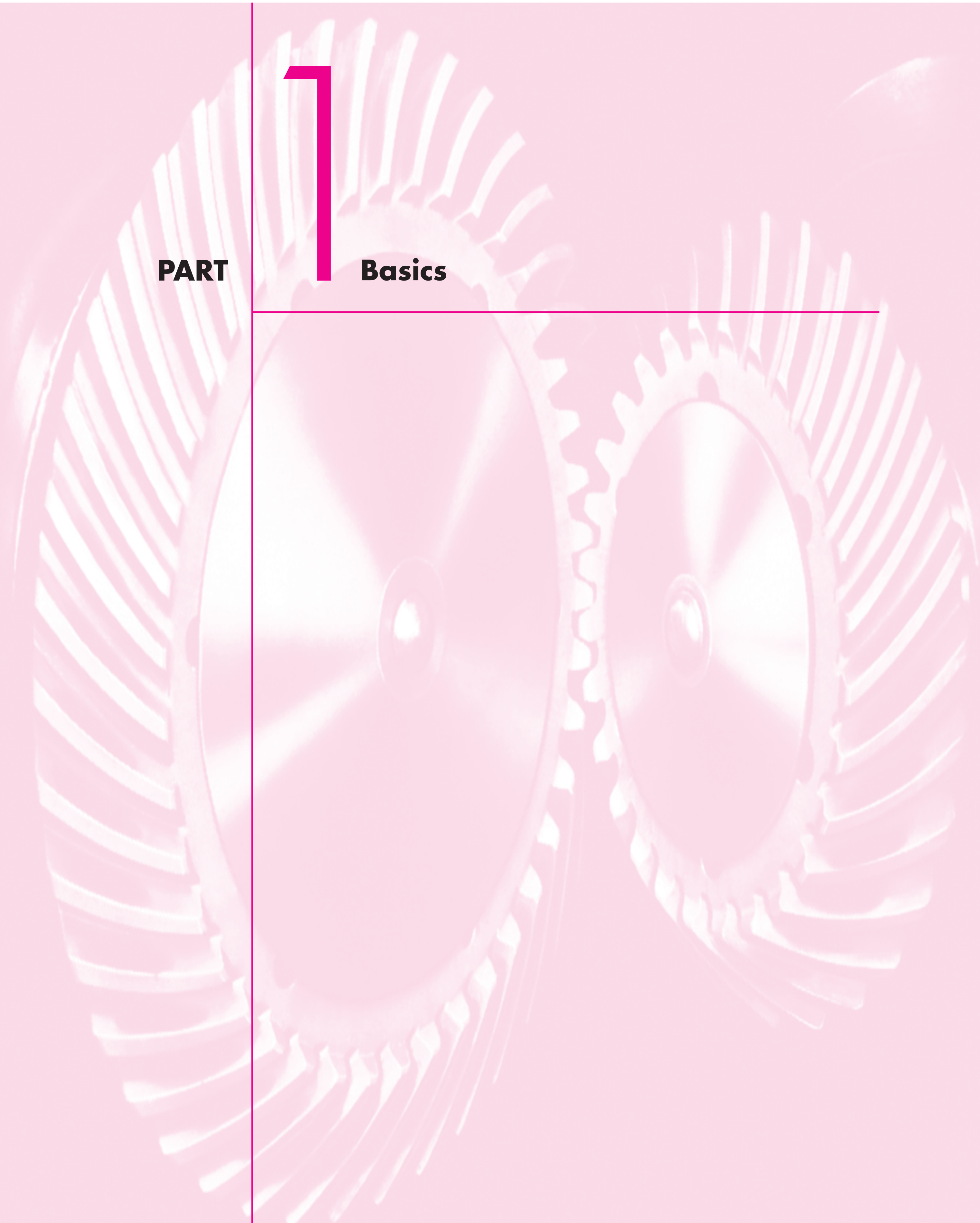


PART 