$$

Now $k_f = \sigma/S'_e = 1.33S'_e/S'_e = 1.33$. However, a Gerber fatigue locus gives mean values of

$$\frac{S_a}{S'_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$$

Setting $S_a = S_m$ and solving the quadratic in S_a gives

$$S_a = \frac{S_{ut}^2}{2S_e'}\left(-1 + \sqrt{1 + \frac{4S_e'^2}{S_{ut}^2}}\right)$$

Setting $S_a = \sigma/2$, $S_{ut} = S_e'/0.5$ gives

$$\sigma = \frac{S_e'}{0.5^2}\left[-1 + \sqrt{1 + 4(0.5)^2}\right] = 1.66S_e'$$

and $k_f = \sigma/S_e' = 1.66$. Since a Gerber locus runs in and among fatigue data and Goodman does not, we will use $k_f = 1.66$.

The second effect to be accounted for in using the miscellaneous-effects Marin factor k_f is stress concentration, for which we will use our fundamentals from Chap. 6. For a 20° full-depth tooth the radius of the root fillet is denoted r_f, where

$$r_f = \frac{0.300}{P} = \frac{0.300}{8} = 0.0375 \text{ in}$$

From Fig. A–15–6

$$\frac{r}{d} = \frac{r_f}{t} = \frac{0.0375}{0.250} = 0.15$$

Since $D/d = \infty$, we approximate with $D/d = 3$, giving $K_t = 1.68$. From Fig. 6–20, $q = 0.62$. From Eq. (6–32)

$$K_f = 1 + (0.62)(1.68 - 1) = 1.42$$

The miscellaneous-effects Marin factor for stress concentration can be expressed as

$$k_f = \frac{1}{K_f} = \frac{1}{1.42} = 0.704$$

The final value of k_f is the product of the two k_f factors, that is, $1.66(0.704) = 1.17$. The Marin equation for the fully corrected endurance strength is

$$\begin{aligned} S_e &= k_a k_b k_c k_d k_e k_f S_e' \\ &= 0.934(0.948)(1)(1)(1)1.17(27.5) = 28.5 \text{ kpsi} \end{aligned}$$

For a design factor of $n_d = 3$, as used in Ex. 14–1, applied to the load or strength, the allowable bending stress is

$$\sigma_{\text{all}} = \frac{S_e}{n_d} = \frac{28.5}{3} = 9.5 \text{ kpsi}$$

The transmitted load W^t is

$$W^t = \frac{FY\sigma_{\text{all}}}{K_v P} = \frac{1.5(0.296)9\,500}{1.52(8)} = 347 \text{ lbf}$$

and the power is, with $V = 628$ ft/min from Ex. 14–1,

$$hp = \frac{W^t V}{33\,000} = \frac{347(628)}{33\,000} = 6.6 \text{ hp}$$

Again, it should be emphasized that these results should be accepted *only* as preliminary estimates to alert you to the nature of bending in gear teeth.

In Ex. 14–2 our resources (Fig. A–15–6) did not directly address stress concentration in gear teeth. A photoelastic investigation by Dolan and Broghamer reported in 1942 constitutes a primary source of information on stress concentration.[3] Mitchiner and Mabie[4] interpret the results in term of fatigue stress-concentration factor K_f as

$$K_f = H + \left(\frac{t}{r}\right)^L \left(\frac{t}{l}\right)^M \tag{14–9}$$

where

$$H = 0.34 - 0.458\,366\,2\phi$$
$$L = 0.316 - 0.458\,366\,2\phi$$
$$M = 0.290 + 0.458\,366\,2\phi$$
$$r = \frac{(b - r_f)^2}{(d/2) + b - r_f}$$

In these equations l and t are from the layout in Fig. 14–1, ϕ is the pressure angle, r_f is the fillet radius, b is the dedendum, and d is the pitch diameter. It is left as an exercise for the reader to compare K_f from Eq. (14–9) with the results of using the approximation of Fig. A–15–6 in Ex. 14–2.

14–2 Surface Durability

In this section we are interested in the failure of the surfaces of gear teeth, which is generally called *wear*. *Pitting,* as explained in Sec. 6–16, is a surface fatigue failure due to many repetitions of high contact stresses. Other surface failures are *scoring,* which is a lubrication failure, and *abrasion,* which is wear due to the presence of foreign material.

To obtain an expression for the surface-contact stress, we shall employ the Hertz theory. In Eq. (3–74) it was shown that the contact stress between two cylinders may be computed from the equation

$$p_{\max} = \frac{2F}{\pi b l} \tag{a}$$

where $p_{\max}$ = largest surface pressure
F = force pressing the two cylinders together
l = length of cylinders

and half-width b is obtained from Eq. (3–73):

$$b = \left\{ \frac{2F}{\pi l} \frac{\left[\left(1 - \nu_1^2\right)/E_1\right] + \left[\left(1 - \nu_2^2\right)/E_2\right]}{(1/d_1) + (1/d_2)} \right\}^{1/2} \tag{14–10}$$

where ν_1, ν_2, E_1, and E_2 are the elastic constants and d_1 and d_2 are the diameters, respectively, of the two contacting cylinders.

To adapt these relations to the notation used in gearing, we replace F by $W^t/\cos\phi$, d by $2r$, and l by the face width F. With these changes, we can substitute the value of b

[3] T. J. Dolan and E. I. Broghamer, *A Photoelastic Study of the Stresses in Gear Tooth Fillets,* Bulletin 335, Univ. Ill. Exp. Sta., March 1942, See also W. D. Pilkey, *Peterson's Stress Concentration Factors,* 2nd ed., John Wiley & Sons, New York, 1997, pp. 383–385, 412–415.

[4] R. G. Mitchiner and H. H. Mabie, "Determination of the Lewis Form Factor and the AGMA Geometry Factor J of External Spur Gear Teeth," *J. Mech. Des.,* Vol. 104, No. 1, Jan. 1982, pp. 148–158.

as given by Eq. (14–10) in Eq. (*a*). Replacing $p_{\max}$ by σ_C, the *surface compressive stress (Hertzian stress)* is found from the equation

$$\sigma_C^2 = \frac{W^t}{\pi F \cos\phi} \frac{(1/r_1) + (1/r_2)}{\left[\left(1-\nu_1^2\right)/E_1\right] + \left[\left(1-\nu_2^2\right)/E_2\right]} \tag{14–11}$$

where r_1 and r_2 are the instantaneous values of the radii of curvature on the pinion- and gear-tooth profiles, respectively, at the point of contact. By accounting for load sharing in the value of W^t used, Eq. (14–11) can be solved for the Hertzian stress for any or all points from the beginning to the end of tooth contact. Of course, pure rolling exists only at the pitch point. Elsewhere the motion is a mixture of rolling and sliding. Equation (14–11) does not account for any sliding action in the evaluation of stress. We note that AGMA uses μ for Poisson's ratio instead of ν as is used here.

We have already noted that the first evidence of wear occurs near the pitch line. The radii of curvature of the tooth profiles at the pitch point are

$$r_1 = \frac{d_P \sin\phi}{2} \qquad r_2 = \frac{d_G \sin\phi}{2} \tag{14–12}$$

where ϕ is the pressure angle and d_P and d_G are the pitch diameters of the pinion and gear, respectively.

Note, in Eq. (14–11), that the denominator of the second group of terms contains four elastic constants, two for the pinion and two for the gear. As a simple means of combining and tabulating the results for various combinations of pinion and gear materials, AGMA defines an *elastic coefficient* C_p by the equation

$$C_p = \left[\frac{1}{\pi\left(\dfrac{1-\nu_P^2}{E_P} + \dfrac{1-\nu_G^2}{E_G}\right)}\right]^{1/2} \tag{14–13}$$

With this simplification, and the addition of a velocity factor K_v, Eq. (14–11) can be written as

$$\sigma_C = -C_p\left[\frac{K_v W^t}{F\cos\phi}\left(\frac{1}{r_1} + \frac{1}{r_2}\right)\right]^{1/2} \tag{14–14}$$

where the sign is negative because σ_C is a compressive stress.

EXAMPLE 14–3

The pinion of Examples 14–1 and 14–2 is to be mated with a 50-tooth gear manufactured of ASTM No. 50 cast iron. Using the tangential load of 382 lbf, estimate the factor of safety of the drive based on the possibility of a surface fatigue failure.

Solution

From Table A–5 we find the elastic constants to be $E_P = 30$ Mpsi, $\nu_P = 0.292$, $E_G = 14.5$ Mpsi, $\nu_G = 0.211$. We substitute these in Eq. (14–13) to get the elastic coefficient as

$$C_p = \left\{\frac{1}{\pi\left[\dfrac{1-(0.292)^2}{30(10^6)} + \dfrac{1-(0.211)^2}{14.5(10^6)}\right]}\right\}^{1/2} = 1817$$

From Example 14–1, the pinion pitch diameter is $d_P = 2$ in. The value for the gear is $d_G = 50/8 = 6.25$ in. Then Eq. (14–12) is used to obtain the radii of curvature at the pitch points. Thus

$$r_1 = \frac{2 \sin 20^\circ}{2} = 0.342 \text{ in} \qquad r_2 = \frac{6.25 \sin 20^\circ}{2} = 1.069 \text{ in}$$

The face width is given as $F = 1.5$ in. Use $K_v = 1.52$ from Example 14–1. Substituting all these values in Eq. (14–14) with $\phi = 20^\circ$ gives the contact stress as

$$\sigma_C = -1817 \left[\frac{1.52(380)}{1.5 \cos 20^\circ} \left(\frac{1}{0.342} + \frac{1}{1.069} \right) \right]^{1/2} = -72\,400 \text{ psi}$$

The surface endurance strength of cast iron can be estimated from

$$S_C = 0.32 H_B \text{ kpsi}$$

for 10^8 cycles, where S_C is in kpsi. Table A–24 gives $H_B = 262$ for ASTM No. 50 cast iron. Therefore $S_C = 0.32(262) = 83.8$ kpsi. Contact stress is not linear with transmitted load [see Eq. (14–14)]. If the factor of safety is defined as the loss-of-function load divided by the imposed load, then the ratio of loads is the ratio of stresses squared. In other words,

$$n = \frac{\text{loss-of-function load}}{\text{imposed load}} = \frac{S_C^2}{\sigma_C^2} = \left(\frac{83.8}{72.4} \right)^2 = 1.34$$

One is free to define factor of safety as S_C/σ_C. Awkwardness comes when one compares the factor of safety in bending fatigue with the factor of safety in surface fatigue for a particular gear. Suppose the factor of safety of this gear in bending fatigue is 1.20 and the factor of safety in surface fatigue is 1.34 as above. The threat, since 1.34 is greater than 1.20, is in bending fatigue since both numbers are based on load ratios. If the factor of safety in surface fatigue is based on $S_C/\sigma_C = \sqrt{1.34} = 1.16$, then 1.20 is greater than 1.16, but the threat is not from surface fatigue. The surface fatigue factor of safety can be defined either way. One way has the burden of requiring a squared number before numbers that instinctively seem comparable can be compared.

In addition to the dynamic factor K_v already introduced, there are transmitted load excursions, nonuniform distribution of the transmitted load over the tooth contact, and the influence of rim thickness on bending stress. Tabulated strength values can be means, ASTM minimums, or of unknown heritage. In surface fatigue there are no endurance limits. Endurance strengths have to be qualified as to corresponding cycle count, and the slope of the *S-N* curve needs to be known. In bending fatigue there is a definite change in slope of the *S-N* curve near 10^6 cycles, but some evidence indicates that an endurance limit does not exist. Gearing experience leads to cycle counts of 10^{11} or more. Evidence of diminishing endurance strengths in bending have been included in AGMA methodology.

14–3 AGMA Stress Equations

Two fundamental stress equations are used in the AGMA methodology, one for bending stress and another for pitting resistance (contact stress). In AGMA terminology, these are called *stress numbers,* as contrasted with actual applied stresses, and are designated by a lowercase letter s instead of the Greek lower case σ we have used in this book (and shall continue to use). The fundamental equations are

$$\sigma = \begin{cases} W^t K_o K_v K_s \dfrac{P_d}{F} \dfrac{K_m K_B}{J} & \text{(U.S. customary units)} \\ W^t K_o K_v K_s \dfrac{1}{b m_t} \dfrac{K_H K_B}{Y_J} & \text{(SI units)} \end{cases} \tag{14–15}$$

where for U.S. customary units (SI units),

W^t is the tangential transmitted load, lbf (N)
K_o is the overload factor
K_v is the dynamic factor
K_s is the size factor
P_d is the transverse diameteral pitch
F (b) is the face width of the narrower member, in (mm)
K_m (K_H) is the load-distribution factor
K_B is the rim-thickness factor
J (Y_J) is the geometry factor for bending strength (which includes root fillet stress-concentration factor K_f)
(m_t) is the transverse metric module

Before you try to digest the meaning of all these terms in Eq. (14–15), view them as advice concerning items the designer should consider *whether he or she follows the voluntary standard or not*. These items include issues such as

- Transmitted load magnitude
- Overload
- Dynamic augmentation of transmitted load
- Size
- Geometry: pitch and face width
- Distribution of load across the teeth
- Rim support of the tooth
- Lewis form factor and root fillet stress concentration

The fundamental equation for pitting resistance (contact stress) is

$$\sigma_c = \begin{cases} C_p \sqrt{W^t K_o K_v K_s \dfrac{K_m}{d_P F} \dfrac{C_f}{I}} & \text{(U.S. customary units)} \\ Z_E \sqrt{W^t K_o K_v K_s \dfrac{K_H}{d_{w1} b} \dfrac{Z_R}{Z_I}} & \text{(SI units)} \end{cases} \tag{14–16}$$

where W^t, K_o, K_v, K_s, K_m, F, and b are the same terms as defined for Eq. (14–15). For U.S. customary units (SI units), the additional terms are

C_p (Z_E) is an elastic coefficient, $\sqrt{\text{lbf/in}^2}$ ($\sqrt{\text{N/mm}^2}$)
C_f (Z_R) is the surface condition factor
d_P (d_{w1}) is the pitch diameter of the *pinion,* in (mm)
I (Z_I) is the geometry factor for pitting resistance

The evaluation of all these factors is explained in the sections that follow. The development of Eq. (14–16) is clarified in the second part of Sec. 14–5.

14–4 AGMA Strength Equations

Instead of using the term *strength,* AGMA uses data termed *allowable stress numbers* and designates these by the symbols s_{at} and s_{ac}. It will be less confusing here if we continue the practice in this book of using the uppercase letter S to designate strength and the lowercase Greek letters σ and τ for stress. To make it perfectly clear we shall use the term *gear strength* as a replacement for the phrase *allowable stress numbers* as used by AGMA.

Following this convention, values for *gear bending strength,* designated here as S_t, are to be found in Figs. 14–2, 14–3, and 14–4, and in Tables 14–3 and 14–4. Since gear strengths are not identified with other strengths such as S_{ut}, S_e, or S_y as used elsewhere in this book, their use should be restricted to gear problems.

In this approach the strengths are modified by various factors that produce limiting values of the bending stress and the contact stress.

Figure 14–2

Allowable bending stress number for through-hardened steels. The SI equations are $S_t = 0.533H_B + 88.3$ MPa, grade 1, and $S_t = 0.703H_B + 113$ MPa, grade 2. *(Source: ANSI/AGMA 2001-D04 and 2101-D04.)*

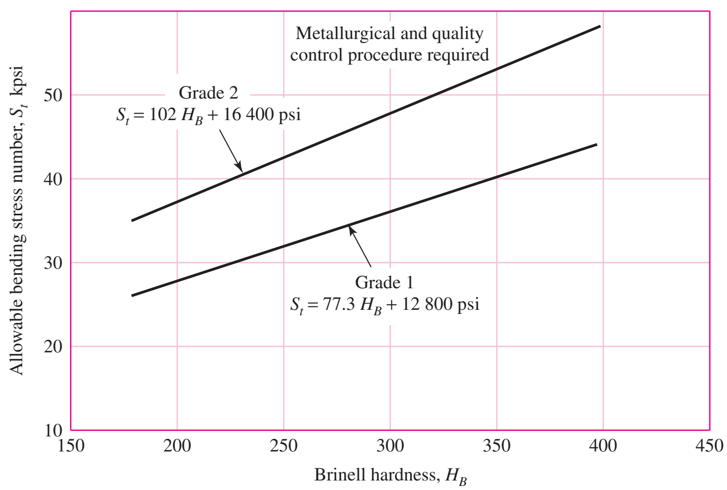
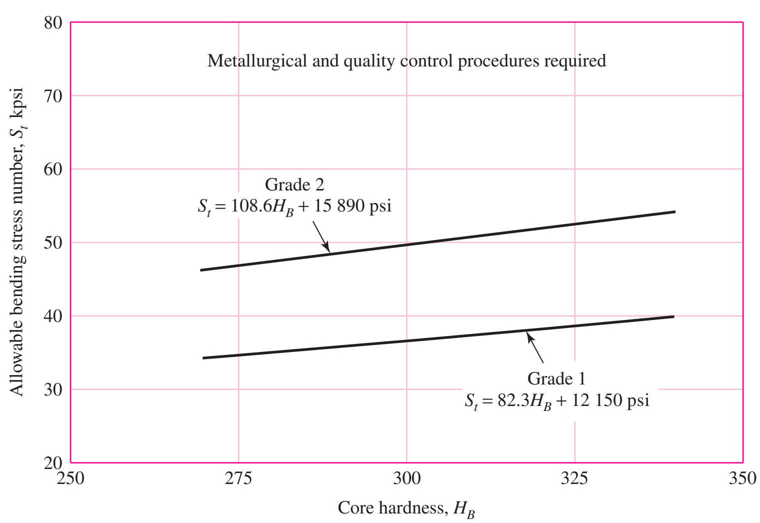


Figure 14–3

Allowable bending stress number for nitrided through-hardened steel gears (i.e., AISI 4140, 4340), S_t. The SI equations are $S_t = 0.568H_B + 83.8$ MPa, grade 1, and $S_t = 0.749H_B + 110$ MPa, grade 2. *(Source: ANSI/AGMA 2001-D04 and 2101-D04.)*



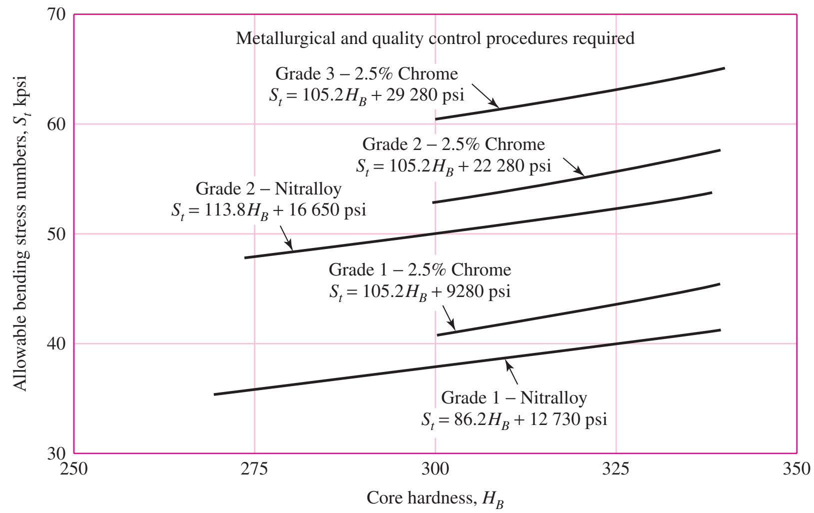


Figure 14–4

Allowable bending stress numbers for nitriding steel gears S_t. The SI equations are $S_t = 0.594H_B + 87.76$ MPa Nitralloy grade 1 $S_t = 0.784H_B + 114.81$ MPa Nitralloy grade 2 $S_t = 0.7255H_B + 63.89$ MPa 2.5% chrome, grade 1 $S_t = 0.7255H_B + 153.63$ MPa 2.5% chrome, grade 2 $S_t = 0.7255H_B + 201.91$ MPa 2.5% chrome, grade 3 *(Source: ANSI/AGMA 2001-D04,2101-D04.)*

Table 14–3

Repeatedly Applied Bending Strength S_t at 10^7 Cycles and 0.99 Reliability for Steel Gears

Source: ANSI/AGMA 2001-D04.

Material Designation	Heat Treatment	Minimum Surface Hardness[1]	Allowable Bending Stress Number S_t,[2] psi Grade 1	Grade 2	Grade 3
Steel[3]	Through-hardened	See Fig. 14–2	See Fig. 14–2	See Fig. 14–2	—
	Flame[4] or induction hardened[4] with type A pattern[5]	See Table 8*	45 000	55 000	—
	Flame[4] or induction hardened[4] with type B pattern[5]	See Table 8*	22 000	22 000	—
	Carburized and hardened	See Table 9*	55 000	65 000 or 70 000[6]	75 000
	Nitrided[4,7] (through-hardened steels)	83.5 HR15N	See Fig. 14–3	See Fig. 14–3	—
Nitralloy 135M, Nitralloy N, and 2.5% chrome (no aluminum)	Nitrided[4,7]	87.5 HR15N	See Fig. 14–4	See Fig. 14–4	See Fig. 14–4

Notes: See ANSI/AGMA 2001-D04 for references cited in notes 1–7.

[1]Hardness to be equivalent to that at the root diameter in the center of the tooth space and face width.

[2]See tables 7 through 10 for major metallurgical factors for each stress grade of steel gears.

[3]The steel selected must be compatible with the heat treatment process selected and hardness required.

[4]The allowable stress numbers indicated may be used with the case depths prescribed in 16.1.

[5]See figure 12 for type A and type B hardness patterns.

[6]If bainite and microcracks are limited to grade 3 levels, 70,000 psi may be used.

[7]The overload capacity of nitrided gears is low. Since the shape of the effective S-N curve is flat, the sensitivity to shock should be investigated before proceeding with the design. [7]

*Tables 8 and 9 of ANSI/AGMA 2001-D04 are comprehensive tabulations of the major metallurgical factors affecting S_t and S_c of flame-hardened and induction-hardened (Table 8) and carburized and hardened (Table 9) steel gears.

Table 14–4

Repeatedly Applied Bending Strength S_t for Iron and Bronze Gears at 10^7 Cycles and 0.99 Reliability
Source: ANSI/AGMA 2001-D04.

Material	Material Designation[1]	Heat Treatment	Typical Minimum Surface Hardness[2]	Allowable Bending Stress Number, S_t,[3] psi
ASTM A48 gray cast iron	Class 20	As cast	—	5000
	Class 30	As cast	174 HB	8500
	Class 40	As cast	201 HB	13 000
ASTM A536 ductile (nodular) Iron	Grade 60–40–18	Annealed	140 HB	22 000–33 000
	Grade 80–55–06	Quenched and tempered	179 HB	22 000–33 000
	Grade 100–70–03	Quenched and tempered	229 HB	27 000–40 000
	Grade 120–90–02	Quenched and tempered	269 HB	31 000–44 000
Bronze		Sand cast	Minimum tensile strength 40 000 psi	5700
	ASTM B–148 Alloy 954	Heat treated	Minimum tensile strength 90 000 psi	23 600

Notes:
[1]See ANSI/AGMA 2004-B89, *Gear Materials and Heat Treatment Manual.*
[2]Measured hardness to be equivalent to that which would be measured at the root diameter in the center of the tooth space and face width.
[3]The lower values should be used for general design purposes. The upper values may be used when:
High quality material is used.
Section size and design allow maximum response to heat treatment.
Proper quality control is effected by adequate inspection.
Operating experience justifies their use.

The equation for the allowable bending stress is

$$\sigma_{\text{all}} = \begin{cases} \dfrac{S_t}{S_F}\dfrac{Y_N}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_t}{S_F}\dfrac{Y_N}{Y_\theta Y_Z} & \text{(SI units)} \end{cases} \qquad (14\text{–}17)$$

where for U.S. customary units (SI units),

S_t is the allowable bending stress, lbf/in^2 (N/mm^2)
Y_N is the stress cycle factor for bending stress
K_T (Y_θ) are the temperature factors
K_R (Y_Z) are the reliability factors
S_F is the AGMA factor of safety, a stress ratio

The equation for the allowable contact stress $\sigma_{c,\text{all}}$ is

$$\sigma_{c,\text{all}} = \begin{cases} \dfrac{S_c}{S_H}\dfrac{Z_N C_H}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_c}{S_H}\dfrac{Z_N Z_W}{Y_\theta Y_Z} & \text{(SI units)} \end{cases} \tag{14–18}$$

where the upper equation is in U.S. customary units and the lower equation is in SI units, Also,

S_c is the allowable contact stress, lbf/in^2 (N/mm^2)
Z_N is the stress cycle life factor
C_H (Z_W) are the hardness ratio factors for pitting resistance
K_T (Y_θ) are the temperature factors
K_R (Y_Z) are the reliability factors
S_H is the AGMA factor of safety, a stress ratio

The values for the allowable contact stress, designated here as S_c, are to be found in Fig. 14–5 and Tables 14–5, 14–6, and 14–7.

AGMA allowable stress numbers (strengths) for bending and contact stress are for

- Unidirectional loading
- 10 million stress cycles
- 99 percent reliability

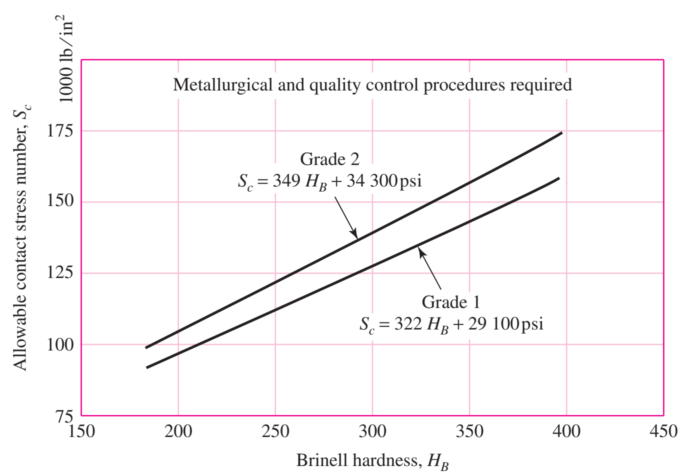


Figure 14–5

Contact-fatigue strength S_c at 10^7 cycles and 0.99 reliability for through-hardened steel gears. The SI equations are $S_c = 2.22H_B + 200$ MPa, grade 1, and $S_c = 2.41H_B + 237$ MPa, grade 2. *(Source: ANSI/AGMA 2001-D04 and 2101-D04.)*

Table 14–5

Nominal Temperature Used in Nitriding and Hardnesses Obtained

Source: Darle W. Dudley, *Handbook of Practical Gear Design*, rev. ed., McGraw-Hill, New York, 1984.

Steel	Temperature before nitriding, °F	Nitriding, °F	Hardness, Rockwell C Scale: Case	Hardness, Rockwell C Scale: Core
Nitralloy 135*	1150	975	62–65	30–35
Nitralloy 135M	1150	975	62–65	32–36
Nitralloy N	1000	975	62–65	40–44
AISI 4340	1100	975	48–53	27–35
AISI 4140	1100	975	49–54	27–35
31 Cr Mo V 9	1100	975	58–62	27–33

*Nitralloy is a trademark of the Nitralloy Corp., New York.

Table 14–6

Repeatedly Applied Contact Strength S_c at 10^7 Cycles and 0.99 Reliability for Steel Gears

Source: ANSI/AGMA 2001-D04.

Material Designation	Heat Treatment	Minimum Surface Hardness[1]	Allowable Contact Stress Number,[2] S_c, psi Grade 1	Grade 2	Grade 3
Steel[3]	Through hardened[4]	See Fig. 14–5	See Fig. 14–5	See Fig. 14–5	—
	Flame[5] or induction hardened[5]	50 HRC	170 000	190 000	—
		54 HRC	175 000	195 000	—
	Carburized and hardened[5]	See Table 9*	180 000	225 000	275 000
	Nitrided[5] (through hardened steels)	83.5 HR15N	150 000	163 000	175 000
		84.5 HR15N	155 000	168 000	180 000
2.5% chrome (no aluminum)	Nitrided[5]	87.5 HR15N	155 000	172 000	189 000
Nitralloy 135M	Nitrided[5]	90.0 HR15N	170 000	183 000	195 000
Nitralloy N	Nitrided[5]	90.0 HR15N	172 000	188 000	205 000
2.5% chrome (no aluminum)	Nitrided[5]	90.0 HR15N	176 000	196 000	216 000

Notes: See ANSI/AGMA 2001-D04 for references cited in notes 1–5.

[1]Hardness to be equivalent to that at the start of active profile in the center of the face width.

[2]See Tables 7 through 10 for major metallurgical factors for each stress grade of steel gears.

[3]The steel selected must be compatible with the heat treatment process selected and hardness required.

[4]These materials must be annealed or normalized as a minimum.

[5]The allowable stress numbers indicated may be used with the case depths prescribed in 16.1.

*Table 9 of ANSI/AGMA 2001-D04 is a comprehensive tabulation of the major metallurgical factors affecting S_t and S_c of carburized and hardened steel gears.

The factors in this section, too, will be evaluated in subsequent sections.

When two-way (reversed) loading occurs, as with idler gears, AGMA recommends using 70 percent of S_t values. This is equivalent to $1/0.70 = 1.43$ as a value of k_e in Ex. 14–2. The recommendation falls between the value of $k_e = 1.33$ for a Goodman failure locus and $k_e = 1.66$ for a Gerber failure locus.

14–5 Geometry Factors *I* and *J* (Z_I and Y_J)

We have seen how the factor Y is used in the Lewis equation to introduce the effect of tooth form into the stress equation. The AGMA factors[5] I and J are intended to accomplish the same purpose in a more involved manner.

The determination of I and J depends upon the *face-contact ratio* m_F. This is defined as

$$m_F = \frac{F}{p_x} \tag{14–19}$$

where p_x is the axial pitch and F is the face width. For spur gears, $m_F = 0$.

[5]A useful reference is AGMA 908-B89, *Geometry Factors for Determining Pitting Resistance and Bending Strength of Spur, Helical and Herringbone Gear Teeth.*

Table 14–7

Repeatedly Applied Contact Strength S_c 10^7 Cycles and 0.99 Reliability for Iron and Bronze Gears
Source: ANSI/AGMA 2001-D04.

Material	Material Designation[1]	Heat Treatment	Typical Minimum Surface Hardness[2]	Allowable Contact Stress Number,[3] S_c, psi
ASTM A48 gray cast iron	Class 20	As cast	—	50 000–60 000
	Class 30	As cast	174 HB	65 000–75 000
	Class 40	As cast	201 HB	75 000–85 000
ASTM A536 ductile (nodular) iron	Grade 60–40–18	Annealed	140 HB	77 000–92 000
	Grade 80–55–06	Quenched and tempered	179 HB	77 000–92 000
	Grade 100–70–03	Quenched and tempered	229 HB	92 000–112 000
	Grade 120–90–02	Quenched and tempered	269 HB	103 000–126 000
Bronze	—	Sand cast	Minimum tensile strength 40 000 psi	30 000
	ASTM B-148 Alloy 954	Heat treated	Minimum tensile strength 90 000 psi	65 000

Notes:
[1]See ANSI/AGMA 2004-B89, *Gear Materials and Heat Treatment Manual.*
[2]Hardness to be equivalent to that at the start of active profile in the center of the face width.
[3]The lower values should be used for general design purposes. The upper values may be used when:
High-quality material is used.
Section size and design allow maximum response to heat treatment.
Proper quality control is effected by adequate inspection.
Operating experience justifies their use.

Low-contact-ratio (LCR) helical gears having a small helix angle or a thin face width, or both, have face-contact ratios less than unity ($m_F \leq 1$), and will not be considered here. Such gears have a noise level not too different from that for spur gears. Consequently we shall consider here only spur gears with $m_F = 0$ and conventional helical gears with $m_F > 1$.

Bending-Strength Geometry Factor J (Y_J)

The AGMA factor J employs a modified value of the Lewis form factor, also denoted by Y; a *fatigue stress-concentration factor* K_f; and a tooth *load-sharing ratio* m_N. The resulting equation for J for spur and helical gears is

$$J = \frac{Y}{K_f m_N} \qquad (14\text{–}20)$$

It is important to note that the form factor Y in Eq. (14–20) is *not* the Lewis factor at all. The value of Y here is obtained from calculations within AGMA 908-B89, and is often based on the highest point of single-tooth contact.

The factor K_f in Eq. (14–20) is called a *stress correction factor* by AGMA. It is based on a formula deduced from a photoelastic investigation of stress concentration in gear teeth over 50 years ago.

The load-sharing ratio m_N is equal to the face width divided by the minimum total length of the lines of contact. This factor depends on the transverse contact ratio m_p, the face-contact ratio m_F, the effects of any profile modifications, and the tooth deflection. For spur gears, $m_N = 1.0$. For helical gears having a face-contact ratio $m_F > 2.0$, a conservative approximation is given by the equation

$$m_N = \frac{p_N}{0.95Z} \tag{14–21}$$

where p_N is the normal base pitch and Z is the length of the line of action in the transverse plane (distance L_{ab} in Fig. 13–15).

Use Fig. 14–6 to obtain the geometry factor J for spur gears having a 20° pressure angle and full-depth teeth. Use Figs. 14–7 and 14–8 for helical gears having a 20° normal pressure angle and face-contact ratios of $m_F = 2$ or greater. For other gears, consult the AGMA standard.

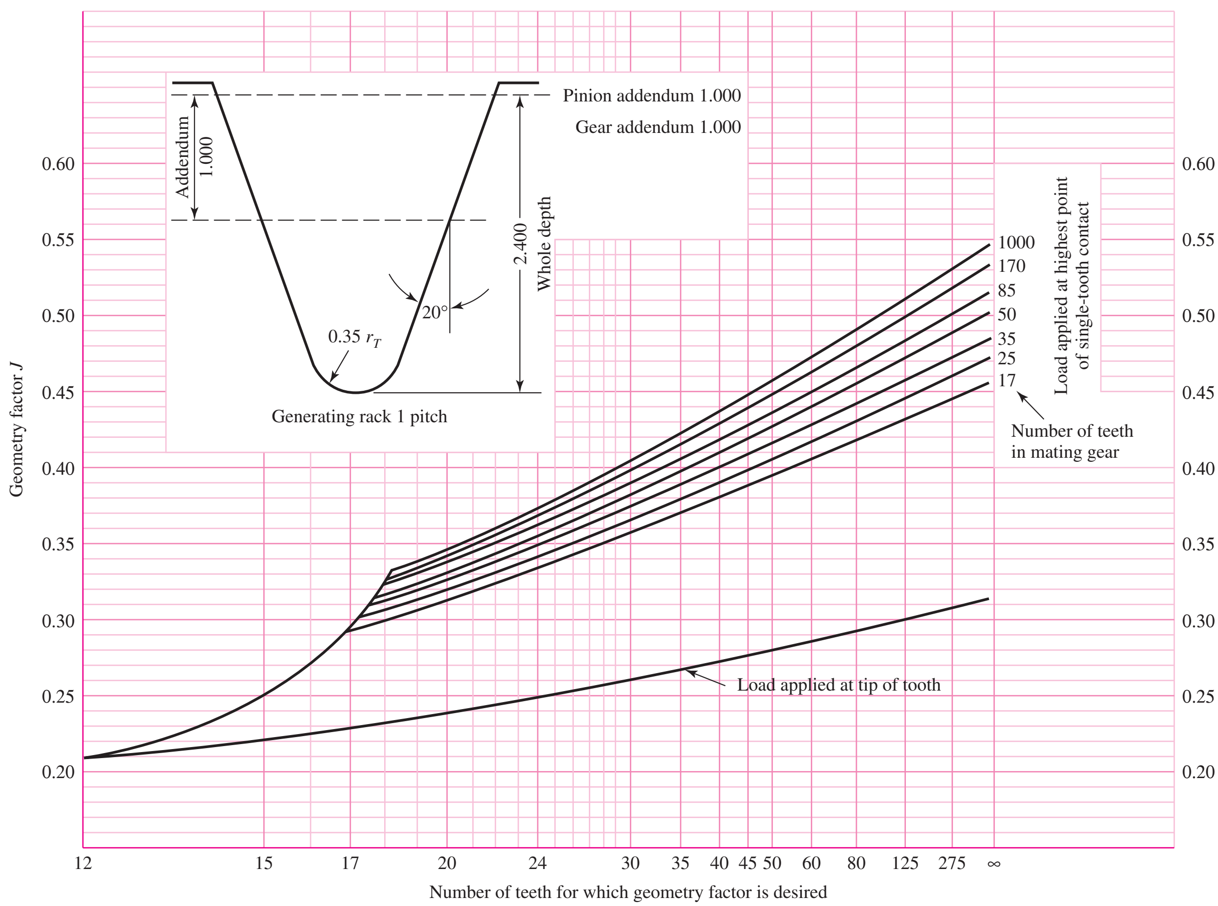


Figure 14–6

Spur-gear geometry factors *J*. *Source:* The graph is from AGMA 218.01, which is consistent with tabular data from the current AGMA 908-B89. The graph is convenient for design purposes.

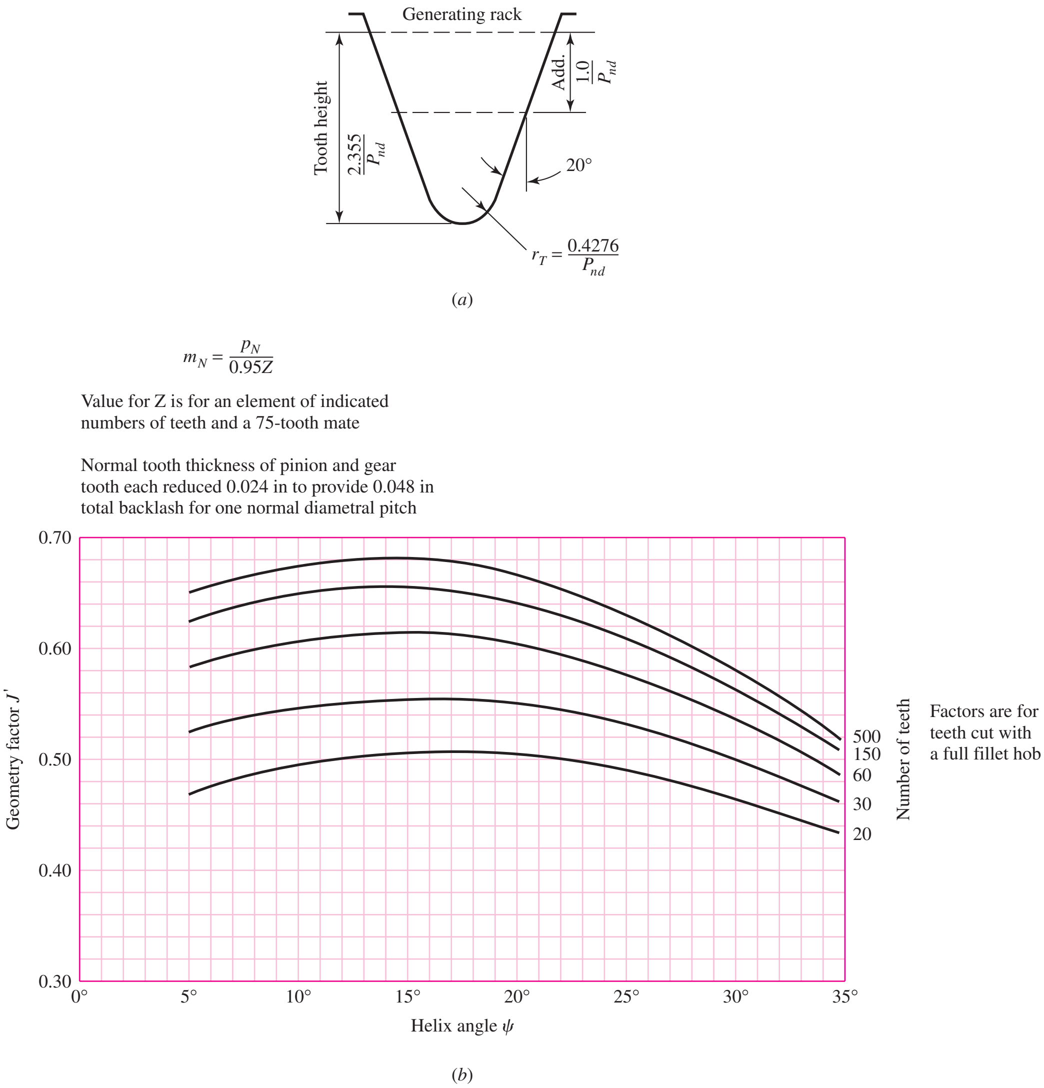


Figure 14–7

Helical-gear geometry factors J'. *Source:* The graph is from AGMA 218.01, which is consistent with tabular data from the current AGMA 908-B89. The graph is convenient for design purposes.

Surface-Strength Geometry Factor I (Z_I)

The factor I is also called the *pitting-resistance geometry factor* by AGMA. We will develop an expression for I by noting that the sum of the reciprocals of Eq. (14–14), from Eq. (14–12), can be expressed as

$$\frac{1}{r_1} + \frac{1}{r_2} = \frac{2}{\sin\phi_t}\left(\frac{1}{d_P} + \frac{1}{d_G}\right) \tag{a}$$

where we have replaced ϕ by ϕ_t, the transverse pressure angle, so that the relation will apply to helical gears too. Now define *speed ratio* m_G as

$$m_G = \frac{N_G}{N_P} = \frac{d_G}{d_P} \tag{14–22}$$

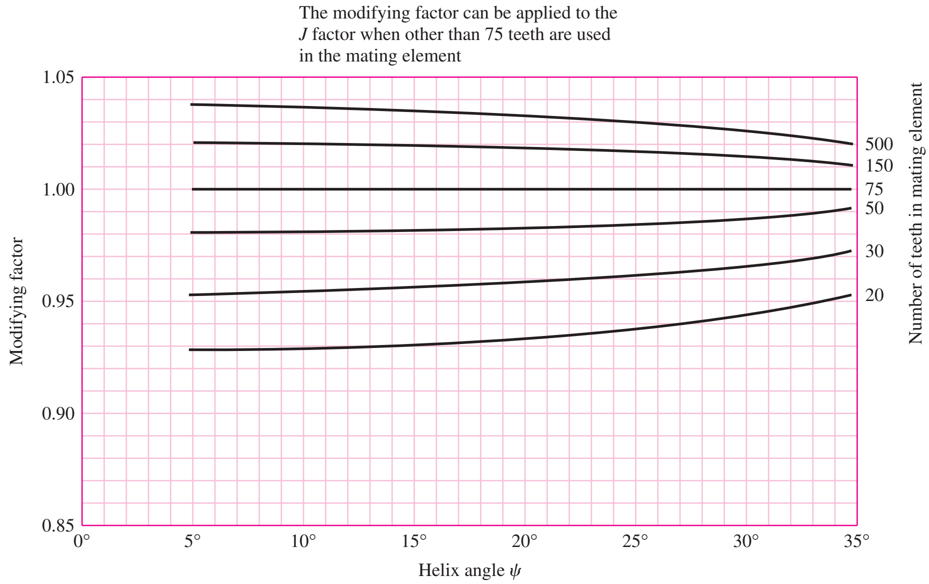


Figure 14–8

J′-factor multipliers for use with Fig. 14–7 to find *J*. *Source:* The graph is from AGMA 218.01, which is consistent with tabular data from the current AGMA 908-B89. The graph is convenient for design purposes.

Equation (*a*) can now be written

$$\frac{1}{r_1} + \frac{1}{r_2} = \frac{2}{d_P \sin\phi_t} \frac{m_G + 1}{m_G} \tag{b}$$

Now substitute Eq. (*b*) for the sum of the reciprocals in Eq. (14–14). The result is found to be

$$\sigma_c = -\sigma_C = C_p \left[\frac{K_V W^t}{d_P F} \frac{1}{\dfrac{\cos\phi_t \sin\phi_t}{2} \dfrac{m_G}{m_G + 1}} \right]^{1/2} \tag{c}$$

The geometry factor *I* for external spur and helical gears is the denominator of the second term in the brackets in Eq. (*c*). By adding the load-sharing ratio m_N, we obtain a factor valid for both spur and helical gears. The equation is then written as

$$I = \begin{cases} \dfrac{\cos\phi_t \sin\phi_t}{2m_N} \dfrac{m_G}{m_G + 1} & \text{external gears} \\[2ex] \dfrac{\cos\phi_t \sin\phi_t}{2m_N} \dfrac{m_G}{m_G - 1} & \text{internal gears} \end{cases} \tag{14–23}$$

where $m_N = 1$ for spur gears. In solving Eq. (14–21) for m_N, note that

$$p_N = p_n \cos\phi_n \tag{14–24}$$

where p_n is the normal circular pitch. The quantity *Z*, for use in Eq. (14–21), can be obtained from the equation

$$Z = \left[(r_P + a)^2 - r_{bP}^2\right]^{1/2} + \left[(r_G + a)^2 - r_{bG}^2\right]^{1/2} - (r_P + r_G)\sin\phi_t \tag{14–25}$$

where r_P and r_G are the pitch radii and r_{bP} and r_{bG} the base-circle radii of the pinion and gear, respectively.[6] Recall from Eq. (13–6), the radius of the base circle is

$$r_b = r\cos\phi_t \tag{14–26}$$

[6]For a development, see Joseph E. Shigley and John J. Uicker Jr., *Theory of Machines and Mechanisms,* McGraw-Hill, New York, 1980, p. 262.

Certain precautions must be taken in using Eq. (14–25). The tooth profiles are not conjugate below the base circle, and consequently, if either one or the other of the first two terms in brackets is larger than the third term, then it should be replaced by the third term. In addition, the effective outside radius is sometimes less than $r + a$, owing to removal of burrs or rounding of the tips of the teeth. When this is the case, always use the effective outside radius instead of $r + a$.

14–6 The Elastic Coefficient C_p (Z_E)

Values of C_p may be computed directly from Eq. (14–13) or obtained from Table 14–8.

14–7 Dynamic Factor K_v

As noted earlier, dynamic factors are used to account for inaccuracies in the manufacture and meshing of gear teeth in action. *Transmission error* is defined as the departure from uniform angular velocity of the gear pair. Some of the effects that produce transmission error are:

- Inaccuracies produced in the generation of the tooth profile; these include errors in tooth spacing, profile lead, and runout
- Vibration of the tooth during meshing due to the tooth stiffness
- Magnitude of the pitch-line velocity
- Dynamic unbalance of the rotating members
- Wear and permanent deformation of contacting portions of the teeth
- Gearshaft misalignment and the linear and angular deflection of the shaft
- Tooth friction

In an attempt to account for these effects, AGMA has defined a set of *quality numbers*.[7] These numbers define the tolerances for gears of various sizes manufactured to a specified accuracy. Quality numbers 3 to 7 will include most commercial-quality gears. Quality numbers 8 to 12 are of precision quality. The AGMA *transmission accuracy-level number* Q_v could be taken as the same as the quality number. The following equations for the dynamic factor are based on these Q_v numbers:

$$K_v = \begin{cases} \left(\dfrac{A + \sqrt{V}}{A}\right)^B & V \text{ in ft/min} \\[2ex] \left(\dfrac{A + \sqrt{200V}}{A}\right)^B & V \text{ in m/s} \end{cases} \tag{14–27}$$

where

$$\begin{aligned} A &= 50 + 56(1 - B) \\ B &= 0.25(12 - Q_v)^{2/3} \end{aligned} \tag{14–28}$$

and the maximum velocity, representing the end point of the Q_v curve, is given by

$$(V_t)_{\max} = \begin{cases} [A + (Q_v - 3)]^2 & \text{ft/min} \\[1ex] \dfrac{[A + (Q_v - 3)]^2}{200} & \text{m/s} \end{cases} \tag{14–29}$$

[7]AGMA 2000-A88. ANSI/AGMA 2001-D04, adopted in 2004, replaced Q_v with A_v and incorporated ANSI/AGMA 2015-1-A01. A_v ranges from 6 to 12, with lower numbers representing greater accuracy. The Q_v approach was maintained as an alternate approach, and resulting K_v values are comparable.

Table 14–8

Elastic Coefficient C_p (Z_E), $\sqrt{\text{psi}}$ ($\sqrt{\text{MPa}}$) *Source:* AGMA 218.01

Pinion Material	Pinion Modulus of Elasticity E_p psi (MPa)*	Gear Material and Modulus of Elasticity E_G, lbf/in² (MPa)*: Steel 30×10^6 (2×10^5)	Malleable Iron 25×10^6 (1.7×10^5)	Nodular Iron 24×10^6 (1.7×10^5)	Cast Iron 22×10^6 (1.5×10^5)	Aluminum Bronze 17.5×10^6 (1.2×10^5)	Tin Bronze 16×10^6 (1.1×10^5)
Steel	30×10^6 (2×10^5)	2300 (191)	2180 (181)	2160 (179)	2100 (174)	1950 (162)	1900 (158)
Malleable iron	25×10^6 (1.7×10^5)	2180 (181)	2090 (174)	2070 (172)	2020 (168)	1900 (158)	1850 (154)
Nodular iron	24×10^6 (1.7×10^5)	2160 (179)	2070 (172)	2050 (170)	2000 (166)	1880 (156)	1830 (152)
Cast iron	22×10^6 (1.5×10^5)	2100 (174)	2020 (168)	2000 (166)	1960 (163)	1850 (154)	1800 (149)
Aluminum bronze	17.5×10^6 (1.2×10^5)	1950 (162)	1900 (158)	1880 (156)	1850 (154)	1750 (145)	1700 (141)
Tin bronze	16×10^6 (1.1×10^5)	1900 (158)	1850 (154)	1830 (152)	1800 (149)	1700 (141)	1650 (137)

Poisson's ratio = 0.30.

*When more exact values for modulus of elasticity are obtained from roller contact tests, they may be used.

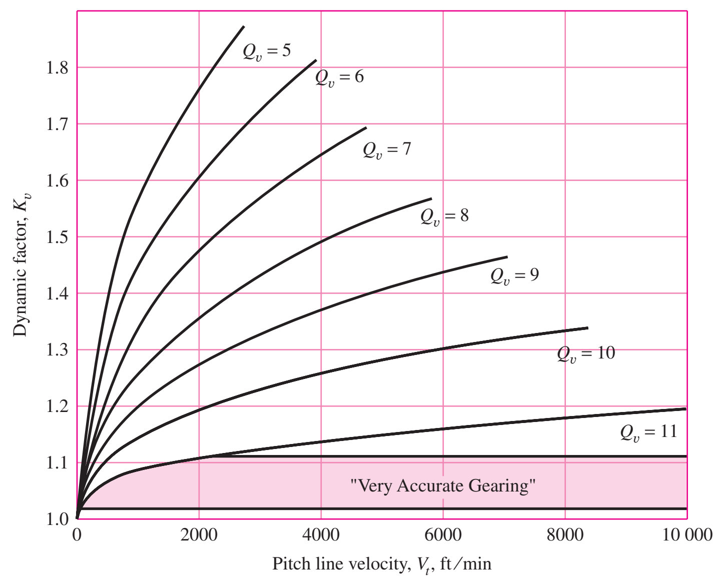


Figure 14–9

Dynamic factor K_v. The equations to these curves are given by Eq. (14–27) and the end points by Eq. (14–29). *(ANSI/AGMA 2001-D04, Annex A)*

Figure 14–9 is a graph of K_v, the dynamic factor, as a function of pitch-line speed for graphical estimates of K_v.

14–8 Overload Factor K_o

The overload factor K_o is intended to make allowance for all externally applied loads in excess of the nominal tangential load W^t in a particular application (see Figs. 14–17 and 14–18). Examples include variations in torque from the mean value due to firing of cylinders in an internal combustion engine or reaction to torque variations in a piston pump drive. There are other similar factors such as application factor or service factor. These factors are established after considerable field experience in a particular application.[8]

14–9 Surface Condition Factor C_f (Z_R)

The surface condition factor C_f or Z_R is used only in the pitting resistance equation, Eq. (14–16). It depends on

- Surface finish as affected by, but not limited to, cutting, shaving, lapping, grinding, shotpeening
- Residual stress
- Plastic effects (work hardening)

Standard surface conditions for gear teeth have not yet been established. When a detrimental surface finish effect is known to exist, AGMA specifies a value of C_f greater than unity.

[8]An extensive list of service factors appears in Howard B. Schwerdlin, "Couplings," Chap. 16 in Joseph E. Shigley, Charles R. Mischke, and Thomas H. Brown, Jr. (eds.), *Standard Handbook of Machine Design,* 3rd ed., McGraw-Hill, New York, 2004.

14–10 Size Factor K_s

The size factor reflects nonuniformity of material properties due to size. It depends upon

- Tooth size
- Diameter of part
- Ratio of tooth size to diameter of part
- Face width
- Area of stress pattern
- Ratio of case depth to tooth size
- Hardenability and heat treatment

Standard size factors for gear teeth have not yet been established for cases where there is a detrimental size effect. In such cases AGMA recommends a size factor greater than unity. If there is no detrimental size effect, use unity.

AGMA has identified and provided a symbol for size factor. Also, AGMA suggests $K_s = 1$, which makes K_s a placeholder in Eqs. (14–15) and (14–16) until more information is gathered. Following the standard in this manner is a failure to apply all of your knowledge. From Table 13–1, $l = a + b = 2.25/P$. The tooth thickness t in Fig. 14–6 is given in Sec. 14–1, Eq. (*b*), as $t = \sqrt{4lx}$ where $x = 3Y/(2P)$ from Eq. (14–3). From Eq. (6–25) the equivalent diameter d_e of a rectangular section in bending is $d_e = 0.808\sqrt{Ft}$. From Eq. (6–20) $k_b = (d_e/0.3)^{-0.107}$. Noting that K_s is the reciprocal of k_b, we find the result of all the algebraic substitution is

$$K_s = \frac{1}{k_b} = 1.192\left(\frac{F\sqrt{Y}}{P}\right)^{0.0535} \tag{a}$$

K_s can be viewed as Lewis's geometry incorporated into the Marin size factor in fatigue. You may set $K_s = 1$, or you may elect to use the preceding Eq. (*a*). This is a point to discuss with your instructor. We will use Eq. (*a*) to remind you that you have a choice. If K_s in Eq. (*a*) is less than 1, use $K_s = 1$.

14–11 Load-Distribution Factor Km (KH)

The load-distribution factor modified the stress equations to reflect nonuniform distribution of load across the line of contact. The ideal is to locate the gear "midspan" between two bearings at the zero slope place when the load is applied. However, this is not always possible. The following procedure is applicable to

- Net face width to pinion pitch diameter ratio $F/d \le 2$
- Gear elements mounted between the bearings
- Face widths up to 40 in
- Contact, when loaded, across the full width of the narrowest member

The load-distribution factor under these conditions is currently given by the *face load distribution factor*, C_{mf}, where

$$K_m = C_{mf} = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e) \tag{14–30}$$

where

$$C_{mc} = \begin{cases} 1 & \text{for uncrowned teeth} \\ 0.8 & \text{for crowned teeth} \end{cases} \qquad (14\text{–}31)$$

$$C_{pf} = \begin{cases} \dfrac{F}{10d} - 0.025 & F \le 1 \text{ in} \\ \dfrac{F}{10d} - 0.0375 + 0.0125F & 1 < F \le 17 \text{ in} \\ \dfrac{F}{10d} - 0.1109 + 0.0207F - 0.000\,228F^2 & 17 < F \le 40 \text{ in} \end{cases} \qquad (14\text{–}32)$$

Note that for values of $F/(10d) < 0.05$, $F/(10d) = 0.05$ is used.

$$C_{pm} = \begin{cases} 1 & \text{for straddle-mounted pinion with } S_1/S < 0.175 \\ 1.1 & \text{for straddle-mounted pinion with } S_1/S \ge 0.175 \end{cases} \qquad (14\text{–}33)$$

$$C_{ma} = A + BF + CF^2 \qquad \text{(see Table 14–9 for values of } A, B, \text{ and } C) \qquad (14\text{–}34)$$

$$C_e = \begin{cases} 0.8 & \text{for gearing adjusted at assembly, or compatibility is improved by lapping, or both} \\ 1 & \text{for all other conditions} \end{cases} \qquad (14\text{–}35)$$

See Fig. 14–10 for definitions of S and S_1 for use with Eq. (14–33), and see Fig. 14–11 for graph of C_{ma}.

Table 14–9

Empirical Constants A, B, and C for Eq. (14–34), Face Width F in Inches*

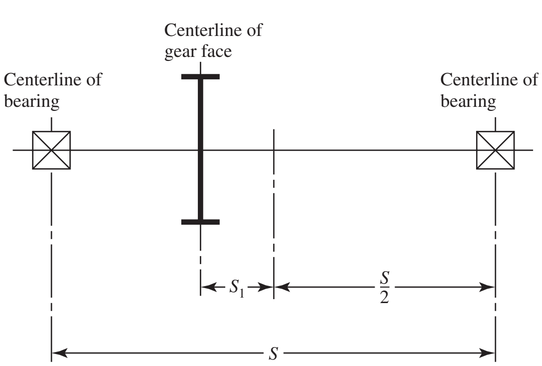
Source: ANSI/AGMA 2001-*D04.*

Condition	*A*	*B*	*C*
Open gearing	0.247	0.0167	$-0.765(10^{-4})$
Commercial, enclosed units	0.127	0.0158	$-0.930(10^{-4})$
Precision, enclosed units	0.0675	0.0128	$-0.926(10^{-4})$
Extraprecision enclosed gear units	0.00360	0.0102	$-0.822(10^{-4})$

*See ANSI/AGMA 2101-D04, pp. 20–22, for SI formulation.

Figure 14–10

Definition of distances S and S_1 used in evaluating C_{pm}, Eq. (14–33). *(ANSI/AGMA 2001-D04.)*



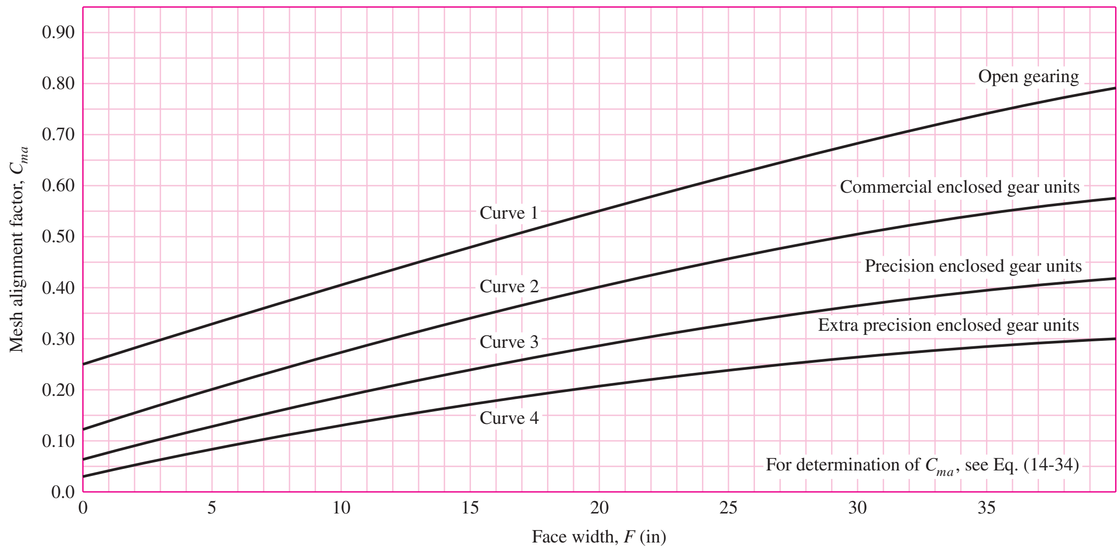


Figure 14–11

Mesh alignment factor C_{ma}. Curve-fit equations in Table 14–9. *(ANSI/AGMA 2001-D04.)*

14–12 Hardness-Ratio Factor C_H

The pinion generally has a smaller number of teeth than the gear and consequently is subjected to more cycles of contact stress. If both the pinion and the gear are through-hardened, then a uniform surface strength can be obtained by making the pinion harder than the gear. A similar effect can be obtained when a surface-hardened pinion is mated with a through-hardened gear. The hardness-ratio factor C_H is used *only for the gear*. Its purpose is to adjust the surface strengths for this effect. The values of C_H are obtained from the equation

$$C_H = 1.0 + A'(m_G - 1.0) \tag{14–36}$$

where

$$A' = 8.98(10^{-3})\left(\frac{H_{BP}}{H_{BG}}\right) - 8.29(10^{-3})\; 1.2 \le \frac{H_{BP}}{H_{BG}} \le 1.7$$

The terms H_{BP} and H_{BG} are the Brinell hardness (10-mm ball at 3000-kg load) of the pinion and gear, respectively. The term m_G is the speed ratio and is given by Eq. (14–22). See Fig. 14–12 for a graph of Eq. (14–36). For

$$\frac{H_{BP}}{H_{BG}} < 1.2, \quad A' = 0$$

$$\frac{H_{BP}}{H_{BG}} > 1.7, \quad A' = 0.006\,98$$

When surface-hardened pinions with hardnesses of 48 Rockwell C scale (Rockwell C48) or harder are run with through-hardened gears (180–400 Brinell), a work hardening occurs. The C_H factor is a function of pinion surface finish f_P and the mating gear hardness. Figure 14–13 displays the relationships:

$$C_H = 1 + B'(450 - H_{BG}) \tag{14–37}$$

Figure 14–12

Hardness ratio factor C_H (through-hardened steel). *(ANSI/AGMA 2001-D04.)*

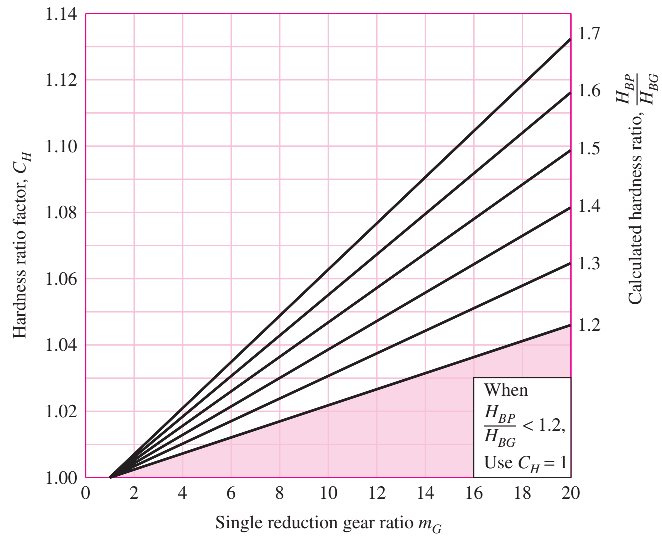
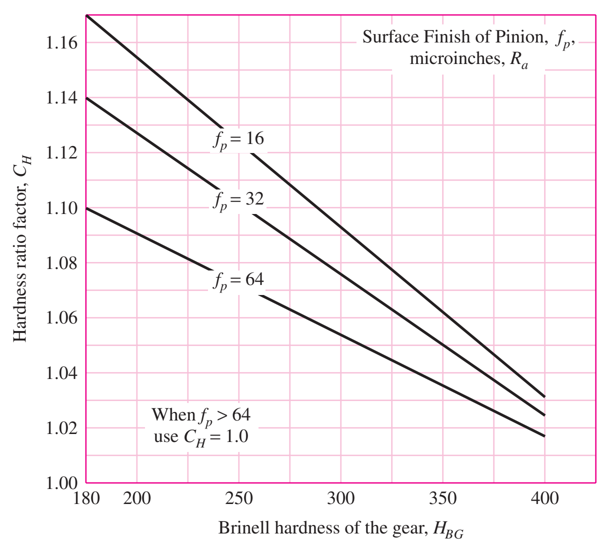


Figure 14–13

Hardness ratio factor C_H (surface-hardened steel pinion). *(ANSI/AGMA 2001-D04.)*



where $B' = 0.000\,75 \exp[-0.0112 f_P]$ and f_P is the surface finish of the pinion expressed as root-mean-square roughness R_a in μ in.

14–13 Stress Cycle Factors Y_N and Z_N

The AGMA strengths as given in Figs. 14–2 through 14–4, in Tables 14–3 and 14–4 for bending fatigue, and in Fig. 14–5 and Tables 14–5 and 14–6 for contact-stress fatigue are based on 10^7 load cycles applied. The purpose of the load cycle factors Y_N and Z_N is to modify the gear strength for lives other than 10^7 cycles. Values for these factors are given in Figs. 14–14 and 14–15. Note that for 10^7 cycles $Y_N = Z_N = 1$ on each graph. Note also that the equations for Y_N and Z_N change on either side of 10^7 cycles. For life goals slightly higher than 10^7 cycles, the mating gear may be experiencing fewer than 10^7 cycles and the equations for $(Y_N)_P$ and $(Y_N)_G$ can be different. The same comment applies to $(Z_N)_P$ and $(Z_N)_G$.

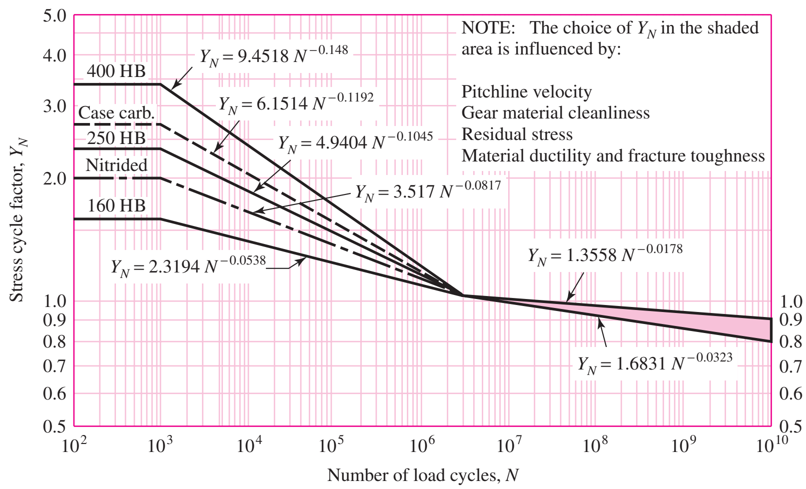


Figure 14–14

Repeatedly applied bending strength stress-cycle factor Y_N. *(ANSI/AGMA 2001-D04.)*

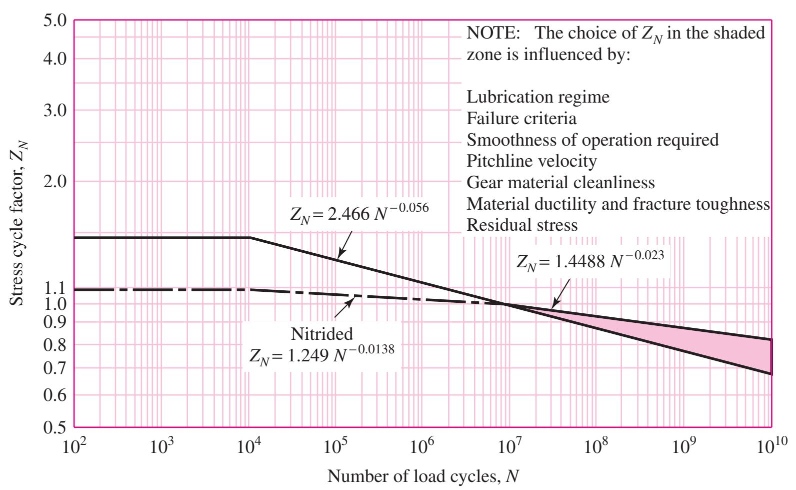


Figure 14–15

Pitting resistance stress-cycle factor Z_N. *(ANSI/AGMA 2001-D04.)*

14–14 Reliability Factor K_R (Y_Z)

The reliability factor accounts for the effect of the statistical distributions of material fatigue failures. Load variation is not addressed here. The gear strengths S_t and S_c are based on a reliability of 99 percent. Table 14–10 is based on data developed by the U.S. Navy for bending and contact-stress fatigue failures.

The functional relationship between K_R and reliability is highly nonlinear. When interpolation is required, linear interpolation is too crude. A log transformation to each quantity produces a linear string. A least-squares regression fit is

$$K_R = \begin{cases} 0.658 - 0.0759\ln(1-R) & 0.5 < R < 0.99 \\ 0.50 - 0.109\ln(1-R) & 0.99 \le R \le 0.9999 \end{cases} \tag{14–38}$$

For cardinal values of R, take K_R from the table. Otherwise use the logarithmic interpolation afforded by Eqs. (14–38).

Table 14–10

Reliability Factors K_R (Y_Z)

Source: ANSI/AGMA 2001-D04.

Reliability	K_R (Y_Z)
0.9999	1.50
0.999	1.25
0.99	1.00
0.90	0.85
0.50	0.70

14–15 Temperature Factor K_T (Y_θ)

For oil or gear-blank temperatures up to 250°F (120°C), use $K_T = Y_\theta = 1.0$. For higher temperatures, the factor should be greater than unity. Heat exchangers may be used to ensure that operating temperatures are considerably below this value, as is desirable for the lubricant.

14–16 Rim-Thickness Factor K_B

When the rim thickness is not sufficient to provide full support for the tooth root, the location of bending fatigue failure may be through the gear rim rather than at the tooth fillet. In such cases, the use of a stress-modifying factor K_B or (t_R) is recommended. This factor, the *rim-thickness factor* K_B, adjusts the estimated bending stress for the thin-rimmed gear. It is a function of the backup ratio m_B,

$$m_B = \frac{t_R}{h_t} \tag{14–39}$$

where t_R = rim thickness below the tooth, in, and h_t = the tooth height. The geometry is depicted in Fig. 14–16. The rim-thickness factor K_B is given by

$$K_B = \begin{cases} 1.6 \ln \dfrac{2.242}{m_B} & m_B < 1.2 \\ 1 & m_B \geq 1.2 \end{cases} \tag{14–40}$$

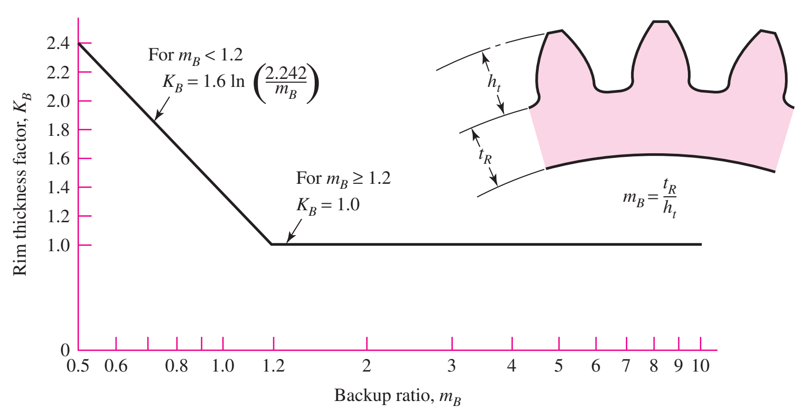


Figure 14–16

Rim thickness factor K_B. *(ANSI/AGMA 2001-D04.)*

Figure 14–16 also gives the value of K_B graphically. The rim-thickness factor K_B is applied in addition to the 0.70 reverse-loading factor when applicable.

14–17 Safety Factors S_F and S_H

The ANSI/AGMA standards 2001-D04 and 2101-D04 contain a safety factor S_F guarding against bending fatigue failure and safety factor S_H guarding against pitting failure.

The definition of S_F, from Eq. (14–17), is

$$S_F = \frac{S_t Y_N/(K_T K_R)}{\sigma} = \frac{\text{fully corrected bending strength}}{\text{bending stress}} \tag{14–41}$$

where σ is estimated from Eq. (14–15). It is a strength-over-stress definition in a case where the stress is linear with the transmitted load.

The definition of S_H, from Eq. (14–18), is

$$S_H = \frac{S_c Z_N C_H/(K_T K_R)}{\sigma_c} = \frac{\text{fully corrected contact strength}}{\text{contact stress}} \tag{14–42}$$

when σ_c is estimated from Eq. (14–16). This, too, is a strength-over-stress definition but in a case where the stress is *not* linear with the transmitted load W^t.

While the definition of S_H does not interfere with its intended function, a caution is required when comparing S_F with S_H in an analysis in order to ascertain the nature and severity of the threat to loss of function. To render S_H linear with the transmitted load, W^t it could have been defined as

$$S_H = \left(\frac{\text{fully corrected contact strength}}{\text{contact stress imposed}}\right)^2 \tag{14–43}$$

with the exponent 2 for linear or helical contact, or an exponent of 3 for crowned teeth (spherical contact). With the definition, Eq. (14–42), compare S_F with S_H^2 (or S_H^3 for crowned teeth) when trying to identify the threat to loss of function with confidence.

The role of the overload factor K_o is to include predictable excursions of load beyond W^t based on experience. A safety factor is intended to account for unquantifiable elements in addition to K_o. When designing a gear mesh, the quantity S_F becomes a design factor $(S_F)_d$ within the meanings used in this book. The quantity S_F evaluated as part of a design assessment is a factor of safety. This applies equally well to the quantity S_H.

14–18 Analysis

Description of the procedure based on the AGMA standard is highly detailed. The best review is a "road map" for bending fatigue and contact-stress fatigue. Figure 14–17 identifies the bending stress equation, the endurance strength in bending equation, and the factor of safety S_F. Figure 14–18 displays the contact-stress equation, the contact fatigue endurance strength equation, and the factor of safety S_H. When analyzing a gear problem, this figure is a useful reference.

The following example of a gear mesh analysis is intended to make all the details presented concerning the AGMA method more familiar.

SPUR GEAR BENDING
BASED ON ANSI/AGMA 2001-D04

$$d_P = \frac{N_P}{P_d}$$

$$V = \frac{\pi d n}{12}$$

$$W^t = \frac{33\,000\ \mathrm{H}}{V}$$

1 [or Eq. (a), Sec. 14–10]; p. 739

Eq. (14–30); p. 739

Eq. (14–40); p. 744

Gear bending stress equation Eq. (14–15)

$$\sigma = W^t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_B}{J}$$

Fig. 14–6; p. 733

Eq. (14–27); p. 736

Table below

$_{0.99}(S_t)_{10^7}$ Tables 14–3, 14–4; pp. 728, 729

Fig. 14–14; p. 743

Gear bending endurance strength equation Eq. (14–17)

$$\sigma_{\mathrm{all}} = \frac{S_t}{S_F} \frac{Y_N}{K_T K_R}$$

Table 14–10, Eq. (14–38); pp. 744, 743

1 if $T < 250°\mathrm{F}$

Bending factor of safety Eq. (14–41)

$$S_F = \frac{S_t Y_N/(K_T K_R)}{\sigma}$$

Remember to compare S_F with S_H^2 when deciding whether bending or wear is the threat to function. For crowned gears compare S_F with S_H^3.

Table of Overload Factors, K_o

	Driven Machine		
Power source	Uniform	Moderate shock	Heavy shock
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

Figure 14–17

Roadmap of gear bending equations based on AGMA standards. *(ANSI/AGMA 2001-D04.)*

SPUR GEAR WEAR
BASED ON ANSI/AGMA 2001-D04

$$d_P = \frac{N_P}{P_d}$$

$$V = \frac{\pi d n}{12}$$

$$W^t = \frac{33\,000\ \mathrm{H}}{V}$$

Gear contact stress equation Eq. (14–16)

$$\sigma_c = C_p \left(W^t K_o K_v K_s \frac{K_m}{d_P F} \frac{C_f}{I} \right)^{1/2}$$

1 [or Eq. (a), Sec. 14–10]; p. 739

Eq. (14–30); p. 739

1

Eq. (14–23); p. 735

Eq. (14–27); p. 736

Table below

Eq. (14–13), Table 14–8; pp. 724, 737

Gear contact endurance strength Eq. (14–18)

$$\sigma_{c,\text{all}} = \frac{S_c Z_N C_H}{S_H K_T K_R}$$

${}_{0.99}(S_c)_{10^7}$ Tables, 14–6, 14–7; pp. 731, 732

Fig. 14–15; p. 743

Section 14–12, gear only; pp. 741, 742

Table 14–10, Eqs. (14–38); pp. 744, 743

1 if $T < 250°$ F

Wear factor of safety Eq. (14–42)

$$S_H = \frac{S_c Z_N C_H / (K_T K_R)}{\sigma_c}$$

Gear only

Remember to compare S_F with S_H^2 when deciding whether bending or wear is the threat to function. For crowned gears compare S_F with S_H^3.

Table of Overload Factors, K_o

	Driven Machine		
Power source	Uniform	Moderate shock	Heavy shock
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

Figure 14–18

Roadmap of gear wear equations based on AGMA standards. *(ANSI/AGMA 2001-D04.)*

EXAMPLE 14–4 A 17-tooth 20° pressure angle spur pinion rotates at 1800 rev/min and transmits 4 hp to a 52-tooth disk gear. The diametral pitch is 10 teeth/in, the face width 1.5 in, and the quality standard is No. 6. The gears are straddle-mounted with bearings immediately adjacent. The pinion is a grade 1 steel with a hardness of 240 Brinell tooth surface and through-hardened core. The gear is steel, through-hardened also, grade 1 material, with a Brinell hardness of 200, tooth surface and core. Poisson's ratio is 0.30, $J_P = 0.30$, $J_G = 0.40$, and Young's modulus is $30(10^6)$ psi. The loading is smooth because of motor and load. Assume a pinion life of 10^8 cycles and a reliability of 0.90, and use $Y_N = 1.3558N^{-0.0178}$, $Z_N = 1.4488N^{-0.023}$. The tooth profile is uncrowned. This is a commercial enclosed gear unit.

(*a*) Find the factor of safety of the gears in bending.

(*b*) Find the factor of safety of the gears in wear.

(*c*) By examining the factors of safety, identify the threat to each gear and to the mesh.

Solution There will be many terms to obtain so use Figs. 14–17 and 14–18 as guides to what is needed.

$$d_P = N_P/P_d = 17/10 = 1.7 \text{ in} \qquad d_G = 52/10 = 5.2 \text{ in}$$

$$V = \frac{\pi d_P n_P}{12} = \frac{\pi(1.7)1800}{12} = 801.1 \text{ ft/min}$$

$$W^t = \frac{33\,000\,H}{V} = \frac{33\,000(4)}{801.1} = 164.8 \text{ lbf}$$

Assuming uniform loading, $K_o = 1$. To evaluate K_v, from Eq. (14–28) with a quality number $Q_v = 6$,

$$B = 0.25(12 - 6)^{2/3} = 0.8255$$

$$A = 50 + 56(1 - 0.8255) = 59.77$$

Then from Eq. (14–27) the dynamic factor is

$$K_v = \left(\frac{59.77 + \sqrt{801.1}}{59.77}\right)^{0.8255} = 1.377$$

To determine the size factor, K_s, the Lewis form factor is needed. From Table 14–2, with $N_P = 17$ teeth, $Y_P = 0.303$. Interpolation for the gear with $N_G = 52$ teeth yields $Y_G = 0.412$. Thus from Eq. (*a*) of Sec. 14–10, with $F = 1.5$ in,

$$(K_s)_P = 1.192\left(\frac{1.5\sqrt{0.303}}{10}\right)^{0.0535} = 1.043$$

$$(K_s)_G = 1.192\left(\frac{1.5\sqrt{0.412}}{10}\right)^{0.0535} = 1.052$$

The load distribution factor K_m is determined from Eq. (14–30), where five terms are needed. They are, where $F = 1.5$ in when needed:

Uncrowned, Eq. (14–30): $C_{mc} = 1$,
Eq. (14–32): $C_{pf} = 1.5/[10(1.7)] - 0.0375 + 0.0125(1.5) = 0.0695$
Bearings immediately adjacent, Eq. (14–33): $C_{pm} = 1$
Commercial enclosed gear units (Fig. 14–11): $C_{ma} = 0.15$
Eq. (14–35): $C_e = 1$

Thus,

$$K_m = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e) = 1 + (1)[0.0695(1) + 0.15(1)] = 1.22$$

Assuming constant thickness gears, the rim-thickness factor $K_B = 1$. The speed ratio is $m_G = N_G/N_P = 52/17 = 3.059$. The load cycle factors given in the problem statement, with N(pinion) $= 10^8$ cycles and N(gear) $= 10^8/m_G = 10^8/3.059$ cycles, are

$$(Y_N)_P = 1.3558(10^8)^{-0.0178} = 0.977$$

$$(Y_N)_G = 1.3558(10^8/3.059)^{-0.0178} = 0.996$$

From Table 14.10, with a reliability of 0.9, $K_R = 0.85$. From Fig. 14–18, the temperature and surface condition factors are $K_T = 1$ and $C_f = 1$. From Eq. (14–23), with $m_N = 1$ for spur gears,

$$I = \frac{\cos 20^\circ \sin 20^\circ}{2} \frac{3.059}{3.059 + 1} = 0.121$$

From Table 14–8, $C_p = 2300\sqrt{\text{psi}}$.

Next, we need the terms for the gear endurance strength equations. From Table 14–3, for grade 1 steel with $H_{BP} = 240$ and $H_{BG} = 200$, we use Fig. 14–2, which gives

$$(S_t)_P = 77.3(240) + 12\,800 = 31\,350 \text{ psi}$$

$$(S_t)_G = 77.3(200) + 12\,800 = 28\,260 \text{ psi}$$

Similarly, from Table 14–6, we use Fig. 14–5, which gives

$$(S_c)_P = 322(240) + 29\,100 = 106\,400 \text{ psi}$$

$$(S_c)_G = 322(200) + 29\,100 = 93\,500 \text{ psi}$$

From Fig. 14–15,

$$(Z_N)_P = 1.4488(10^8)^{-0.023} = 0.948$$

$$(Z_N)_G = 1.4488(10^8/3.059)^{-0.023} = 0.973$$

For the hardness ratio factor C_H, the hardness ratio is $H_{BP}/H_{BG} = 240/200 = 1.2$. Then, from Sec. 14–12,

$$A' = 8.98(10^{-3})(H_{BP}/H_{BG}) - 8.29(10^{-3})$$
$$= 8.98(10^{-3})(1.2) - 8.29(10^{-3}) = 0.002\,49$$

Thus, from Eq. (14–36),

$$C_H = 1 + 0.002\,49(3.059 - 1) = 1.005$$

(*a*) **Pinion tooth bending.** Substituting the appropriate terms for the pinion into Eq. (14–15) gives

$$(\sigma)_P = \left(W^t K_o K_v K_s \frac{P_d}{F}\frac{K_m K_B}{J}\right)_P = 164.8(1)1.377(1.043)\frac{10}{1.5}\frac{1.22\,(1)}{0.30}$$

$$= 6417 \text{ psi}$$

Substituting the appropriate terms for the pinion into Eq. (14–41) gives

Answer
$$(S_F)_P = \left(\frac{S_t Y_N/(K_T K_R)}{\sigma}\right)_P = \frac{31\,350(0.977)/[1(0.85)]}{6417} = 5.62$$

Gear tooth bending. Substituting the appropriate terms for the gear into Eq. (14–15) gives

$$(\sigma)_G = 164.8(1)1.377(1.052)\frac{10}{1.5}\frac{1.22(1)}{0.40} = 4854 \text{ psi}$$

Substituting the appropriate terms for the gear into Eq. (14–41) gives

Answer
$$(S_F)_G = \frac{28\ 260(0.996)/[1(0.85)]}{4854} = 6.82$$

(*b*) **Pinion tooth wear.** Substituting the appropriate terms for the pinion into Eq. (14–16) gives

$$(\sigma_c)_P = C_p \left(W^t K_o K_v K_s \frac{K_m}{d_P F}\frac{C_f}{I}\right)_P^{1/2}$$

$$= 2300\left[164.8(1)1.377(1.043)\frac{1.22}{1.7(1.5)}\frac{1}{0.121}\right]^{1/2} = 70\,360 \text{ psi}$$

Substituting the appropriate terms for the pinion into Eq. (14–42) gives

Answer
$$(S_H)_P = \left[\frac{S_c Z_N/(K_T K_R)}{\sigma_c}\right]_P = \frac{106\,400(0.948)/[1(0.85)]}{70\,360} = 1.69$$

Gear tooth wear. The only term in Eq. (14–16) that changes for the gear is K_s. Thus,

$$(\sigma_c)_G = \left[\frac{(K_s)_G}{(K_s)_P}\right]^{1/2}(\sigma_c)_P = \left(\frac{1.052}{1.043}\right)^{1/2} 70\,360 = 70\,660 \text{ psi}$$

Substituting the appropriate terms for the gear into Eq. (14–42) with $C_H = 1.005$ gives

Answer
$$(S_H)_G = \frac{93\,500(0.973)1.005/[1(0.85)]}{70\,660} = 1.52$$

(*c*) For the pinion, we compare $(S_F)_P$ with $(S_H)_P^2$, or 5.73 with $1.69^2 = 2.86$, so the threat in the pinion is from wear. For the gear, we compare $(S_F)_G$ with $(S_H)_G^2$, or 6.96 with $1.52^2 = 2.31$, so the threat in the gear is also from wear.

There are perspectives to be gained from Ex. 14–4. First, the pinion is overly strong in bending compared to wear. The performance in wear can be improved by surface-hardening techniques, such as flame or induction hardening, nitriding, or carburizing and case hardening, as well as shot peening. This in turn permits the gearset to be made smaller. Second, in bending, the gear is stronger than the pinion, indicating that both the gear core hardness and tooth size could be reduced; that is, we may increase P and reduce diameter of the gears, or perhaps allow a cheaper material. Third, in wear, surface strength equations have the ratio $(Z_N)/K_R$. The values of $(Z_N)_P$ and $(Z_N)_G$ are affected by gear ratio m_G. The designer can control strength by specifying surface hardness. This point will be elaborated later.

Having followed a spur-gear analysis in detail in Ex. 14–4, it is timely to analyze a helical gearset under similar circumstances to observe similarities and differences.

EXAMPLE 14–5

A 17-tooth 20° normal pitch-angle helical pinion with a right-hand helix angle of 30° rotates at 1800 rev/min when transmitting 4 hp to a 52-tooth helical gear. The normal diametral pitch is 10 teeth/in, the face width is 1.5 in, and the set has a quality number of 6. The gears are straddle-mounted with bearings immediately adjacent. The pinion and gear are made from a through-hardened steel with surface and core hardnesses of 240 Brinell on the pinion and surface and core hardnesses of 200 Brinell on the gear. The transmission is smooth, connecting an electric motor and a centrifugal pump. Assume a pinion life of 10^8 cycles and a reliability of 0.9 and use the upper curves in Figs. 14–14 and 14–15.

(*a*) Find the factors of safety of the gears in bending.
(*b*) Find the factors of safety of the gears in wear.
(*c*) By examining the factors of safety identify the threat to each gear and to the mesh.

Solution

All of the parameters in this example are the same as in Ex. 14–4 with the exception that we are using helical gears. Thus, several terms will be the same as Ex. 14–4. The reader should verify that the following terms remain unchanged: $K_o = 1$, $Y_P = 0.303$, $Y_G = 0.412$, $m_G = 3.059$, $(K_s)_P = 1.043$, $(K_s)_G = 1.052$, $(Y_N)_P = 0.977$, $(Y_N)_G = 0.996$, $K_R = 0.85$, $K_T = 1$, $C_f = 1$, $C_p = 2300\sqrt{\text{psi}}$, $(S_t)_P = 31\,350$ psi, $(S_t)_G = 28\,260$ psi, $(S_c)_P = 106\,380$ psi, $(S_c)_G = 93\,500$ psi, $(Z_N)_P = 0.948$, $(Z_N)_G = 0.973$, and $C_H = 1.005$.

For helical gears, the transverse diametral pitch, given by Eq. (13–18), is

$$P_t = P_n \cos\psi = 10 \cos 30^\circ = 8.660 \text{ teeth/in}$$

Thus, the pitch diameters are $d_P = N_P/P_t = 17/8.660 = 1.963$ in and $d_G = 52/8.660 = 6.005$ in. The pitch-line velocity and transmitted force are

$$V = \frac{\pi d_P n_P}{12} = \frac{\pi(1.963)1800}{12} = 925 \text{ ft/min}$$

$$W^t = \frac{33\,000H}{V} = \frac{33\,000(4)}{925} = 142.7 \text{ lbf}$$

As in Ex. 14–4, for the dynamic factor, $B = 0.8255$ and $A = 59.77$. Thus, Eq. (14–27) gives

$$K_v = \left(\frac{59.77 + \sqrt{925}}{59.77}\right)^{0.8255} = 1.404$$

The geometry factor I for helical gears requires a little work. First, the transverse pressure angle is given by Eq. (13–19)

$$\phi_t = \tan^{-1}\left(\frac{\tan\phi_n}{\cos\psi}\right) = \tan^{-1}\left(\frac{\tan 20^\circ}{\cos 30^\circ}\right) = 22.80^\circ$$

The radii of the pinion and gear are $r_P = 1.963/2 = 0.9815$ in and $r_G = 6.004/2 = 3.002$ in, respectively. The addendum is $a = 1/P_n = 1/10 = 0.1$, and the base-circle radii of the pinion and gear are given by Eq. (13–6) with $\phi = \phi_t$:

$$(r_b)_P = r_P \cos\phi_t = 0.9815 \cos 22.80^\circ = 0.9048 \text{ in}$$

$$(r_b)_G = 3.002 \cos 22.80^\circ = 2.767 \text{ in}$$

From Eq. (14–25), the surface strength geometry factor

$$\begin{aligned} Z &= \sqrt{(0.9815 + 0.1)^2 - 0.9048^2} + \sqrt{(3.004 + 0.1)^2 - 2.769^2} \\ &\quad - (0.9815 + 3.004)\sin 22.80^\circ \\ &= 0.5924 + 1.4027 - 1.5444 = 0.4507 \text{ in} \end{aligned}$$

Since the first two terms are less than 1.5444, the equation for Z stands. From Eq. (14–24) the normal circular pitch p_N is

$$p_N = p_n \cos\phi_n = \frac{\pi}{P_n}\cos 20^\circ = \frac{\pi}{10}\cos 20^\circ = 0.2952 \text{ in}$$

From Eq. (14–21), the load sharing ratio

$$m_N = \frac{p_N}{0.95Z} = \frac{0.2952}{0.95(0.4507)} = 0.6895$$

Substituting in Eq. (14–23), the geometry factor I is

$$I = \frac{\sin 22.80^\circ \cos 22.80^\circ}{2(0.6895)} \frac{3.06}{3.06 + 1} = 0.195$$

From Fig. 14–7, geometry factors $J'_P = 0.45$ and $J'_G = 0.54$. Also from Fig. 14–8 the J-factor multipliers are 0.94 and 0.98, correcting J'_P and J'_G to

$$J_P = 0.45(0.94) = 0.423$$

$$J_G = 0.54(0.98) = 0.529$$

The load-distribution factor K_m is estimated from Eq. (14–32):

$$C_{pf} = \frac{1.5}{10(1.963)} - 0.0375 + 0.0125(1.5) = 0.0577$$

with $C_{mc} = 1$, $C_{pm} = 1$, $C_{ma} = 0.15$ from Fig. 14–11, and $C_e = 1$. Therefore, from Eq. (14–30),

$$K_m = 1 + (1)[0.0577(1) + 0.15(1)] = 1.208$$

(*a*) **Pinion tooth bending.** Substituting the appropriate terms into Eq. (14–15) using P_t gives

$$\begin{aligned} (\sigma)_P &= \left(W^t K_o K_v K_s \frac{P_t}{F}\frac{K_m K_B}{J}\right)_P = 142.7(1)1.404(1.043)\frac{8.66}{1.5}\frac{1.208(1)}{0.423} \\ &= 3445 \text{ psi} \end{aligned}$$

Substituting the appropriate terms for the pinion into Eq. (14–41) gives

Answer
$$(S_F)_P = \left(\frac{S_t Y_N/(K_T K_R)}{\sigma}\right)_P = \frac{31\,350(0.977)/[1(0.85)]}{3445} = 10.5$$

Gear tooth bending. Substituting the appropriate terms for the gear into Eq. (14–15) gives

$$(\sigma)_G = 142.7(1)1.404(1.052)\frac{8.66}{1.5}\frac{1.208(1)}{0.529} = 2779 \text{ psi}$$

Substituting the appropriate terms for the gear into Eq. (14–41) gives

Answer
$$(S_F)_G = \frac{28\,260(0.996)/[1(0.85)]}{2779} = 11.9$$

(*b*) **Pinion tooth wear.** Substituting the appropriate terms for the pinion into Eq. (14–16) gives

$$(\sigma_c)_P = C_p\left(W^t K_o K_v K_s \frac{K_m}{d_P F}\frac{C_f}{I}\right)_P^{1/2}$$

$$= 2300\left[142.7(1)1.404(1.043)\frac{1.208}{1.963(1.5)}\frac{1}{0.195}\right]^{1/2} = 48\,230 \text{ psi}$$

Substituting the appropriate terms for the pinion into Eq. (14–42) gives

Answer
$$(S_H)_P = \left(\frac{S_c Z_N/(K_T K_R)}{\sigma_c}\right)_P = \frac{106\,400(0.948)/[1(0.85)]}{48\,230} = 2.46$$

Gear tooth wear. The only term in Eq. (14–16) that changes for the gear is K_s. Thus,

$$(\sigma_c)_G = \left[\frac{(K_s)_G}{(K_s)_P}\right]^{1/2}(\sigma_c)_P = \left(\frac{1.052}{1.043}\right)^{1/2} 48\,230 = 48\,440 \text{ psi}$$

Substituting the appropriate terms for the gear into Eq. (14–42) with $C_H = 1.005$ gives

Answer
$$(S_H)_G = \frac{93\,500(0.973)1.005/[1(0.85)]}{48\,440} = 2.22$$

(*c*) For the pinion we compare S_F with S_H^2, or 10.5 with $2.46^2 = 6.05$, so the threat in the pinion is from wear. For the gear we compare S_F with S_H^2, or 11.9 with $2.22^2 = 4.93$, so the threat is also from wear in the gear. For the meshing gearset wear controls.

It is worthwhile to compare Ex. 14–4 with Ex. 14–5. The spur and helical gearsets were placed in nearly identical circumstances. The helical gear teeth are of greater length because of the helix and identical face widths. The pitch diameters of the helical gears are larger. The J factors and the I factor are larger, thereby reducing stresses. The result is larger factors of safety. In the design phase the gearsets in Ex. 14–4 and Ex. 14–5 can be made smaller with control of materials and relative hardnesses.

Now that examples have given the AGMA parameters substance, it is time to examine some desirable (and necessary) relationships between material properties of spur gears in mesh. In bending, the AGMA equations are displayed side by side:

$$\sigma_P = \left(W^t K_o K_v K_s \frac{P_d}{F}\frac{K_m K_B}{J}\right)_P \qquad \sigma_G = \left(W^t K_o K_v K_s \frac{P_d}{F}\frac{K_m K_B}{J}\right)_G$$

$$(S_F)_P = \left(\frac{S_t Y_N/(K_T K_R)}{\sigma}\right)_P \qquad (S_F)_G = \left(\frac{S_t Y_N/(K_T K_R)}{\sigma}\right)_G$$

Equating the factors of safety, substituting for stress and strength, canceling identical terms (K_s virtually equal or exactly equal), and solving for $(S_t)_G$ gives

$$(S_t)_G = (S_t)_P \frac{(Y_N)_P}{(Y_N)_G}\frac{J_P}{J_G} \tag{a}$$

The stress-cycle factor Y_N comes from Fig. 14–14, where for a particular hardness, $Y_N = \alpha N^\beta$. For the pinion, $(Y_N)_P = \alpha N_P^\beta$, and for the gear, $(Y_N)_G = \alpha(N_P/m_G)^\beta$. Substituting these into Eq. (*a*) and simplifying gives

$$(S_t)_G = (S_t)_P m_G^\beta \frac{J_P}{J_G} \tag{14–44}$$

Normally, $m_G > 1$ and $J_G > J_P$, so equation (14–44) shows that the gear can be less strong (lower Brinell hardness) than the pinion for the same safety factor.

EXAMPLE 14–6

In a set of spur gears, a 300-Brinell 18-tooth 16-pitch 20° full-depth pinion meshes with a 64-tooth gear. Both gear and pinion are of grade 1 through-hardened steel. Using $\beta = -0.023$, what hardness can the gear have for the same factor of safety?

Solution

For through-hardened grade 1 steel the pinion strength $(S_t)_P$ is given in Fig. 14–2:

$$(S_t)_P = 77.3(300) + 12\,800 = 35\,990 \text{ psi}$$

From Fig. 14–6 the form factors are $J_P = 0.32$ and $J_G = 0.41$. Equation (14–44) gives

$$(S_t)_G = 35\,990\left(\frac{64}{18}\right)^{-0.023}\frac{0.32}{0.41} = 27\,280 \text{ psi}$$

Use the equation in Fig. 14–2 again.

Answer

$$(H_B)_G = \frac{27\,280 - 12\,800}{77.3} = 187 \text{ Brinell}$$

The AGMA contact-stress equations also are displayed side by side:

$$(\sigma_c)_P = C_p\left(W^t K_o K_v K_s \frac{K_m}{d_P F}\frac{C_f}{I}\right)_P^{1/2} \qquad (\sigma_c)_G = C_p\left(W^t K_o K_v K_s \frac{K_m}{d_P F}\frac{C_f}{I}\right)_G^{1/2}$$

$$(S_H)_P = \left(\frac{S_c Z_N/(K_T K_R)}{\sigma_c}\right)_P \qquad (S_H)_G = \left(\frac{S_c Z_N C_H/(K_T K_R)}{\sigma_c}\right)_G$$

Equating the factors of safety, substituting the stress relations, and canceling identical terms including K_s gives, after solving for $(S_c)_G$,

$$(S_c)_G = (S_c)_P \frac{(Z_N)_P}{(Z_N)_G} \left(\frac{1}{C_H}\right)_G = (S_C)_P m_G^{\beta} \left(\frac{1}{C_H}\right)_G$$

where, as in the development of Eq. (14–44), $(Z_N)_P/(Z_N)_G = m_G^{\beta}$ and the value of β for wear comes from Fig. 14–15. Since C_H is so close to unity, it is usually neglected; therefore

$$(S_c)_G = (S_c)_P m_G^{\beta} \tag{14–45}$$

EXAMPLE 14–7 For $\beta = -0.056$ for a through-hardened steel, grade 1, continue Ex. 14–6 for wear.

Solution From Fig. 14–5,

$$(S_c)_P = 322(300) + 29\,100 = 125\,700 \text{ psi}$$

From Eq. (14–45),

$$(S_c)_G = (S_c)_P \left(\frac{64}{18}\right)^{-0.056} = 125\,700 \left(\frac{64}{18}\right)^{-0.056} = 117\,100 \text{ psi}$$

Answer

$$(H_B)_G = \frac{117\,100 - 29\,200}{322} = 273 \text{ Brinell}$$

which is slightly less than the pinion hardness of 300 Brinell.

Equations (14–44) and (14–45) apply as well to helical gears.

14–19 Design of a Gear Mesh

A useful decision set for spur and helical gears includes

a priori decisions:

- Function: load, speed, reliability, life, K_o
- Unquantifiable risk: design factor n_d
- Tooth system: ϕ, ψ, addendum, dedendum, root fillet radius
- Gear ratio m_G, N_p, N_G
- Quality number Q_v

design decisions:

- Diametral pitch P_d
- Face width F
- Pinion material, core hardness, case hardness
- Gear material, core hardness, case hardness

The first item to notice is the dimensionality of the decision set. There are four design decision categories, eight different decisions if you count them separately. This is a larger number than we have encountered before. It is important to use a design strategy that is convenient in either longhand execution or computer implementation. The design decisions have been placed in order of importance (impact on the amount of work to be redone in iterations). The steps are, after the a priori decisions have been made,

- Choose a diametral pitch.
- Examine implications on face width, pitch diameters, and material properties. If not satisfactory, return to pitch decision for change.
- Choose a pinion material and examine core and case hardness requirements. If not satisfactory, return to pitch decision and iterate until no decisions are changed.
- Choose a gear material and examine core and case hardness requirements. If not satisfactory, return to pitch decision and iterate until no decisions are changed.

With these plan steps in mind, we can consider them in more detail.

First select a trial diametral pitch.

Pinion bending:

- Select a median face width for this pitch, $4\pi/P$
- Find the range of necessary ultimate strengths
- Choose a material and a core hardness
- Find face width to meet factor of safety in bending
- Choose face width
- Check factor of safety in bending

Gear bending:

- Find necessary companion core hardness
- Choose a material and core hardness
- Check factor of safety in bending

Pinion wear:

- Find necessary S_c and attendant case hardness
- Choose a case hardness
- Check factor of safety in wear

Gear wear:

- Find companion case hardness
- Choose a case hardness
- Check factor of safety in wear

Completing this set of steps will yield a satisfactory design. Additional designs with diametral pitches adjacent to the first satisfactory design will produce several among which to choose. A figure of merit is necessary in order to choose the best. Unfortunately, a figure of merit in gear design is complex in an academic environment because material and processing cost vary. The possibility of using a process depends on the manufacturing facility if gears are made in house.

After examining Ex. 14–4 and Ex. 14–5 and seeing the wide range of factors of safety, one might entertain the notion of setting all factors of safety equal.[9] In steel

[9]In designing gears it makes sense to define the factor of safety in wear as $(S)_H^2$ for uncrowned teeth, so that there is no mix-up. ANSI, in the preface to ANSI/AGMA 2001-D04 and 2101-D04, states "the use is completely voluntary. . . does not preclude anyone from using . . . procedures . . . not conforming to the standards."

gears, wear is usually controlling and $(S_H)_P$ and $(S_H)_G$ can be brought close to equality. The use of softer cores can bring down $(S_F)_P$ and $(S_F)_G$, but there is value in keeping them higher. A tooth broken by bending fatigue not only can destroy the gear set, but can bend shafts, damage bearings, and produce inertial stresses up- and downstream in the power train, causing damage elsewhere if the gear box locks.

EXAMPLE 14–8

Design a 4:1 spur-gear reduction for a 100-hp, three-phase squirrel-cage induction motor running at 1120 rev/min. The load is smooth, providing a reliability of 0.95 at 10^9 revolutions of the pinion. Gearing space is meager. Use Nitralloy 135M, grade 1 material to keep the gear size small. The gears are heat-treated first then nitrided.

Solution

Make the a priori decisions:

- Function: 100 hp, 1120 rev/min, $R = 0.95$, $N = 10^9$ cycles, $K_o = 1$
- Design factor for unquantifiable exingencies: $n_d = 2$
- Tooth system: $\phi_n = 20°$
- Tooth count: $N_P = 18$ teeth, $N_G = 72$ teeth (no interference)
- Quality number: $Q_v = 6$, use grade 1 material
- Assume $m_B \geq 1.2$ in Eq. (14–40), $K_B = 1$

Pitch: Select a trial diametral pitch of $P_d = 4$ teeth/in. Thus, $d_P = 18/4 = 4.5$ in and $d_G = 72/4 = 18$ in. From Table 14–2, $Y_P = 0.309$, $Y_G = 0.4324$ (interpolated). From Fig. 14–6, $J_P = 0.32$, $J_G = 0.415$.

$$V = \frac{\pi d_P n_P}{12} = \frac{\pi(4.5)1120}{12} = 1319 \text{ ft/min}$$

$$W^t = \frac{33\,000H}{V} = \frac{33\,000(100)}{1319} = 2502 \text{ lbf}$$

From Eqs. (14–28) and (14–27),

$$B = 0.25(12 - Q_v)^{2/3} = 0.25(12 - 6)^{2/3} = 0.8255$$

$$A = 50 + 56(1 - 0.8255) = 59.77$$

$$K_v = \left(\frac{59.77 + \sqrt{1319}}{59.77}\right)^{0.8255} = 1.480$$

From Eq. (14–38), $K_R = 0.658 - 0.0759 \ln(1 - 0.95) = 0.885$. From Fig. 14–14,

$$(Y_N)_P = 1.3558(10^9)^{-0.0178} = 0.938$$

$$(Y_N)_G = 1.3558(10^9/4)^{-0.0178} = 0.961$$

From Fig. 14–15,

$$(Z_N)_P = 1.4488(10^9)^{-0.023} = 0.900$$

$$(Z_N)_G = 1.4488(10^9/4)^{-0.023} = 0.929$$

From the recommendation after Eq. (14–8), $3p \le F \le 5p$. Try $F = 4p = 4\pi/P = 4\pi/4 = 3.14$ in. From Eq. (*a*), Sec. 14–10,

$$K_s = 1.192\left(\frac{F\sqrt{Y}}{P}\right)^{0.0535} = 1.192\left(\frac{3.14\sqrt{0.309}}{4}\right)^{0.0535} = 1.140$$

From Eqs. (14–31), (14–33), (14–35), $C_{mc} = C_{pm} = C_e = 1$. From Fig. 14–11, $C_{ma} = 0.175$ for commercial enclosed gear units. From Eq. (14–32), $F/(10d_P) = 3.14/[10(4.5)] = 0.0698$. Thus,

$$C_{pf} = 0.0698 - 0.0375 + 0.0125(3.14) = 0.0715$$

From Eq. (14–30),

$$K_m = 1 + (1)[0.0715(1) + 0.175(1)] = 1.247$$

From Table 14–8, for steel gears, $C_p = 2300\sqrt{\text{psi}}$. From Eq. (14–23), with $m_G = 4$ and $m_N = 1$,

$$I = \frac{\cos 20^\circ \sin 20^\circ}{2}\,\frac{4}{4+1} = 0.1286$$

Pinion tooth bending. With the above estimates of K_s and K_m from the trial diametral pitch, we check to see if the mesh width F is controlled by bending or wear considerations. Equating Eqs. (14–15) and (14–17), substituting $n_d W^t$ for W^t, and solving for the face width $(F)_{\text{bend}}$ necessary to resist bending fatigue, we obtain

$$(F)_{\text{bend}} = n_d W^t K_o K_v K_s P_d \frac{K_m K_B}{J_P}\,\frac{K_T K_R}{S_t Y_N} \tag{1}$$

Equating Eqs. (14–16) and (14–18), substituting $n_d W^t$ for W^t, and solving for the face width $(F)_{\text{wear}}$ necessary to resist wear fatigue, we obtain

$$(F)_{\text{wear}} = \left(\frac{C_p Z_N}{S_c K_T K_R}\right)^2 n_d W^t K_o K_v K_s \frac{K_m C_f}{d_P I} \tag{2}$$

From Table 14–5 the hardness range of Nitralloy 135M is Rockwell C32–36 (302–335 Brinell). Choosing a midrange hardness as attainable, using 320 Brinell. From Fig. 14–4,

$$S_t = 86.2(320) + 12\,730 = 40\,310 \text{ psi}$$

Inserting the numerical value of S_t in Eq. (1) to estimate the face width gives

$$(F)_{\text{bend}} = 2(2502)(1)1.48(1.14)4\frac{1.247(1)(1)0.885}{0.32(40\,310)0.938} = 3.08 \text{ in}$$

From Table 14–6 for Nitralloy 135M, $S_c = 170\,000$ psi. Inserting this in Eq. (2), we find

$$(F)_{\text{wear}} = \left(\frac{2300(0.900)}{170\,000(1)0.885}\right)^2 2(2502)1(1.48)1.14\frac{1.247(1)}{4.5(0.1286)} = 3.44 \text{ in}$$

Decision Make face width 3.50 in. Correct K_s and K_m:

$$K_s = 1.192\left(\frac{3.50\sqrt{0.309}}{4}\right)^{0.0535} = 1.147$$

$$\frac{F}{10d_P} = \frac{3.50}{10(4.5)} = 0.0778$$

$$C_{pf} = 0.0778 - 0.0375 + 0.0125(3.50) = 0.0841$$

$$K_m = 1 + (1)[0.0841(1) + 0.175(1)] = 1.259$$

The bending stress induced by W^t in bending, from Eq. (14–15), is

$$(\sigma)_P = 2502(1)1.48(1.147)\frac{4}{3.50}\frac{1.259(1)}{0.32} = 19\,100 \text{ psi}$$

The AGMA factor of safety in bending of the pinion, from Eq. (14–41), is

$$(S_F)_P = \frac{40\,310(0.938)/[1(0.885)]}{19\,100} = 2.24$$

Decision **Gear tooth bending.** Use cast gear blank because of the 18-in pitch diameter. Use the same material, heat treatment, and nitriding. The load-induced bending stress is in the ratio of J_P/J_G. Then

$$(\sigma)_G = 19\,100\frac{0.32}{0.415} = 14\,730 \text{ psi}$$

The factor of safety of the gear in bending is

$$(S_F)_G = \frac{40\,310(0.961)/[1(0.885)]}{14\,730} = 2.97$$

Pinion tooth wear. The contact stress, given by Eq. (14–16), is

$$(\sigma_c)_P = 2300\left[2502(1)1.48(1.147)\frac{1.259}{4.5(3.5)}\frac{1}{0.129}\right]^{1/2} = 118\,000 \text{ psi}$$

The factor of safety from Eq. (14–42), is

$$(S_H)_P = \frac{170\,000(0.900)/[1(0.885)]}{118\,000} = 1.465$$

By our definition of factor of safety, pinion bending is $(S_F)_P = 2.24$, and wear is $(S_H)_P^2 = (1.465)^2 = 2.15$.

Gear tooth wear. The hardness of the gear and pinion are the same. Thus, from Fig. 14–12, $C_H = 1$, the contact stress on the gear is the same as the pinion, $(\sigma_c)_G = 118\,000$ psi. The wear strength is also the same, $S_c = 170\,000$ psi. The factor of safety of the gear in wear is

$$(S_H)_G = \frac{170\,000(0.929)/[1(0.885)]}{118\,000} = 1.51$$

So, for the gear in bending, $(S_F)_G = 2.97$, and wear $(S_H)_G^2 = (1.51)^2 = 2.29$.

Rim. Keep $m_B \geq 1.2$. The whole depth is h_t = addendum + dedendum $= 1/P_d + 1.25/P_d = 2.25/P_d = 2.25/4 = 0.5625$ in. The rim thickness t_R is

$$t_R \geq m_B h_t = 1.2(0.5625) = 0.675 \text{ in}$$

In the design of the gear blank, be sure the rim thickness exceeds 0.675 in; if it does not, review and modify this mesh design.

This design example showed a satisfactory design for a four-pitch spur-gear mesh. Material could be changed, as could pitch. There are a number of other satisfactory designs, thus a figure of merit is needed to identify the best.

One can appreciate that gear design was one of the early applications of the digital computer to mechanical engineering. A design program should be interactive, presenting results of calculations, pausing for a decision by the designer, and showing the consequences of the decision, with a loop back to change a decision for the better. The program can be structured in totem-pole fashion, with the most influential decision at the top, then tumbling down, decision after decision, ending with the ability to change the current decision or to begin again. Such a program would make a fine class project. Troubleshooting the coding will reinforce your knowledge, adding flexibility as well as bells and whistles in subsequent terms.

Standard gears may not be the most economical design that meets the functional requirements, because no application is standard in all respects.[10] Methods of designing custom gears are well-understood and frequently used in mobile equipment to provide good weight-to-performance index. The required calculations including optimizations are within the capability of a personal computer.

PROBLEMS

Because gearing problems can be difficult, the problems are presented by section.

Section 14–1

14–1 A steel spur pinion has a pitch of 6 teeth/in, 22 full-depth teeth, and a 20° pressure angle. The pinion runs at a speed of 1200 rev/min and transmits 15 hp to a 60-tooth gear. If the face width is 2 in, estimate the bending stress.

14–2 A steel spur pinion has a diametral pitch of 12 teeth/in, 16 teeth cut full-depth with a 20° pressure angle, and a face width of $\frac{3}{4}$ in. This pinion is expected to transmit 1.5 hp at a speed of 700 rev/min. Determine the bending stress.

14–3 A steel spur pinion has a module of 1.25 mm, 18 teeth cut on the 20° full-depth system, and a face width of 12 mm. At a speed of 1800 rev/min, this pinion is expected to carry a steady load of 0.5 kW. Determine the resulting bending stress.

14–4 A steel spur pinion has 15 teeth cut on the 20° full-depth system with a module of 5 mm and a face width of 60 mm. The pinion rotates at 200 rev/min and transmits 5 kW to the mating steel gear. What is the resulting bending stress?

[10]See H. W. Van Gerpen, C. K. Reece, and J. K. Jensen, *Computer Aided Design of Custom Gears,* Van Gerpen–Reece Engineering, Cedar Falls, Iowa, 1996.

14–5 A steel spur pinion has a module of 1 mm and 16 teeth cut on the 20° full-depth system and is to carry 0.15 kW at 400 rev/min. Determine a suitable face width based on an allowable bending stress of 150 MPa.

14–6 A 20° full-depth steel spur pinion has 17 teeth and a module of 1.5 mm and is to transmit 0.25 kW at a speed of 400 rev/min. Find an appropriate face width if the bending stress is not to exceed 75 MPa.

14–7 A 20° full-depth steel spur pinion has a diametral pitch of 5 teeth/in and 24 teeth and transmits 6 hp at a speed of 50 rev/min. Find an appropriate face width if the allowable bending stress is 20 kpsi.

14–8 A steel spur pinion is to transmit 15 hp at a speed of 600 rev/min. The pinion is cut on the 20° full-depth system and has a diametral pitch of 5 teeth/in and 16 teeth. Find a suitable face width based on an allowable stress of 10 kpsi.

14–9 A 20° full-depth steel spur pinion with 18 teeth is to transmit 2.5 hp at a speed of 600 rev/min. Determine appropriate values for the face width and diametral pitch based on an allowable bending stress of 10 kpsi.

14–10 A 20° full-depth steel spur pinion is to transmit 1.5 kW hp at a speed of 900 rev/min. If the pinion has 18 teeth, determine suitable values for the module and face width. The bending stress should not exceed 75 MPa.

Section 14–2

14–11 A speed reducer has 20° full-depth teeth and consists of a 22-tooth steel spur pinion driving a 60-tooth cast-iron gear. The horsepower transmitted is 15 at a pinion speed of 1200 rev/min. For a diametral pitch of 6 teeth/in and a face width of 2 in, find the contact stress.

14–12 A gear drive consists of a 16-tooth 20° steel spur pinion and a 48-tooth cast-iron gear having a pitch of 12 teeth/in. For a power input of 1.5 hp at a pinion speed of 700 rev/min, select a face width based on an allowable contact stress of 100 kpsi.

14–13 A gearset has a diametral pitch of 5 teeth/in, a 20° pressure angle, and a 24-tooth cast-iron spur pinion driving a 48-tooth cast-iron gear. The pinion is to rotate at 50 rev/min. What horsepower input can be used with this gearset if the contact stress is limited to 100 kpsi and $F = 2.5$ in?

14–14 A 20° 20-tooth cast-iron spur pinion having a module of 4 mm drives a 32-tooth cast-iron gear. Find the contact stress if the pinion speed is 1000 rev/min, the face width is 50 mm, and 10 kW of power is transmitted.

14–15 A steel spur pinion and gear have a diametral pitch of 12 teeth/in, milled teeth, 17 and 30 teeth, respectively, a 20° pressure angle, and a pinion speed of 525 rev/min. The tooth properties are $S_{ut} = 76$ kpsi, $S_y = 42$ kpsi and the Brinell hardness is 149. For a design factor of 2.25, a face width of $\frac{7}{8}$ in, what is the power rating of the gearset?

14–16 A milled-teeth steel pinion and gear pair have $S_{ut} = 113$ kpsi, $S_y = 86$ kpsi and a hardness at the involute surface of 262 Brinell. The diametral pitch is 3 teeth/in, the face width is 2.5 in, and the pinion speed is 870 rev/min. The tooth counts are 20 and 100. For a design factor of 1.5, rate the gearset for power considering both bending and wear.

14–17 A 20° full-depth steel spur pinion rotates at 1145 rev/min. It has a module of 6 mm, a face width of 75 mm, and 16 milled teeth. The ultimate tensile strength at the involute is 900 MPa exhibiting a Brinell hardness of 260. The gear is steel with 30 teeth and has identical material strengths. For a design factor of 1.3 find the power rating of the gearset based on the pinion and the gear resisting bending and wear fatigue.

14–18 A steel spur pinion has a pitch of 6 teeth/in, 17 full-depth milled teeth, and a pressure angle of 20°. The pinion has an ultimate tensile strength at the involute surface of 116 kpsi, a Brinell hardness of 232, and a yield strength of 90 kpsi. Its shaft speed is 1120 rev/min, its face width is 2 in, and its mating gear has 51 teeth. Rate the pinion for power transmission if the design factor is 2.
(*a*) Pinion bending fatigue imposes what power limitation?
(*b*) Pinion surface fatigue imposes what power limitation? The gear has identical strengths to the pinion with regard to material properties.
(*c*) Consider power limitations due to gear bending and wear.
(*d*) Rate the gearset.

Section 14–3 to 14–19

14–19 A commercial enclosed gear drive consists of a 20° spur pinion having 16 teeth driving a 48-tooth gear. The pinion speed is 300 rev/min, the face width 2 in, and the diametral pitch 6 teeth/in. The gears are grade 1 steel, through-hardened at 200 Brinell, made to No. 6 quality standards, uncrowned, and are to be accurately and rigidly mounted. Assume a pinion life of 10^8 cycles and a reliability of 0.90. Determine the AGMA bending and contact stresses and the corresponding factors of safety if 5 hp is to be transmitted.

14–20 A 20° spur pinion with 20 teeth and a module of 2.5 mm transmits 120 W to a 36-tooth gear. The pinion speed is 100 rev/min, and the gears are grade 1, 18 mm face width, through-hardened steel at 200 Brinell, uncrowned, manufactured to a No. 6 quality standard, and considered to be of open gearing quality installation. Find the AGMA bending and contact stresses and the corresponding factors of safety for a pinion life of 10^8 cycles and a reliability of 0.95.

14–21 Repeat Prob. 14–19 using helical gears each with a 20° normal pitch angle and a helix angle of 30° and a normal diametral pitch of 6 teeth/in.

14–22 A spur gearset has 17 teeth on the pinion and 51 teeth on the gear. The pressure angle is 20° and the overload factor $K_o = 1$. The diametral pitch is 6 teeth/in and the face width is 2 in. The pinion speed is 1120 rev/min and its cycle life is to be 10^8 revolutions at a reliability $R = 0.99$. The quality number is 5. The material is a through-hardened steel, grade 1, with Brinell hardnesses of 232 core and case of both gears. For a design factor of 2, rate the gearset for these conditions using the AGMA method.

14–23 In Sec. 14–10, Eq. (*a*) is given for K_s based on the procedure in Ex. 14–2. Derive this equation.

14–24 A speed-reducer has 20° full-depth teeth, and the single-reduction spur-gear gearset has 22 and 60 teeth. The diametral pitch is 4 teeth/in and the face width is $3\frac{1}{4}$ in. The pinion shaft speed is 1145 rev/min. The life goal of 5-year 24-hour-per-day service is about $3(10^9)$ pinion revolutions. The absolute value of the pitch variation is such that the transmission accuracy level number is 6. The materials are 4340 through-hardened grade 1 steels, heat-treated to 250 Brinell, core and case, both gears. The load is moderate shock and the power is smooth. For a reliability of 0.99, rate the speed reducer for power.

14–25 The speed reducer of Prob. 14–24 is to be used for an application requiring 40 hp at 1145 rev/min. Estimate the stresses of pinion bending, gear bending, pinion wear, and gear wear and the attendant AGMA factors of safety $(S_F)_P$, $(S_F)_G$, $(S_H)_P$, and $(S_H)_G$. For the reducer, what is the factor of safety for unquantifiable exingencies in W^t? What mode of failure is the most threatening?

14–26 The gearset of Prob. 14–24 needs improvement of wear capacity. Toward this end the gears are nitrided so that the grade 1 materials have hardnesses as follows: The pinion core is 250 and the pinion case hardness is 390 Brinell, and the gear core hardness is 250 core and 390 case. Estimate the power rating for the new gearset.

14–27 The gearset of Prob. 14–24 has had its gear specification changed to 9310 for carburizing and surface hardening with the result that the pinion Brinell hardnesses are 285 core and 580–600 case, and the gear hardnesses are 285 core and 580–600 case. Estimate the power rating for the new gearset.

14–28 The gearset of Prob. 14–27 is going to be upgraded in material to a quality of grade 2 9310 steel. Estimate the power rating for the new gearset.

14–29 Matters of scale always improve insight and perspective. Reduce the physical size of the gearset in Prob. 14–24 by one-half and note the result on the estimates of transmitted load W^t and power.

14–30 AGMA procedures with cast-iron gear pairs differ from those with steels because life predictions are difficult; consequently $(Y_N)_P$, $(Y_N)_G$, $(Z_N)_P$, and $(Z_N)_G$ are set to unity. The consequence of this is that the fatigue strengths of the pinion and gear materials are the same. The reliability is 0.99 and the life is 10^7 revolution of the pinion $(K_R = 1)$. For longer lives the reducer is derated in power. For the pinion and gear set of Prob. 14–24, use grade 40 cast iron for both gears $(H_B = 201$ Brinell). Rate the reducer for power with S_F and S_H equal to unity.

14–31 Spur-gear teeth have rolling and slipping contact (often about 8 percent slip). Spur gears tested to wear failure are reported at 10^8 cycles as Buckingham's surface fatigue load-stress factor K. This factor is related to Hertzian contact strength S_C by

$$S_C = \sqrt{\frac{1.4K}{(1/E_1 + 1/E_2)\sin\phi}}$$

where ϕ is the normal pressure angle. Cast iron grade 20 gears with $\phi = 14\frac{1}{2}^\circ$ and 20° pressure angle exhibit a minimum K of 81 and 112 psi, respectively. How does this compare with $S_C = 0.32H_B$ kpsi?

14–32 You've probably noticed that although the AGMA method is based on two equations, the details of assembling all the factors is computationally intensive. To reduce error and omissions, a computer program would be useful. Write a program to perform a power rating of an existing gearset, then use Prob. 14–24, 14–26, 14–27, 14–28, and 14–29 to test your program by comparing the results to your longhand solutions.

14–33 In Ex. 14–5 use nitrided grade 1 steel (4140) which produces Brinell hardnesses of 250 core and 500 at the surface (case). Use the upper fatigue curves on Figs. 14–14 and 14–15. Estimate the power capacity of the mesh with factors of safety of $S_F = S_H = 1$.

14–34 In Ex. 14–5 use carburized and case-hardened gears of grade 1. Carburizing and case-hardening can produce a 550 Brinell case. The core hardnesses are 200 Brinell. Estimate the power capacity of the mesh with factors of safety of $S_F = S_H = 1$, using the lower fatigue curves in Figs. 14–14 and 14–15.

14–35 In Ex. 14–5, use carburized and case-hardened gears of grade 2 steel. The core hardnesses are 200, and surface hardnesses are 600 Brinell. Use the lower fatigue curves of Figs. 14–14 and 14–15. Estimate the power capacity of the mesh using $S_F = S_H = 1$. Compare the power capacity with the results of Prob. 14–34.

15 Bevel and Worm Gears

Chapter Outline

The American Gear Manufacturers Association (AGMA) has established standards for the analysis and design of the various kinds of bevel and worm gears. Chapter 14 was an introduction to the AGMA methods for spur and helical gears. AGMA has established similar methods for other types of gearing, which all follow the same general approach.

15–1 Bevel Gearing—General

Bevel gears may be classified as follows:

- Straight bevel gears
- Spiral bevel gears
- Zerol bevel gears
- Hypoid gears
- Spiroid gears

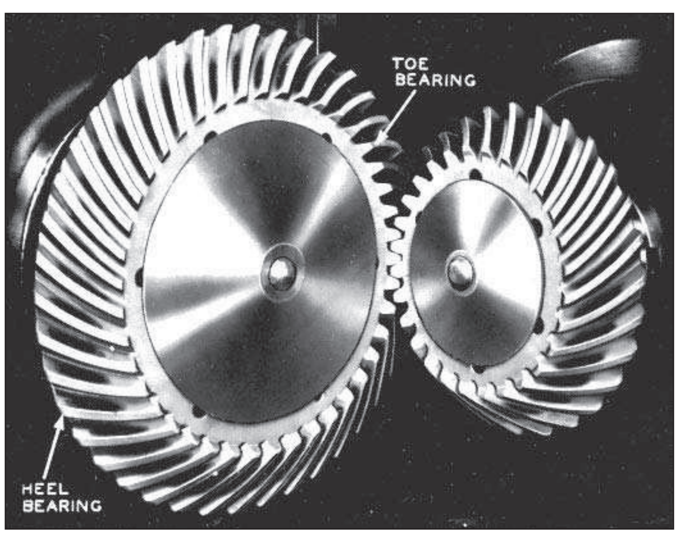
A straight bevel gear was illustrated in Fig. 13–35. These gears are usually used for pitch-line velocities up to 1000 ft/min (5 m/s) when the noise level is not an important consideration. They are available in many stock sizes and are less expensive to produce than other bevel gears, especially in small quantities.

A *spiral bevel* gear is shown in Fig. 15–1; the definition of the *spiral angle* is illustrated in Fig. 15–2. These gears are recommended for higher speeds and where the noise level is an important consideration. Spiral bevel gears are the bevel counterpart of the helical gear; it can be seen in Fig. 15–1 that the pitch surfaces and the nature of contact are the same as for straight bevel gears except for the differences brought about by the spiral-shaped teeth.

The *Zerol bevel gear* is a patented gear having curved teeth but with a zero spiral angle. The axial thrust loads permissible for Zerol bevel gears are not as large as those for the spiral bevel gear, and so they are often used instead of straight bevel gears. The Zerol bevel gear is generated by the same tool used for regular spiral bevel gears. For design purposes, use the same procedure as for straight bevel gears and then simply substitute a Zerol bevel gear.

Figure 15–1

Spiral bevel gears. *(Courtesy of Gleason Works, Rochester, N.Y.)*



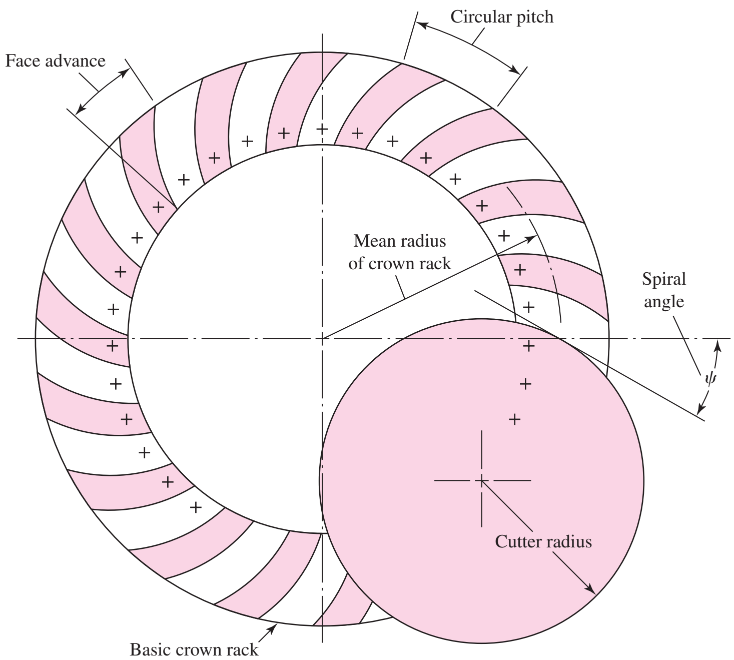


Figure 15–2

Cutting spiral-gear teeth on the basic crown rack.

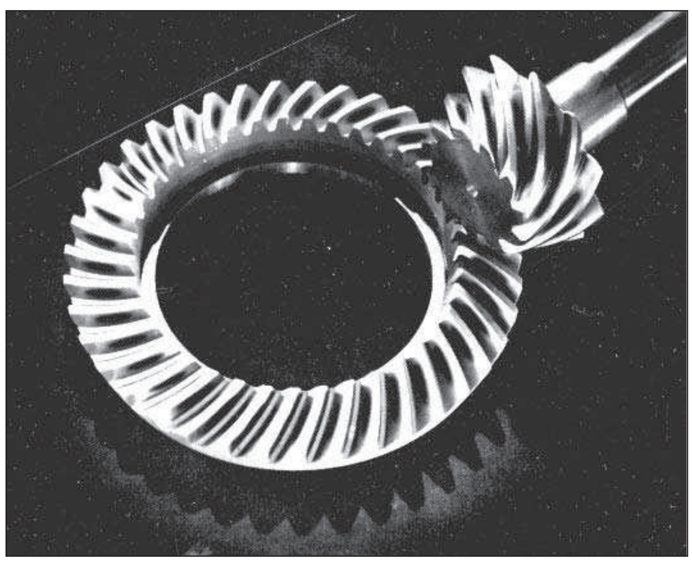


Figure 15–3

Hypoid gears. *(Courtesy of Gleason Works, Rochester, N.Y.)*

It is frequently desirable, as in the case of automotive differential applications, to have gearing similar to bevel gears but with the shafts offset. Such gears are called *hypoid gears,* because their pitch surfaces are hyperboloids of revolution. The tooth action between such gears is a combination of rolling and sliding along a straight line and has much in common with that of worm gears. Figure 15–3 shows a pair of hypoid gears in mesh.

Figure 15–4 is included to assist in the classification of spiral bevel gearing. It is seen that the hypoid gear has a relatively small shaft offset. For larger offsets, the pinion begins to resemble a tapered worm and the set is then called *spiroid gearing*.

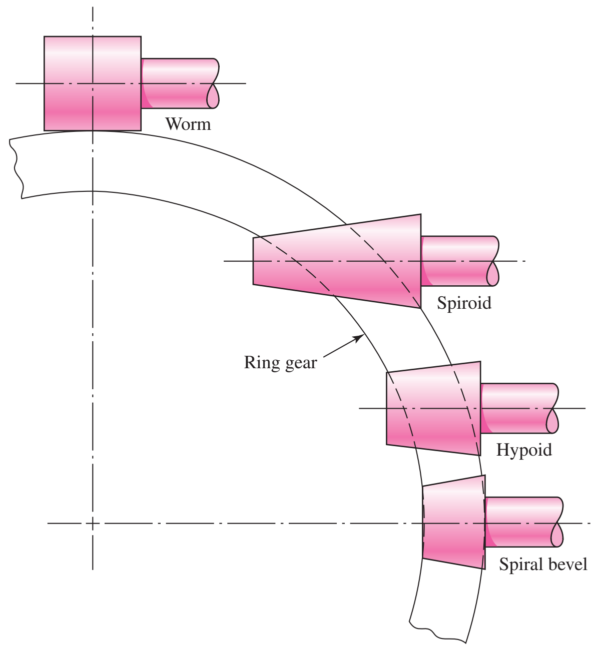


Figure 15–4

Comparison of intersecting- and offset-shaft bevel-type gearings. *(From* Gear Handbook *by Darle W. Dudley, 1962, p. 2–24.)*

15–2 Bevel-Gear Stresses and Strengths

In a typical bevel-gear mounting, Fig. 13–36, for example, one of the gears is often mounted outboard of the bearings. This means that the shaft deflections can be more pronounced and can have a greater effect on the nature of the tooth contact. Another difficulty that occurs in predicting the stress in bevel-gear teeth is the fact that the teeth are tapered. Thus, to achieve perfect line contact passing through the cone center, the teeth ought to bend more at the large end than at the small end. To obtain this condition requires that the load be proportionately greater at the large end. Because of this varying load across the face of the tooth, it is desirable to have a fairly short face width.

Because of the complexity of bevel, spiral bevel, Zerol bevel, hypoid, and spiroid gears, as well as the limitations of space, only a portion of the applicable standards that refer to straight-bevel gears is presented here.[1] Table 15–1 gives the symbols used in ANSI/AGMA 2003-B97.

Fundamental Contact Stress Equation

$$s_c = \sigma_c = C_p \left(\frac{W^t}{F d_P I} K_o K_v K_m C_s C_{xc} \right)^{1/2} \qquad \text{(U.S. customary units)}$$

$$\sigma_H = Z_E \left(\frac{1000 W^t}{b d Z_1} K_A K_v K_{H\beta} Z_x Z_{xc} \right)^{1/2} \qquad \text{(SI units)} \qquad (15\text{–}1)$$

The first term in each equation is the AGMA symbol, whereas; σ_c, our normal notation, is directly equivalent.

[1]Figures 15–5 to 15–13 and Tables 15–1 to 15–7 have been extracted from ANSI/AGMA 2003-B97, *Rating the Pitting Resistance and Bending Strength of Generated Straight Bevel, Zerol Bevel and Spiral Bevel Gear Teeth* with the permission of the publisher, the American Gear Manufacturers Association, 500 Montgomery Street, Suite 350, Alexandria, VA, 22314-1560

Table 15–1

Symbols Used in Bevel Gear Rating Equations, ANSI/AGMA 2003-B97 Standard *Source:* ANSI/AGMA 2003-B97.

AGMA Symbol	ISO Symbol	Description	Units
A_m	R_m	Mean cone distance	in (mm)
A_0	R_e	Outer cone distance	in (mm)
C_H	Z_W	Hardness ratio factor for pitting resistance	
C_i	Z_i	Inertia factor for pitting resistance	
C_L	Z_{NT}	Stress cycle factor for pitting resistance	
C_p	Z_E	Elastic coefficient	$[\text{lbf/in}^2]^{0.5}$ ($[\text{N/mm}^2]^{0.5}$)
C_R	Z_Z	Reliability factor for pitting	
C_{SF}		Service factor for pitting resistance	
C_S	Z_x	Size factor for pitting resistance	
C_{xc}	Z_{xc}	Crowning factor for pitting resistance	
D, d	d_{e2}, d_{e1}	Outer pitch diameters of gear and pinion, respectively	in (mm)
E_G, E_P	E_2, E_1	Young's modulus of elasticity for materials of gear and pinion, respectively	lbf/in^2 (N/mm^2)
e	e	Base of natural (Napierian) logarithms	
F	b	Net face width	in (mm)
F_{eG}, F_{eP}	b'_2, b'_1	Effective face widths of gear and pinion, respectively	in (mm)
f_P	R_{a1}	Pinion surface roughness	μin (μm)
H_{BG}	H_{B2}	Minimum Brinell hardness number for gear material	HB
H_{BP}	H_{B1}	Minimum Brinell hardness number for pinion material	HB
h_c	$E_{ht\,\min}$	Minimum total case depth at tooth middepth	in (mm)
h_e	h'_c	Minimum effective case depth	in (mm)
$h_{e\,\lim}$	$h'_{c\,\lim}$	Suggested maximum effective case depth limit at tooth middepth	in (mm)
I	Z_I	Geometry factor for pitting resistance	
J	Y_J	Geometry factor for bending strength	
J_G, J_P	Y_{J2}, Y_{J1}	Geometry factor for bending strength for gear and pinion, respectively	
K_F	Y_F	Stress correction and concentration factor	
K_i	Y_i	Inertia factor for bending strength	
K_L	Y_{NT}	Stress cycle factor for bending strength	
K_m	$K_{H\beta}$	Load distribution factor	
K_o	K_A	Overload factor	
K_R	Y_z	Reliability factor for bending strength	
K_S	Y_X	Size factor for bending strength	
K_{SF}		Service factor for bending strength	
K_T	K_θ	Temperature factor	
K_v	K_v	Dynamic factor	
K_x	Y_β	Lengthwise curvature factor for bending strength	
	m_{et}	Outer transverse module	(mm)
	m_{mt}	Mean transverse module	(mm)
	m_{mn}	Mean normal module	(mm)
m_{NI}	ε_{NI}	Load sharing ratio, pitting	
m_{NJ}	ε_{NJ}	Load sharing ratio, bending	
N	z_2	Number of gear teeth	
N_L	n_L	Number of load cycles	
n	z_1	Number of pinion teeth	
n_P	n_1	Pinion speed	rev/min

(Continued)

Table 15–1

Symbols Used in Gear Rating Equations, ANSI/AGMA 2003-B97 Standard (*Continued*)

AGMA Symbol	ISO Symbol	Description	Units
P	P	Design power through gear pair	hp (kW)
P_a	P_a	Allowable transmitted power	hp (kW)
P_{ac}	P_{az}	Allowable transmitted power for pitting resistance	hp (kW)
P_{acu}	P_{azu}	Allowable transmitted power for pitting resistance at unity service factor	hp (kW)
P_{at}	P_{ay}	Allowable transmitted power for bending strength	hp (kW)
P_{atu}	P_{ayu}	Allowable transmitted power for bending strength at unity service factor	hp (kW)
P_d		Outer transverse diametral pitch	in^{-1}
P_m		Mean transverse diametral pitch	in^{-1}
P_{mn}		Mean normal diametral pitch	in^{-1}
Q_v	Q_v	Transmission accuracy number	
q	q	Exponent used in formula for lengthwise curvature factor	
R, r	r_{mpt2}, r_{mpt1}	Mean transverse pitch radii for gear and pinion, respectively	in (mm)
R_t, r_t	r_{myo2}, r_{myo1}	Mean transverse radii to point of load application for gear and pinion, respectively	in (mm)
r_c	r_{c0}	Cutter radius used for producing Zerol bevel and spiral bevel gears	in (mm)
s	g_c	Length of the instantaneous line of contact between mating tooth surfaces	in (mm)
s_{ac}	$\sigma_{H\lim}$	Allowable contact stress number	lbf/in^2 (N/mm^2)
s_{at}	$\sigma_{F\lim}$	Bending stress number (allowable)	lbf/in^2 (N/mm^2)
s_c	σ_H	Calculated contact stress number	lbf/in^2 (N/mm^2)
s_F	s_F	Bending safety factor	
s_H	s_H	Contact safety factor	
s_t	σ_F	Calculated bending stress number	lbf/in^2 (N/mm^2)
s_{wc}	σ_{HP}	Permissible contact stress number	lbf/in^2 (N/mm^2)
s_{wt}	σ_{FP}	Permissible bending stress number	lbf/in^2 (N/mm^2)
T_P	T_1	Operating pinion torque	lbf in (Nm)
T_T	θ_T	Operating gear blank temperature	°F(°C)
t_O	s_{ai}	Normal tooth top land thickness at narrowest point	in (mm)
U_c	U_c	Core hardness coefficient for nitrided gear	lbf/in^2 (N/mm^2)
U_H	U_H	Hardening process factor for steel	lbf/in^2 (N/mm^2)
v_t	v_{et}	Pitch-line velocity at outer pitch circle	ft/min (m/s)
Y_{KG}, Y_{KP}	Y_{K2}, Y_{K1}	Tooth form factors including stress-concentration factor for gear and pinion, respectively	
μ_G, μ_P	ν_2, ν_1	Poisson's ratio for materials of gear and pinion, respectively	
ρ_O	ρ_{Y_O}	Relative radius of profile curvature at point of maximum contact stress between mating tooth surfaces	in (mm)
ϕ	α_n	Normal pressure angle at pitch surface	
ϕ_t	α_{wt}	Transverse pressure angle at pitch point	
ψ	β_m	Mean spiral angle at pitch surface	
ψ_b	β_{mb}	Mean base spiral angle	

Permissible Contact Stress Number (Strength) Equation

$$s_{wc} = (\sigma_c)_{\text{all}} = \frac{s_{ac} C_L C_H}{S_H K_T C_R} \qquad \text{(U.S. customary units)}$$

$$\sigma_{HP} = \frac{\sigma_{H\,\text{lim}} Z_{NT} Z_W}{S_H K_\theta Z_Z} \qquad \text{(SI units)} \tag{15–2}$$

Bending Stress

$$s_t = \frac{W^t}{F} P_d K_o K_v \frac{K_s K_m}{K_x J} \qquad \text{(U.S. customary units)}$$

$$\sigma_F = \frac{1000 W^t}{b} \frac{K_A K_v}{m_{et}} \frac{Y_x K_{H\beta}}{Y_\beta Y_J} \qquad \text{(SI units)} \tag{15–3}$$

Permissible Bending Stress Equation

$$s_{wt} = \frac{s_{at} K_L}{S_F K_T K_R} \qquad \text{(U.S. customary units)}$$

$$\sigma_{FP} = \frac{\sigma_{F\,\text{lim}} Y_{NT}}{S_F K_\theta Y_z} \qquad \text{(SI units)} \tag{15–4}$$

15–3 AGMA Equation Factors

Overload Factor K_o (K_A)

The overload factor makes allowance for any externally applied loads in excess of the nominal transmitted load. Table 15–2, from Appendix A of 2003-B97, is included for your guidance.

Safety Factors S_H and S_F

The factors of safety S_H and S_F as defined in 2003-B97 are adjustments to strength, not load, and consequently cannot be used as is to assess (by comparison) whether the threat is from wear fatigue or bending fatigue. Since W^t is the same for the pinion and gear, the comparison of $\sqrt{S_H}$ to S_F allows direct comparison.

Dynamic Factor K_v

In 2003-C87 AGMA changed the definition of K_v to its reciprocal but used the same symbol. Other standards have yet to follow this move. The dynamic factor K_v makes

Table 15–2

Overload Factors K_o (K_A)

Source: ANSI/AGMA 2003-B97.

Character of Prime Mover	Character of Load on Driven Machine: Uniform	Light Shock	Medium Shock	Heavy Shock
Uniform	1.00	1.25	1.50	1.75 or higher
Light shock	1.10	1.35	1.60	1.85 or higher
Medium shock	1.25	1.50	1.75	2.00 or higher
Heavy shock	1.50	1.75	2.00	2.25 or higher

Note: This table is for speed-decreasing drives. For speed-increasing drives, add $0.01(N/n)^2$ or $0.01(z_2/z_1)^2$ to the above factors.

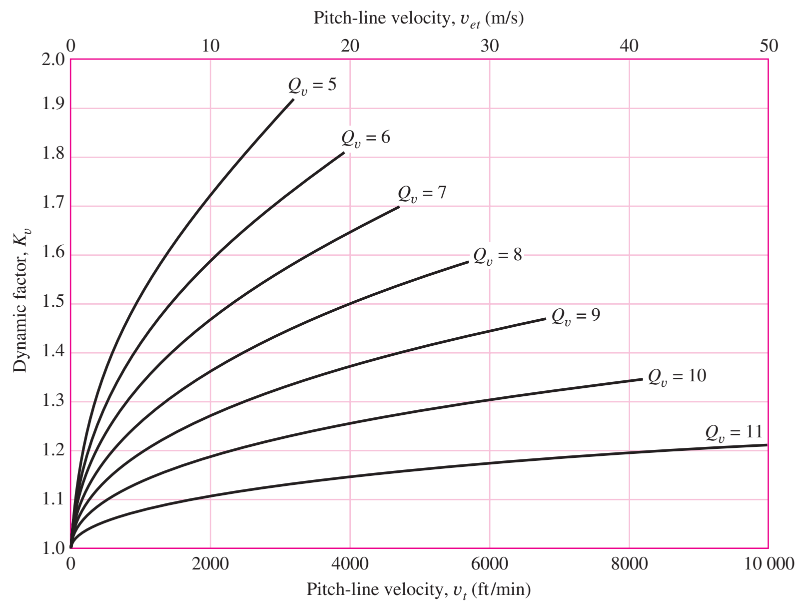


Figure 15–5

Dynamic factor K_v. *(Source: ANSI/AGMA 2003-B97.)*

allowance for the effect of gear-tooth quality related to speed and load, and the increase in stress that follows. AGMA uses a *transmission accuracy number* Q_v to describe the precision with which tooth profiles are spaced along the pitch circle. Figure 15–5 shows graphically how pitch-line velocity and transmission accuracy number are related to the dynamic factor K_v. Curve fits are

$$K_v = \left(\frac{A + \sqrt{v_t}}{A}\right)^B \qquad \text{(U.S. customary units)}$$

$$K_v = \left(\frac{A + \sqrt{200 v_{et}}}{A}\right)^B \qquad \text{(SI units)} \tag{15–5}$$

where

$$A = 50 + 56(1 - B)$$
$$B = 0.25(12 - Q_v)^{2/3} \tag{15–6}$$

and $v_t\,(v_{et})$ is the pitch-line velocity at outside pitch diameter, expressed in ft/min (m/s):

$$v_t = \pi d_P n_P / 12 \qquad \text{(U.S. customary units)}$$
$$v_{et} = 5.236(10^{-5}) d_1 n_1 \qquad \text{(SI units)} \tag{15–7}$$

The maximum recommended pitch-line velocity is associated with the abscissa of the terminal points of the curve in Fig. 15–5:

$$v_{t\,\max} = [A + (Q_v - 3)]^2 \qquad \text{(U.S. customary units)}$$

$$v_{te\,\max} = \frac{[A + (Q_v - 3)]^2}{200} \qquad \text{(SI units)} \tag{15–8}$$

where $v_{t\,\max}$ and $v_{et\,\max}$ are in ft/min and m/s, respectively.

Size Factor for Pitting Resistance C_s (Z_x)

$$C_s = \begin{cases} 0.5 & F < 0.5 \text{ in} \\ 0.125F + 0.4375 & 0.5 \le F \le 4.5 \text{ in} \\ 1 & F > 4.5 \text{ in} \end{cases} \qquad \text{(U.S. customary units)} \tag{15–9}$$

$$Z_x = \begin{cases} 0.5 & b < 12.7 \text{ mm} \\ 0.004\,92b + 0.4375 & 12.7 \le b \le 114.3 \text{ mm} \\ 1 & b > 114.3 \text{ mm} \end{cases} \qquad \text{(SI units)}$$

Size Factor for Bending K_s (Y_x)

$$K_s = \begin{cases} 0.4867 + 0.2132/P_d & 0.5 \le P_d \le 16 \text{ in}^{-1} \\ 0.5 & P_d > 16 \text{ in}^{-1} \end{cases} \qquad \text{(U.S. customary units)}$$

$$Y_x = \begin{cases} 0.5 & m_{et} < 1.6 \text{ mm} \\ 0.4867 + 0.008\,339m_{et} & 1.6 \le m_{et} \le 50 \text{ mm} \end{cases} \qquad \text{(SI units)} \tag{15–10}$$

Load-Distribution Factor K_m ($K_{H\beta}$)

$$\begin{aligned} K_m &= K_{mb} + 0.0036F^2 && \text{(U.S. customary units)} \\ K_{H\beta} &= K_{mb} + 5.6(10^{-6})b^2 && \text{(SI units)} \end{aligned} \tag{15–11}$$

where

$$K_{mb} = \begin{cases} 1.00 & \text{both members straddle-mounted} \\ 1.10 & \text{one member straddle-mounted} \\ 1.25 & \text{neither member straddle-mounted} \end{cases}$$

Crowning Factor for Pitting C_{xc} (Z_{xc})

The teeth of most bevel gears are crowned in the lengthwise direction during manufacture to accommodate to the deflection of the mountings.

$$C_{xc} = Z_{xc} = \begin{cases} 1.5 & \text{properly crowned teeth} \\ 2.0 & \text{or larger uncrowned teeth} \end{cases} \tag{15–12}$$

Lengthwise Curvature Factor for Bending Strength K_x (Y_β)

For straight-bevel gears,

$$K_x = Y_\beta = 1 \tag{15–13}$$

Pitting Resistance Geometry Factor I (Z_I)

Figure 15–6 shows the geometry factor I (Z_I) for straight-bevel gears with a 20° pressure angle and 90° shaft angle. Enter the figure ordinate with the number of pinion teeth, move to the number of gear-teeth contour, and read from the abscissa.

Bending Strength Geometry Factor J (Y_J)

Figure 15–7 shows the geometry factor J for straight-bevel gears with a 20° pressure angle and 90° shaft angle.

Figure 15–6

Contact geometry factor I (Z_I) for coniflex straight-bevel gears with a 20° normal pressure angle and a 90° shaft angle. *(Source: ANSI/AGMA 2003-B97.)*

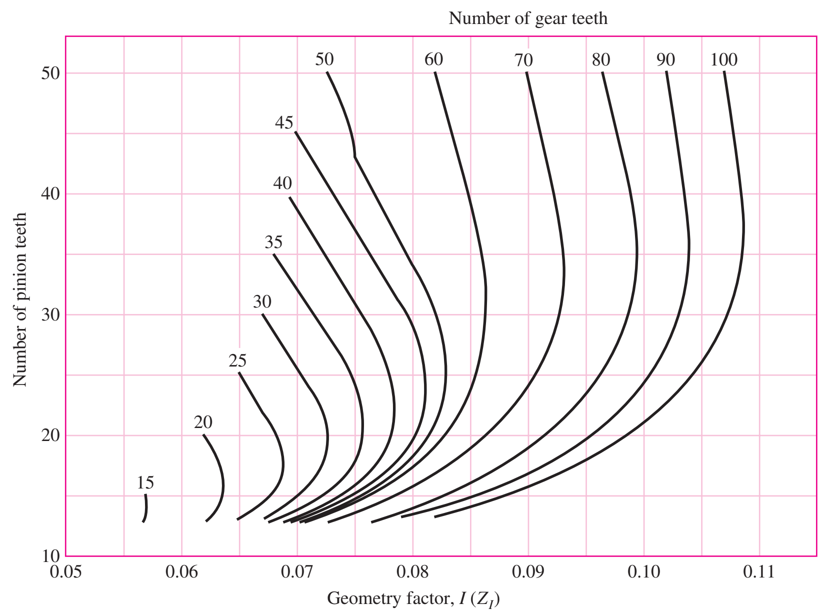
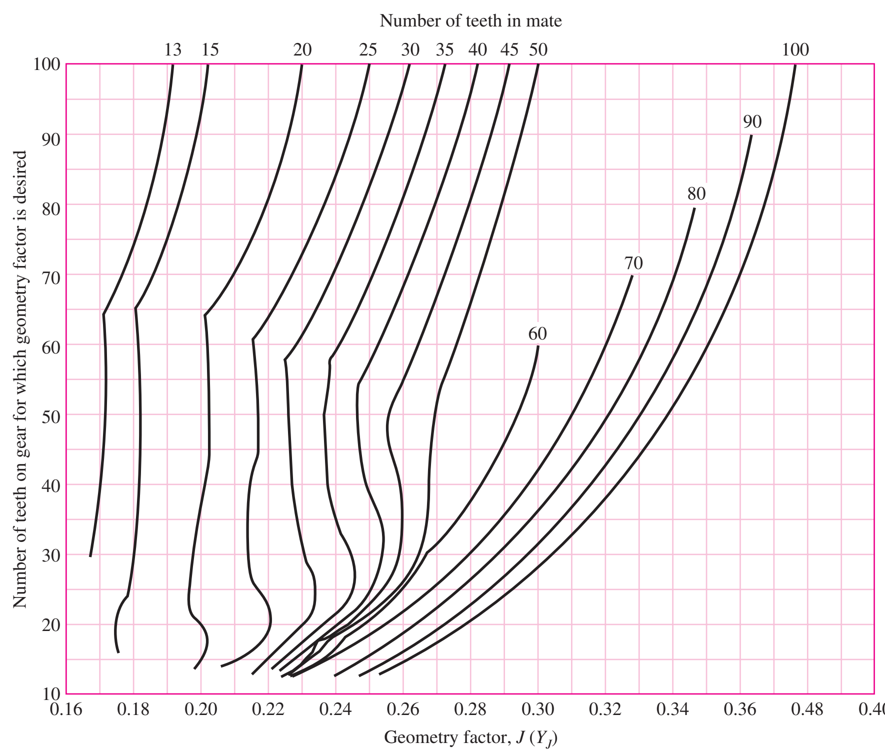


Figure 15–7

Bending factor J (Y_J) for coniflex straight-bevel gears with a 20° normal pressure angle and 90° shaft angle. *(Source: ANSI/AGMA 2003-B97.)*



Stress-Cycle Factor for Pitting Resistance C_L (Z_{NT})

$$C_L = \begin{cases} 2 & 10^3 \le N_L < 10^4 \\ 3.4822 N_L^{-0.0602} & 10^4 \le N_L \le 10^{10} \end{cases}$$

$$Z_{NT} = \begin{cases} 2 & 10^3 \le n_L < 10^4 \\ 3.4822 n_L^{-0.0602} & 10^4 \le n_L \le 10^{10} \end{cases} \tag{15–14}$$

See Fig. 15–8 for a graphical presentation of Eqs. (15–14).

Stress-Cycle Factor for Bending Strength K_L (Y_{NT})

$$K_L = \begin{cases} 2.7 & 10^2 \le N_L < 10^3 \\ 6.1514 N_L^{-0.1182} & 10^3 \le N_L < 3(10^6) \\ 1.6831 N_L^{-0.0323} & 3(10^6) \le N_L \le 10^{10} \quad \text{general} \\ 1.3558 N_L^{-0.0178} & 3(10^6) \le N_L \le 10^{10} \quad \text{critical} \end{cases}$$

$$Y_{NT} = \begin{cases} 2.7 & 10^2 \le n_L < 10^3 \\ 6.1514 n_L^{-0.1182} & 10^3 \le n_L < 3(10^6) \\ 1.6831 n_L^{-0.0323} & 3(10^6) \le n_L \le 10^{10} \quad \text{general} \\ 1.3558 n_L^{-0.0323} & 3(10^6) \le n_L \le 10^{10} \quad \text{critical} \end{cases} \tag{15–15}$$

See Fig. 15–9 for a plot of Eqs. (15–15).

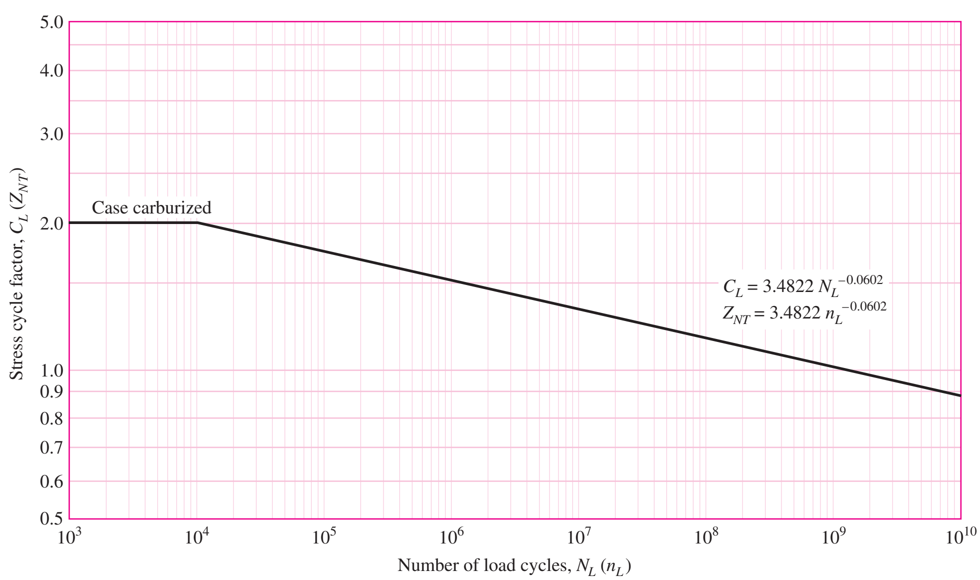


Figure 15–8

Contact stress cycle factor for pitting resistance C_L (Z_{NT}) for carburized case-hardened steel bevel gears. *(Source: ANSI/AGMA 2003-B97.)*

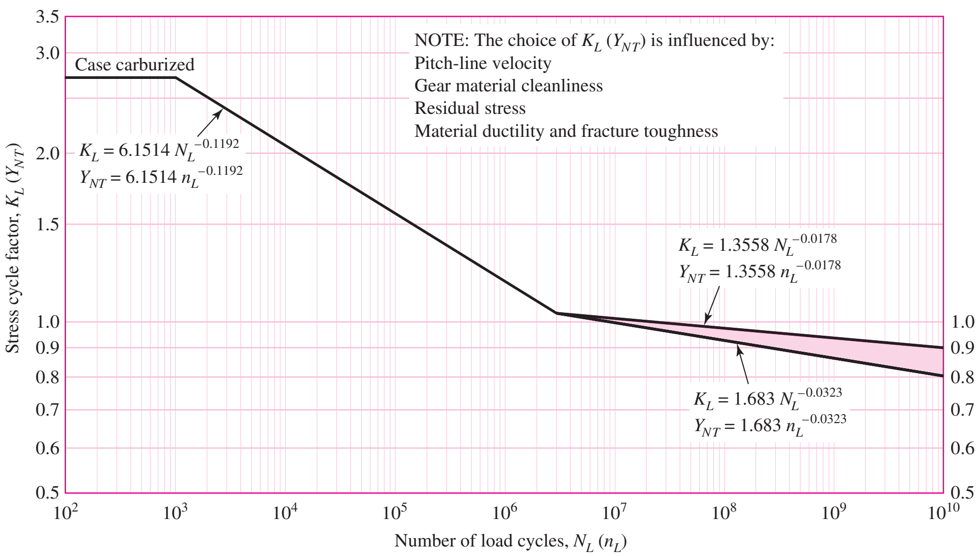


Figure 15–9

Stress cycle factor for bending strength K_L (Y_{NT}) for carburized case-hardened steel bevel gears. *(Source: ANSI/AGMA 2003-B97.)*

Hardness-Ratio Factor C_H (Z_W)

$$C_H = 1 + B_1(N/n - 1) \qquad B_1 = 0.008\,98(H_{BP}/H_{BG}) - 0.008\,29$$
$$Z_W = 1 + B_1(z_1/z_2 - 1) \qquad B_1 = 0.008\,98(H_{B1}/H_{B2}) - 0.008\,29 \tag{15–16}$$

The preceding equations are valid when $1.2 \le H_{BP}/H_{BG} \le 1.7$ ($1.2 \le H_{B1}/H_{B2} \le 1.7$). Figure 15–10 graphically displays Eqs. (15–16). When a surface-hardened pinion (48 HRC or harder) is run with a through-hardened gear ($180 \le H_B \le 400$), a work-hardening effect occurs. The $C_H(Z_W)$ factor varies with pinion surface roughness $f_P(R_{a1})$ and the mating-gear hardness:

$$C_H = 1 + B_2(450 - H_{BG}) \qquad B_2 = 0.000\,75\exp(-0.0122 f_P)$$
$$Z_W = 1 + B_2(450 - H_{B2}) \qquad B_2 = 0.000\,75\exp(-0.52 f_P) \tag{15–17}$$

where $\quad f_P(R_{a1})$ = pinion surface hardness μin (μm)

$H_{BG}(H_{B2})$ = minimum Brinell hardness

See Fig. 15–11 for carburized steel gear pairs of approximately equal hardness $C_H = Z_W = 1$.

Temperature Factor K_T (K_θ)

$$K_T = \begin{cases} 1 & 32°\text{F} \le t \le 250°\text{F} \\ (460 + t)/710 & t > 250°\text{F} \end{cases}$$
$$K_\theta = \begin{cases} 1 & 0°\text{C} \le \theta \le 120°\text{C} \\ (273 + \theta)/393 & \theta > 120°\text{C} \end{cases} \tag{15–18}$$

Figure 15–10

Hardness-ratio factor C_H (Z_W) for through-hardened pinion and gear. *(Source: ANSI/AGMA 2003-B97.)*

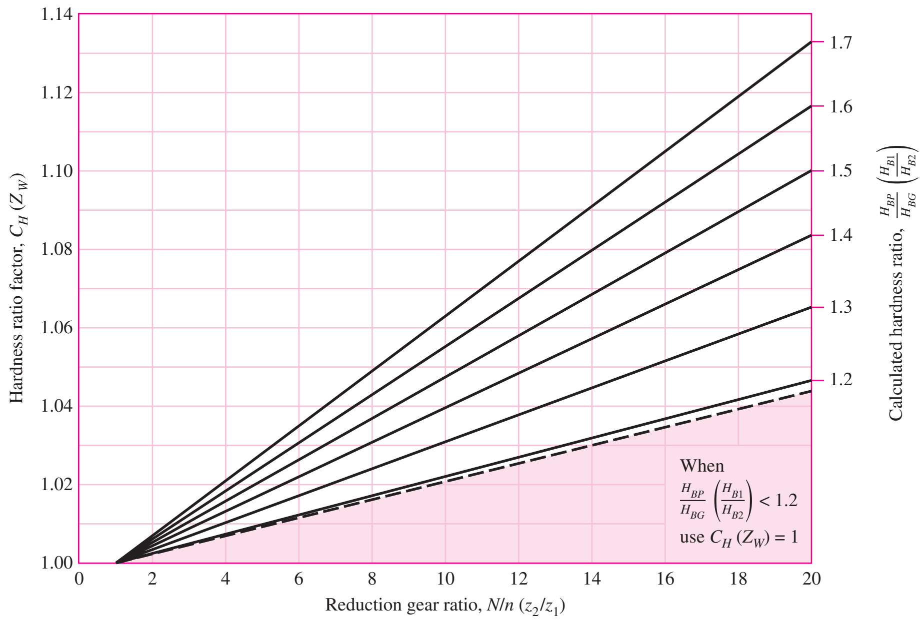
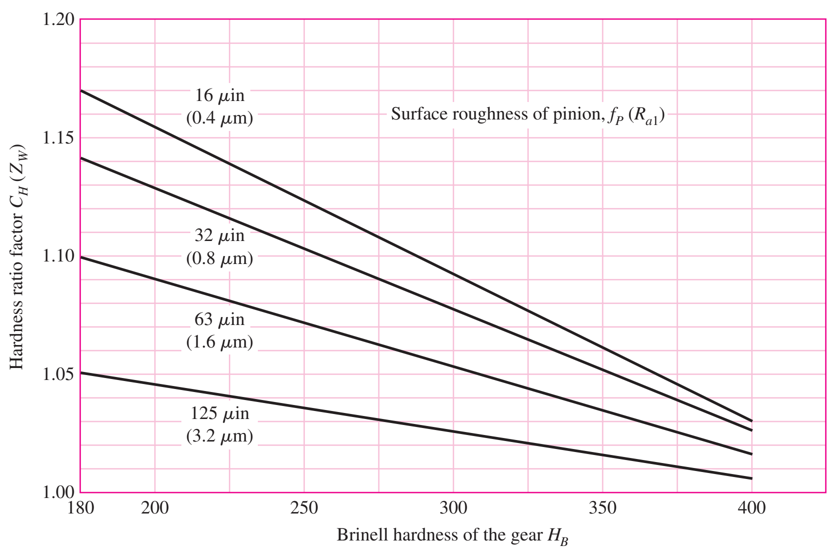


Figure 15–11

Hardness-ratio factor C_H (Z_W) for surface-hardened pinions. *(Source: ANSI/AGMA 2003-B97.)*



Reliability Factors C_R (Z_Z) and K_R (Y_Z)

Table 15–3 displays the reliability factors. Note that $C_R = \sqrt{K_R}$ and $Z_Z = \sqrt{Y_Z}$. Logarithmic interpolation equations are

$$Y_Z = K_R = \begin{cases} 0.50 - 0.25\ \log(1-R) & 0.99 \le R \le 0.999 \\ 0.70 - 0.15\ \log(1-R) & 0.90 \le R < 0.99 \end{cases} \qquad \begin{matrix}(15\text{–}19)\\(15\text{–}20)\end{matrix}$$

The reliability of the stress (fatigue) numbers allowable in Tables 15–4, 15–5, 15–6, and 15–7 is 0.99.

Table 15–3

Reliability Factors

Source: ANSI/AGMA 2003-B97.

Requirements of Application	Reliability Factors for Steel* C_R (Z_Z)	K_R (Y_Z)†
Fewer than one failure in 10 000	1.22	1.50
Fewer than one failure in 1000	1.12	1.25
Fewer than one failure in 100	1.00	1.00
Fewer than one failure in 10	0.92	0.85‡
Fewer than one failure in 2	0.84	0.70§

*At the present time there are insufficient data concerning the reliability of bevel gears made from other materials.

†Tooth breakage is sometimes considered a greater hazard than pitting. In such cases a greater value of K_R (Y_Z) is selected for bending.

‡At this value plastic flow might occur rather than pitting.

§From test data extrapolation.

Table 15–4

Allowable Contact Stress Number for Steel Gears, s_{ac} $(\sigma_{H\,\mathrm{lim}})$ *Source:* ANSI/AGMA 2003-B97.

Material Designation	Heat Treatment	Minimum Surface* Hardness	Allowable Contact Stress Number, s_{ac} $(\sigma_{H\,\mathrm{lim}})$ lbf/in² (N/mm²) Grade 1†	Grade 2†	Grade 3†
Steel	Through-hardened‡	Fig. 15–12	Fig. 15–12	Fig. 15–12	
	Flame or induction hardened§	50 HRC	175 000 (1210)	190 000 (1310)	
	Carburized and case hardened§	2003-B97 Table 8	200 000 (1380)	225 000 (1550)	250 000 (1720)
AISI 4140	Nitrided§	84.5 HR15N		145 000 (1000)	
Nitralloy 135M	Nitrided§	90.0 HR15N		160 000 (1100)	

*Hardness to be equivalent to that at the tooth middepth in the center of the face width.

†See ANSI/AGMA 2003-B97, Tables 8 through 11, for metallurgical factors for each stress grade of steel gears.

‡These materials must be annealed or normalized as a minumum.

§The allowable stress numbers indicated may be used with the case depths prescribed in 21.1, ANSI/AGMA 2003-B97.

Elastic Coefficient for Pitting Resistance C_p (Z_E)

$$C_p = \sqrt{\frac{1}{\pi\left[\left(1-\nu_P^2\right)/E_P + \left(1-\nu_G^2\right)/E_G\right]}}$$

$$Z_E = \sqrt{\frac{1}{\pi\left[\left(1-\nu_1^2\right)/E_1 + \left(1-\nu_2^2\right)/E_2\right]}} \tag{15–21}$$

Table 15–5

Allowable Contact Stress Number for Iron Gears, s_{ac} $(\sigma_{H\,\mathrm{lim}})$ *Source:* ANSI/AGMA 2003-B97.

Material Designation: Material	Material Designation: ASTM	Material Designation: ISO	Heat Treatment	Typical Minimum Surface Hardness	Allowable Contact Stress Number, s_{ac} $(\sigma_{H\,\mathrm{lim}})$ lbf/in^2 (N/mm^2)
Cast iron	ASTM A48	ISO/DR 185			
	Class 30	Grade 200	As cast	175 HB	50 000 (345)
	Class 40	Grade 300	As cast	200 HB	65 000 (450)
Ductile	ASTM A536	ISO/DIS 1083			
(nodular)	Grade 80-55-06	Grade 600-370-03	Quenched	180 HB	94 000 (650)
iron	Grade 120-90-02	Grade 800-480-02	and tempered	300 HB	135 000 (930)

Table 15–6

Allowable Bending Stress Numbers for Steel Gears, s_{at} $(\sigma_{F\,\mathrm{lim}})$ *Source:* ANSI/AGMA 2003-B97.

Material Designation	Heat Treatment	Minimum Surface Hardness	Bending Stress Number (Allowable), s_{at} $(\sigma_{F\,\mathrm{lim}})$ lbf/in^2 (N/mm^2): Grade 1*	Grade 2*	Grade 3*
Steel	Through-hardened	Fig. 15–13	Fig. 15–13	Fig. 15–13	
	Flame or induction hardened				
	Unhardened roots	50 HRC	15 000 (85)	13 500 (95)	
	Hardened roots		22 500 (154)		
	Carburized and case hardened†	2003-B97 Table 8	30 000 (205)	35 000 (240)	40 000 (275)
AISI 4140	Nitrided†,‡	84.5 HR15N		22 000 (150)	
Nitralloy 135M	Nitrided†,‡	90.0 HR15N		24 000 (165)	

*See ANSI/AGMA 2003-B97, Tables 8–11, for metallurgical factors for each stress grade of steel gears.

†The allowable stress numbers indicated may be used with the case depths prescribed in 21.1, ANSI/AGMA 2003-B97.

‡The overload capacity of nitrided gears is low. Since the shape of the effective S-N curve is flat, the sensitivity to shock should be investigated before proceeding with the design.

where

$$C_p = \text{elastic coefficient, } 2290\sqrt{\text{psi}} \text{ for steel}$$

$$Z_E = \text{elastic coefficient, } 190\sqrt{\text{N/mm}^2} \text{ for steel}$$

$$E_P \text{ and } E_G = \text{Young's moduli for pinion and gear respectively, psi}$$

$$E_1 \text{ and } E_2 = \text{Young's moduli for pinion and gear respectively, N/mm}^2$$

Allowable Contact Stress

Tables 15–4 and 15–5 provide values of $s_{ac}(\sigma_H)$ for steel gears and for iron gears, respectively. Figure 15–12 graphically displays allowable stress for grade 1 and 2 materials.

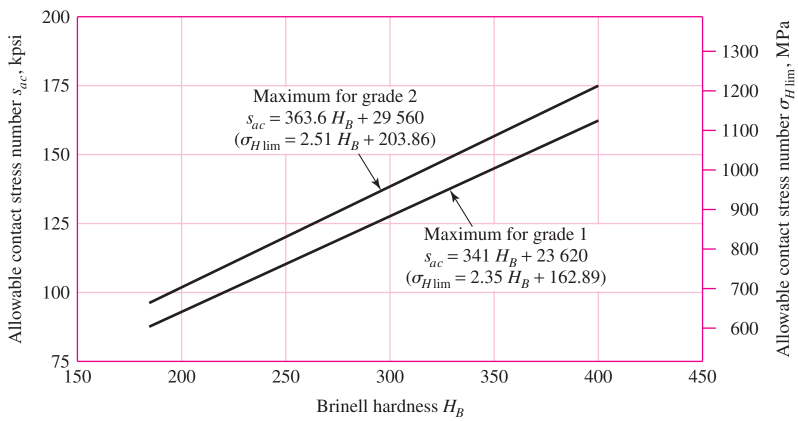
Table 15–7

Allowable Bending Stress Number for Iron Gears, s_{at} ($\sigma_{F\,\mathrm{lim}}$) *Source:* ANSI/AGMA 2003-B97.

Material	ASTM (Material Designation)	ISO (Material Designation)	Heat Treatment	Typical Minimum Surface Hardness	Bending Stress Number (Allowable), s_{at} ($\sigma_{F\,\mathrm{lim}}$) lbf/in^2 (N/mm^2)
Cast iron	ASTM A48	ISO/DR 185			
	Class 30	Grade 200	As cast	175 HB	4500 (30)
	Class 40	Grade 300	As cast	200 HB	6500 (45)
Ductile	ASTM A536	ISO/DIS 1083			
(nodular)	Grade 80-55-06	Grade 600-370-03	Quenched	180 HB	10 000 (70)
iron	Grade 120-90-02	Grade 800-480-02	and tempered	300 HB	13 500 (95)

Figure 15–12

Allowable contact stress number for through-hardened steel gears, $s_{ac}(\sigma_{H\,\mathrm{lim}})$. *(Source: ANSI/AGMA 2003-B97.)*



The equations are

$$\begin{aligned} s_{ac} &= 341H_B + 23\,620 \text{ psi} && \text{grade 1} \\ \sigma_{H\,\mathrm{lim}} &= 2.35H_B + 162.89 \text{ MPa} && \text{grade 1} \\ s_{ac} &= 363.6H_B + 29\,560 \text{ psi} && \text{grade 2} \\ \sigma_{H\,\mathrm{lim}} &= 2.51H_B + 203.86 \text{ MPa} && \text{grade 2} \end{aligned} \tag{15–22}$$

Allowable Bending Stress Numbers

Tables 15–6 and 15–7 provide $s_{at}(\sigma_{F\,\mathrm{lim}})$ for steel gears and for iron gears, respectively. Figure 15–13 shows graphically allowable bending stress $s_{at}(\sigma_{H\,\mathrm{lim}})$ for through-hardened steels. The equations are

$$\begin{aligned} s_{at} &= 44H_B + 2100 \text{ psi} && \text{grade 1} \\ \sigma_{F\,\mathrm{lim}} &= 0.30H_B + 14.48 \text{ MPa} && \text{grade 1} \\ s_{at} &= 48H_B + 5980 \text{ psi} && \text{grade 2} \\ \sigma_{H\,\mathrm{lim}} &= 0.33H_B + 41.24 \text{ MPa} && \text{grade 2} \end{aligned} \tag{15–23}$$

Reversed Loading

AGMA recommends use of 70 percent of allowable strength in cases where tooth load is completely reversed, as in idler gears and reversing mechanisms.

Summary

Figure 15–14 is a "roadmap" for straight-bevel gear wear relations using 2003-B97. Figure 15–15 is a similar guide for straight-bevel gear bending using 2003-B97.

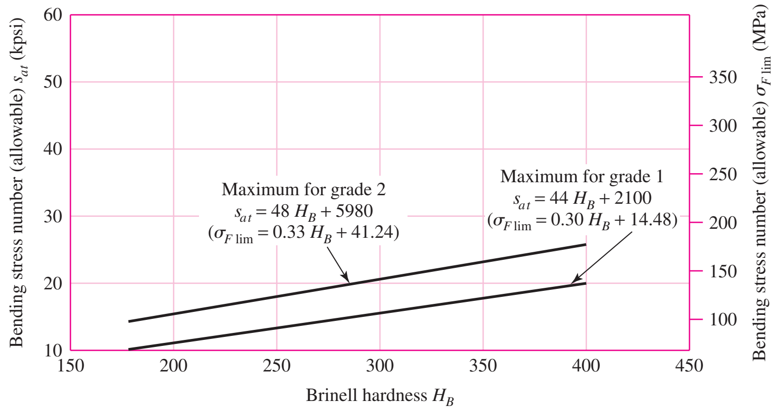


Figure 15–13

Allowable bending stress number for through-hardened steel gears, $s_{at}(\sigma_{F\,\lim})$. *(Source: ANSI/AGMA 2003-B97.)*

Figure 15–14

"Roadmap" summary of principal straight-bevel gear wear equations and their parameters.

STRAIGHT-BEVEL GEAR WEAR

Geometry	Force Analysis	Strength Analysis
$d_p = \dfrac{N_P}{P_d}$	$W^t = \dfrac{2T}{d_{av}}$	$W^t = \dfrac{2T}{d_p}$
$\gamma = \tan^{-1}\dfrac{N_P}{N_G}$	$W^r = W^t \tan\phi \cos\gamma$	$W^r = W^t \tan\phi \cos\gamma$
$\Gamma = \tan^{-1}\dfrac{N_G}{N_P}$	$W^a = W^t \tan\phi \sin\gamma$	$W^a = W^t \tan\phi \sin\gamma$
$d_{av} = d_p - F\cos\Gamma$		

Gear contact stress

At large end of tooth
Table 15-2, p. 771
Eqs. (15-5) to (15-8), p. 772
Eq. (15-11), p. 773

$$S_c = \sigma_c = C_p\left(\frac{W^t}{Fd_pI}K_oK_vK_mC_sC_{xc}\right)^{1/2}$$

Eq. (15-12), p. 773
Eq. (15-9), p. 773
Fig. 15-6, p. 774
Eq. (15-21), p. 778

Gear wear strength

Tables 15-4, 15-5, Fig. 15-12, Eq. (15-22), pp. 778–780
Fig. 15-8, Eq. (15-14), p. 775
Eqs. (15-16), (15-17), gear only, p. 776

$$S_{wc} = (\sigma_c)_{\text{all}} = \frac{s_{ac}C_LC_H}{S_HK_TC_R}$$

Eqs. (15-19), (15-20), Table 15-3, pp. 777, 778
Eq. (15-18), p. 776

Wear factor of safety

$S_H = \dfrac{(\sigma_c)_{\text{all}}}{\sigma_c}$, based on strength

$n_w = \left(\dfrac{(\sigma_c)_{\text{all}}}{\sigma_c}\right)^2$, based on W^t; can be compared directly with S_F

BASED ON ANSI/AGMA 2003-B97

Figure 15–15

"Roadmap" summary of principal straight-bevel gear bending equations and their parameters.

STRAIGHT-BEVEL GEAR BENDING

Geometry	Force Analysis	Strength Analysis
$d_p = \dfrac{N_P}{P}$	$W^t = \dfrac{2T}{d_{av}}$	$W^t = \dfrac{2T}{d_p}$
$\gamma = \tan^{-1}\dfrac{N_P}{N_G}$	$W^r = W^t \tan\phi \cos\gamma$	$W^r = W^t \tan\phi \cos\gamma$
$\Gamma = \tan^{-1}\dfrac{N_G}{N_P}$	$W^a = W^t \tan\phi \sin\gamma$	$W^a = W^t \tan\phi \sin\gamma$
$d_{av} = d_p - F\cos\Gamma$		

Gear bending stress

$$S_t = \sigma = \frac{W^t}{F} P_d K_o K_v \frac{K_s K_m}{K_x J}$$

At large end of tooth

Table 15-2, p. 771

Eqs. (15-5) to (15-8), p. 772

Eq. (15-10), p. 773

Eq. (15-11), p. 773

Fig. 15-7, p. 774

Eq. (15-13), p. 773

Gear bending strength

$$S_{wt} = \sigma_{all} = \frac{s_{at} K_L}{S_F K_T K_R}$$

Table 15-6 or 15-7, pp. 779, 780

Fig. 15-9, Eq. (15-15), pp. 776, 775

Eqs. (15-19), (15-20), Table 15-3 pp. 777, 778

Eq. (15-18) p. 776

Bending factor of safety

$S_F = \dfrac{\sigma_{all}}{\sigma}$, based on strength

$n_B = \dfrac{\sigma_{all}}{\sigma}$, based on W^t, same as S_F

BASED ON ANSI/AGMA 2003-B97

The standard does not mention specific steel but mentions the hardness attainable by heat treatments such as through-hardening, carburizing and case-hardening, flame-hardening, and nitriding. Through-hardening results depend on size (diametral pitch). Through-hardened materials and the corresponding Rockwell C-scale hardness at the 90 percent martensite shown in parentheses following include 1045 (50), 1060 (54), 1335 (46), 2340 (49), 3140 (49), 4047 (52), 4130 (44), 4140 (49), 4340 (49), 5145 (51), E52100 (60), 6150 (53), 8640 (50), and 9840 (49). For carburized case-hard materials the approximate core hardnesses are 1015 (22), 1025 (37), 1118 (33), 1320 (35), 2317 (30), 4320 (35), 4620 (35), 4820 (35), 6120 (35), 8620 (35), and E9310 (30). The conversion from HRC to H_B (300-kg load, 10-mm ball) is

HRC	42	40	38	36	34	32	30	28	26	24	22	20	18	16	14	12	10
H_B	388	375	352	331	321	301	285	269	259	248	235	223	217	207	199	192	187

Most bevel-gear sets are made from carburized case-hardened steel, and the factors incorporated in 2003-B97 largely address these high-performance gears. For through-hardened gears, 2003-B97 is silent on K_L and C_L, and Figs. 15–8 and 15–9 should prudently be considered as approximate.

15–4 Straight-Bevel Gear Analysis

EXAMPLE 15–1

A pair of identical straight-tooth miter gears listed in a catalog has a diametral pitch of 5 at the large end, 25 teeth, a 1.10-in face width, and a 20° normal pressure angle; the gears are grade 1 steel through-hardened with a core and case hardness of 180 Brinell. The gears are uncrowned and intended for general industrial use. They have a quality number of $Q_v = 7$. It is likely that the application intended will require outboard mounting of the gears. Use a safety factor of 1, a 10^7 cycle life, and a 0.99 reliability.
(*a*) For a speed of 600 rev/min find the power rating of this gearset based on AGMA bending strength.
(*b*) For the same conditions as in part (*a*) find the power rating of this gearset based on AGMA wear strength.
(*c*) For a reliability of 0.995, a gear life of 10^9 revolutions, and a safety factor of $S_F = S_H = 1.5$, find the power rating for this gearset using AGMA strengths.

Solution

From Figs. 15–14 and 15–15,

$$d_P = N_P/P = 25/5 = 5.000 \text{ in}$$

$$v_t = \pi d_P n_P/12 = \pi(5)600/12 = 785.4 \text{ ft/min}$$

Overload factor: uniform-uniform loading, Table 15–2, $K_o = 1.00$.
Safety factor: $S_F = 1$, $S_H = 1$.
Dynamic factor K_v: from Eq. (15–6),

$$B = 0.25(12 - 7)^{2/3} = 0.731$$

$$A = 50 + 56(1 - 0.731) = 65.06$$

$$K_v = \left(\frac{65.06 + \sqrt{785.4}}{65.06}\right)^{0.731} = 1.299$$

From Eq. (15–8),

$$v_{t\,\max} = [65.06 + (7 - 3)]^2 = 4769 \text{ ft/min}$$

$v_t < v_{t\,\max}$, that is, $785.4 < 4769$ ft/min, therefore K_v is valid. From Eq. (15–10),

$$K_s = 0.4867 + 0.2132/5 = 0.529$$

From Eq. (15–11),

$$K_{mb} = 1.25 \quad \text{and} \quad K_m = 1.25 + 0.0036(1.10)^2 = 1.254$$

From Eq. (15–13), $K_x = 1$. From Fig. 15–6, $I = 0.065$; from Fig. 15–7, $J_P = 0.216$, $J_G = 0.216$. From Eq. (15–15),

$$K_L = 1.683(10^7)^{-0.0323} = 0.999\,96 \doteq 1$$

From Eq. (15–14),

$$C_L = 3.4822(10^7)^{-0.0602} = 1.32$$

Since $H_{BP}/H_{BG} = 1$, then from Fig. 15–10, $C_H = 1$. From Eqs. (15–13) and (15–18), $K_x = 1$ and $K_T = 1$, respectively. From Eq. (15–20),

$$K_R = 0.70 - 0.15\log(1 - 0.99) = 1, \qquad C_R = \sqrt{K_R} = \sqrt{1} = 1$$

(*a*) *Bending:* From Eq. (15–23),

$$s_{at} = 44(180) + 2100 = 10\,020 \text{ psi}$$

From Eq. (15–3),

$$s_t = \sigma = \frac{W^t}{F} P_d K_o K_v \frac{K_s K_m}{K_x J} = \frac{W^t}{1.10}(5)(1)1.299\frac{0.529(1.254)}{(1)0.216}$$
$$= 18.13 W^t$$

From Eq. (15–4),

$$s_{wt} = \frac{s_{at} K_L}{S_F K_T K_R} = \frac{10\,020(1)}{(1)(1)(1)} = 10\,020 \text{ psi}$$

Equating s_t and s_{wt},

$$18.13 W^t = 10\,020 \qquad W^t = 552.6 \text{ lbf}$$

Answer

$$H = \frac{W^t v_t}{33\,000} = \frac{552.6(785.4)}{33\,000} = 13.2 \text{ hp}$$

(*b*) *Wear:* From Fig. 15–12,

$$s_{ac} = 341(180) + 23\,620 = 85\,000 \text{ psi}$$

From Eq. (15–2),

$$\sigma_{c,\text{all}} = \frac{s_{ac} C_L C_H}{S_H K_T C_R} = \frac{85\,000(1.32)(1)}{(1)(1)(1)} = 112\,200 \text{ psi}$$

Now $C_p = 2290\sqrt{\text{psi}}$ from definitions following Eq. (15–21). From Eq. (15–9),

$$C_s = 0.125(1.1) + 0.4375 = 0.575$$

From Eq. (15–12), $C_{xc} = 2$. Substituting in Eq. (15–1) gives

$$\sigma_c = C_p \left(\frac{W^t}{F d_P I} K_o K_v K_m C_s C_{xc} \right)^{1/2}$$
$$= 2290 \left[\frac{W^t}{1.10(5)0.065}(1)1.299(1.254)0.575(2) \right]^{1/2} = 5242\sqrt{W^t}$$

Equating σ_c and $\sigma_{c,\text{all}}$ gives

$$5242\sqrt{W^t} = 112\,200, \qquad W^t = 458.1 \text{ lbf}$$

$$H = \frac{458.1(785.4)}{33\,000} = 10.9 \text{ hp}$$

Rated power for the gearset is

Answer

$$H = \min(12.9, 10.9) = 10.9 \text{ hp}$$

(*c*) Life goal 10^9 cycles, $R = 0.995$, $S_F = S_H = 1.5$, and from Eq. (15–15),

$$K_L = 1.683(10^9)^{-0.0323} = 0.8618$$

From Eq. (15–19),

$$K_R = 0.50 - 0.25\log(1 - 0.995) = 1.075, \qquad C_R = \sqrt{K_R} = \sqrt{1.075} = 1.037$$

From Eq. (15–14),

$$C_L = 3.4822(10^9)^{-0.0602} = 1$$

Bending: From Eq. (15–23) and part (*a*), $s_{at} = 10\,020$ psi. From Eq. (15–3),

$$s_t = \sigma = \frac{W^t}{1.10} 5(1)1.299 \frac{0.529(1.254)}{(1)0.216} = 18.13W^t$$

From Eq. (15–4),

$$s_{wt} = \frac{s_{at} K_L}{S_F K_T K_R} = \frac{10\,020(0.8618)}{1.5(1)1.075} = 5355 \text{ psi}$$

Equating s_t to s_{wt} gives

$$18.13W^t = 5355 \qquad W^t = 295.4 \text{ lbf}$$

$$H = \frac{295.4(785.4)}{33\,000} = 7.0 \text{ hp}$$

Wear: From Eq. (15–22), and part (*b*), $s_{ac} = 85\,000$ psi.
Substituting into Eq. (15–2) gives

$$\sigma_{c,\text{all}} = \frac{s_{ac} C_L C_H}{S_H K_T C_R} = \frac{85\,000(1)(1)}{1.5(1)1.037} = 54\,640 \text{ psi}$$

Substituting into Eq. (15–1) gives, from part (*b*), $\sigma_c = 5242\sqrt{W^t}$.
Equating σ_c to $\sigma_{c,\text{all}}$ gives

$$\sigma_c = \sigma_{c,\text{all}} = 54\,640 = 5242\sqrt{W^t} \qquad W^t = 108.6 \text{ lbf}$$

The wear power is

$$H = \frac{108.6(785.4)}{33\,000} = 2.58 \text{ hp}$$

Answer

The mesh rated power is $H = \min(7.0, 2.58) = 2.6$ hp.

15–5 Design of a Straight-Bevel Gear Mesh

A useful decision set for straight-bevel gear design is

- Function
- Design factor
- Tooth system
- Tooth count

} A priori decisions

- Pitch and face width
- Quality number
- Gear material, core and case hardness
- Pinion material, core and case hardness

} Design variables

In bevel gears the quality number is linked to the wear strength. The J factor for the gear can be smaller than for the pinion. Bending strength is not linear with face width, because added material is placed at the small end of the teeth. Consequently, face width is roughly prescribed as

$$F = \min(0.3A_0, 10/P_d) \tag{15–24}$$

where A_0 is the cone distance (see Fig. 13–20), given by

$$A_0 = \frac{d_P}{2\sin\gamma} = \frac{d_G}{2\sin\Gamma} \tag{15–25}$$

EXAMPLE 15–2 Design a straight-bevel gear mesh for shaft centerlines that intersect perpendicularly, to deliver 6.85 hp at 900 rev/min with a gear ratio of 3:1, temperature of 300°F, normal pressure angle of 20°, using a design factor of 2. The load is uniform-uniform. Although the minimum number of teeth on the pinion is 13, which will mesh with 31 or more teeth without interference, use a pinion of 20 teeth. The material is to be AGMA grade 1 and the teeth are to be crowned. The reliability goal is 0.995 with a pinion life of 10^9 revolutions.

Solution First we list the a priori decisions and their immediate consequences.

Function: 6.85 hp at 900 rev/min, gear ratio $m_G = 3$, 300°F environment, neither gear straddle-mounted, $K_{mb} = 1.25$ [Eq. (15–11)], $R = 0.995$ at 10^9 revolutions of the pinion,

Eq. (15–14):
$$(C_L)_G = 3.4822(10^9/3)^{-0.0602} = 1.068$$
$$(C_L)_P = 3.4822(10^9)^{-0.0602} = 1$$

Eq. (15–15):
$$(K_L)_G = 1.683(10^9/3)^{-0.0323} = 0.8929$$
$$(K_L)_P = 1.683(10^9)^{-0.0323} = 0.8618$$

Eq. (15–19):
$$K_R = 0.50 - 0.25\log(1 - 0.995) = 1.075$$
$$C_R = \sqrt{K_R} = \sqrt{1.075} = 1.037$$

Eq. (15–18):
$$K_T = C_T = (460 + 300)/710 = 1.070$$

Design factor: $n_d = 2$, $S_F = 2$, $S_H = \sqrt{2} = 1.414$.

Tooth system: crowned, straight-bevel gears, normal pressure angle 20°,

Eq. (15–13): $K_x = 1$

Eq. (15–12): $C_{xc} = 1.5$.

With $N_P = 20$ teeth, $N_G = (3)20 = 60$ teeth and from Fig. 15–14,

$$\gamma = \tan^{-1}(N_P/N_G) = \tan^{-1}(20/60) = 18.43^\circ \qquad \Gamma = \tan^{-1}(60/20) = 71.57^\circ$$

From Figs. 15–6 and 15–7, $I = 0.0825$, $J_P = 0.248$, and $J_G = 0.202$. Note that $J_P > J_G$.

Decision 1: Trial diametral pitch, $P_d = 8$ teeth/in.

Eq. (15–10): $K_s = 0.4867 + 0.2132/8 = 0.5134$

$$d_P = N_P/P_d = 20/8 = 2.5 \text{ in}$$

$$d_G = 2.5(3) = 7.5 \text{ in}$$

$$v_t = \pi d_P n_P/12 = \pi(2.5)900/12 = 589.0 \text{ ft/min}$$

$$W^t = 33\,000\ \text{hp}/v_t = 33\,000(6.85)/589.0 = 383.8 \text{ lbf}$$

Eq. (15–25): $A_0 = d_P/(2\sin\gamma) = 2.5/(2\sin 18.43^\circ) = 3.954$ in

Eq. (15–24):

$$F = \min(0.3A_0, 10/P_d) = \min[0.3(3.954), 10/8] = \min(1.186, 1.25) = 1.186 \text{ in}$$

Decision 2: Let $F = 1.25$ in. Then,

Eq. (15–9): $C_s = 0.125(1.25) + 0.4375 = 0.5937$

Eq. (15–11): $K_m = 1.25 + 0.0036(1.25)^2 = 1.256$

Decision 3: Let the transmission accuracy number be 6. Then, from Eq. (15–6),

$$B = 0.25(12 - 6)^{2/3} = 0.8255$$

$$A = 50 + 56(1 - 0.8255) = 59.77$$

Eq. (15–5): $$K_v = \left(\frac{59.77 + \sqrt{589.0}}{59.77}\right)^{0.8255} = 1.325$$

Decision 4: Pinion and gear material and treatment. Carburize and case-harden grade ASTM 1320 to

Core 21 HRC (H_B is 229 Brinell)
Case 55-64 HRC (H_B is 515 Brinell)

From Table 15–4, $s_{ac} = 200\,000$ psi and from Table 15–6, $s_{at} = 30\,000$ psi.

Gear bending: From Eq. (15–3), the bending stress is

$$(s_t)_G = \frac{W^t}{F} P_d K_o K_v \frac{K_s K_m}{K_x J_G} = \frac{383.8}{1.25} 8(1)1.325 \frac{0.5134(1.256)}{(1)0.202}$$
$$= 10\,390 \text{ psi}$$

The bending strength, from Eq. (15–4), is given by

$$(s_{wt})_G = \left(\frac{s_{at}K_L}{S_F K_T K_R}\right)_G = \frac{30\ 000(0.8929)}{2(1.070)1.075} = 11\,640 \text{ psi}$$

The strength exceeds the stress by a factor of $11640/10390 = 1.12$, giving an actual factor of safety of $(S_F)_G = 2(1.12) = 2.24$.

Pinion bending: The bending stress can be found from

$$(s_t)_P = (s_t)_G \frac{J_G}{J_P} = 10\,390 \frac{0.202}{0.248} = 8463 \text{ psi}$$

The bending strength, again from Eq. (15–4), is given by

$$(s_{wt})_P = \left(\frac{s_{at}K_L}{S_F K_T K_R}\right)_P = \frac{30\ 000\,(0.8618)}{2(1.070)1.075} = 11\ 240 \text{ psi}$$

The strength exceeds the stress by a factor of $11\ 240/8463 = 1.33$, giving an actual factor of safety of $(S_F)_P = 2(1.33) = 2.66$.

Gear wear: The load-induced contact stress for the pinion and gear, from Eq. (15–1), is

$$\begin{aligned} s_c &= C_p \left(\frac{W^t}{F d_P I} K_o K_v K_m C_s C_{xc}\right)^{1/2} \\ &= 2290 \left[\frac{383.8}{1.25(2.5)0.0825}(1)1.325(1.256)0.5937(1.5)\right]^{1/2} \\ &= 107\ 560 \text{ psi} \end{aligned}$$

From Eq. (15–2) the contact strength of the gear is

$$(s_{wc})_G = \left(\frac{s_{ac}C_L C_H}{S_H K_T C_R}\right)_G = \frac{200\ 000(1.068)(1)}{\sqrt{2}(1.070)1.037} = 136\ 120 \text{ psi}$$

The strength exceeds the stress by a factor of $136\ 120/107\ 560 = 1.266$, giving an actual factor of safety of $(S_H)_G^2 = 1.266^2(2) = 3.21$.

Pinion wear: From Eq. (15–2) the contact strength of the pinion is

$$(s_{wc})_P = \left(\frac{s_{ac}C_L C_H}{S_H K_T C_R}\right)_P = \frac{200\ 000(1)(1)}{\sqrt{2}(1.070)1.037} = 127\ 450 \text{ psi}$$

The strength exceeds the stress by a factor of $136\ 120/127\ 450 = 1.068$, giving an actual factor of safety of $(S_H)_P^2 = 1.068^2(2) = 2.28$.

The actual factors of safety are 2.24, 2.66, 3.21, and 2.28. Making a direct comparison of the factors, we note that the threat from gear bending and pinion wear are practically equal. We also note that three of the ratios are comparable. Our goal would be to make changes in the design decisions that drive the factors closer to 2. The next step would be to adjust the design variables. It is obvious that an iterative process is involved. We need a figure of merit to order the designs. A computer program clearly is desirable.

15–6 Worm Gearing—AGMA Equation

Since they are essentially nonenveloping worm gears, the *crossed helical* gears, shown in Fig. 15–16, can be considered with other worm gearing. Because the teeth of worm gears have *point contact* changing to *line contact* as the gears are used, worm gears are said to "wear in," whereas other types "wear out."

Crossed helical gears, and worm gears too, usually have a 90° shaft angle, though this need not be so. The relation between the shaft and helix angles is

$$\sum = \psi_P \pm \psi_G \tag{15–26}$$

where $\sum$ is the shaft angle. The plus sign is used when both helix angles are of the same hand, and the minus sign when they are of opposite hand. The subscript P in Eq. (15–26) refers to the pinion (worm); the subscript W is used for this same purpose. The subscript G refers to the gear, also called *gear wheel, worm wheel,* or simply the *wheel*. Table 15–8 gives cylindrical worm dimensions common to worm and gear.

Section 13–11 introduced worm gears, and Sec. 13–17 developed the force analysis and efficiency of worm gearing to which we will refer. Here our interest is in strength and durability. Good proportions indicate the pitch worm diameter d falls in the range

$$\frac{C^{0.875}}{3} \le d \le \frac{C^{0.875}}{1.6} \tag{15–27}$$

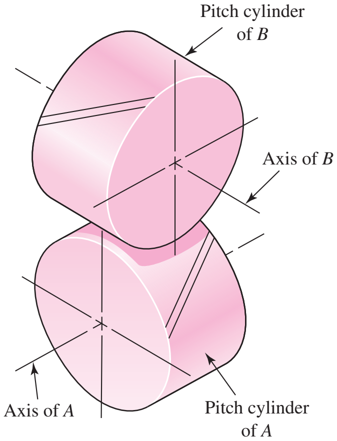


Figure 15–16

View of the pitch cylinders of a pair of crossed helical gears.

Table 15–8

Cylindrical Worm Dimensions Common to Both Worm and Gear*

Quantity	Symbol	ϕ_n 14.5° $N_W \le 2$	20° $N_W \le 2$	25° $N_W > 2$
Addendum	a	$0.3183p_x$	$0.3183p_x$	$0.286p_x$
Dedendum	b	$0.3683p_x$	$0.3683p_x$	$0.349p_x$
Whole depth	h_t	$0.6866p_x$	$0.6866p_x$	$0.635p_x$

*The table entries are for a tangential diametral pitch of the gear of $P_t = 1$.

where C is the center-to-center distance.[2] AGMA relates the allowable tangential force on the worm-gear tooth $(W^t)_{\text{all}}$ to other parameters by

$$(W^t)_{\text{all}} = C_s D_m^{0.8} F_e C_m C_v \tag{15–28}$$

where C_s = materials factor
D_m = mean gear diameter, in (mm)
F_e = effective face width of the gear (actual face width, but not to exceed $0.67d_m$, the mean worm diameter), in (mm)
C_m = ratio correction factor
C_v = velocity factor

The friction force W_f is given by

$$W_f = \frac{f W^t}{\cos\lambda \cos\phi_n} \tag{15–29}$$

where f = coefficient of friction
λ = lead angle at mean worm diameter
ϕ_n = normal pressure angle

The sliding velocity V_s is

$$V_s = \frac{\pi n_W d_m}{12 \cos\lambda} \tag{15–30}$$

where n_W = rotative speed of the worm and d_m = mean worm diameter. The torque at the worm gear is

$$T_G = \frac{W^t D_m}{2} \tag{15–31}$$

where D_m is the mean gear diameter.

The parameters in Eq. (15–28) are, quantitatively,

$$C_s = 270 + 10.37C^3 \qquad C \le 3 \text{ in} \tag{15–32}$$

For sand-cast gears,

$$C_s = \begin{cases} 1000 & C > 3 \quad d_G \le 2.5 \text{ in} \\ 1190 - 477 \log d_G & C > 3 \quad d_G > 2.5 \text{ in} \end{cases} \tag{15–33}$$

For chilled-cast gears,

$$C_s = \begin{cases} 1000 & C > 3 \quad d_G \le 8 \text{ in} \\ 1412 - 456 \log d_G & C > 3 \quad d_G > 8 \text{ in} \end{cases} \tag{15–34}$$

[2]ANSI/AGMA 6034-B92, February 1992, *Practice for Enclosed Cylindrical Wormgear Speed-Reducers and Gear Motors;* and ANSI/AGMA 6022-C93, Dec. 1993, *Design Manual for Cylindrical Wormgearing.* Note: Equations (15–32) to (15–38) are contained in Annex C of 6034-B92 for informational purposes only. To comply with ANSI/AGMA 6034-B92, use the tabulations of these rating factors provided in the standard.

For centrifugally cast gears,

$$C_s = \begin{cases} 1000 & C > 3 \quad d_G \le 25 \text{ in} \\ 1251 - 180 \log d_G & C > 3 \quad d_G > 25 \text{ in} \end{cases} \tag{15–35}$$

The ratio correction factor C_m is given by

$$C_m = \begin{cases} 0.02\sqrt{-m_G^2 + 40m_G - 76} + 0.46 & 3 < m_G \le 20 \\ 0.0107\sqrt{-m_G^2 + 56m_G + 5145} & 20 < m_G \le 76 \\ 1.1483 - 0.006\,58m_G & m_G > 76 \end{cases} \tag{15–36}$$

The velocity factor C_v is given by

$$C_v = \begin{cases} 0.659 \exp(-0.0011V_s) & V_s < 700 \text{ ft/min} \\ 13.31V_s^{-0.571} & 700 \le V_s < 3000 \text{ ft/min} \\ 65.52V_s^{-0.774} & V_s > 3000 \text{ ft/min} \end{cases} \tag{15–37}$$

AGMA reports the coefficient of friction f as

$$f = \begin{cases} 0.15 & V_s = 0 \\ 0.124 \exp\left(-0.074V_s^{0.645}\right) & 0 < V_s \le 10 \text{ ft/min} \\ 0.103 \exp\left(-0.110V_s^{0.450}\right) + 0.012 & V_s > 10 \text{ ft/min} \end{cases} \tag{15–38}$$

Now we examine some worm-gear mesh geometry. The addendum a and dedendum b are

$$a = \frac{p_x}{\pi} = 0.3183p_x \tag{15–39}$$

$$b = \frac{1.157p_x}{\pi} = 0.3683p_x \tag{15–40}$$

The full depth h_t is

$$h_t = \begin{cases} \dfrac{2.157p_x}{\pi} = 0.6866p_x & p_x \ge 0.16 \text{ in} \\ \dfrac{2.200p_x}{\pi} + 0.002 = 0.7003p_x + 0.002 & p_x < 0.16 \text{ in} \end{cases} \tag{15–41}$$

The worm outside diameter d_0 is

$$d_0 = d + 2a \tag{15–42}$$

The worm root diameter d_r is

$$d_r = d - 2b \tag{15–43}$$

The worm-gear throat diameter D_t is

$$D_t = D + 2a \tag{15–44}$$

where D is the worm gear pitch diameter. The worm-gear root diameter D_r is

$$D_r = D - 2b \tag{15–45}$$

The clearance c is

$$c = b - a \tag{15–46}$$

The worm face width (maximum) $(F_W)_{\max}$ is

$$(F_W)_{\max} = 2\sqrt{\left(\frac{D_t}{2}\right)^2 - \left(\frac{D}{2} - a\right)^2} = 2\sqrt{2Da} \tag{15–47}$$

which was simplified using Eq. (15–44). The worm-gear face width F_G is

$$F_G = \begin{cases} 2d_m/3 & p_x > 0.16 \text{ in} \\ 1.125\sqrt{(d_0 + 2c)^2 - (d_0 - 4a)^2} & p_x \le 0.16 \text{ in} \end{cases} \tag{15–48}$$

The heat loss rate H_{loss} from the worm-gear case in ft · lbf/min is

$$H_{\text{loss}} = 33\,000(1 - e)H_{\text{in}} \tag{15–49}$$

where e is efficiency, given by Eq. (13–46), and H_{in} is the input horsepower from the worm. The overall coefficient $\hbar_{\text{CR}}$ for combined convective and radiative heat transfer from the worm-gear case in ft · lbf/(min · in^2 · °F) is

$$\hbar_{\text{CR}} = \begin{cases} \dfrac{n_W}{6494} + 0.13 & \text{no fan on worm shaft} \\ \dfrac{n_W}{3939} + 0.13 & \text{fan on worm shaft} \end{cases} \tag{15–50}$$

When the case lateral area A is expressed in in^2, the temperature of the oil sump t_s is given by

$$t_s = t_a + \frac{H_{\text{loss}}}{\hbar_{\text{CR}} A} = \frac{33\,000(1 - e)(H)_{\text{in}}}{\hbar_{\text{CR}} A} + t_a \tag{15–51}$$

Bypassing Eqs. (15–49), (15–50), and (15–51) one can apply the AGMA recommendation for minimum lateral area $A_{\min}$ in in^2 using

$$A_{\min} = 43.20C^{1.7} \tag{15–52}$$

Because worm teeth are inherently much stronger than worm-gear teeth, they are not considered. The teeth in worm gears are short and thick on the edges of the face; midplane they are thinner as well as curved. Buckingham[3] adapted the Lewis equation for this case:

$$\sigma_a = \frac{W_G^t}{p_n F_e y} \tag{15–53}$$

where $p_n = p_x \cos\lambda$ and y is the Lewis form factor related to circular pitch. For $\phi_n = 14.5°$, $y = 0.100$; $\phi_n = 20°$, $y = 0.125$; $\phi_n = 25°$, $y = 0.150$; $\phi_n = 30°$, $y = 0.175$.

[3] Earle Buckingham, *Analytical Mechanics of Gears,* McGraw-Hill, New York, 1949, p. 495.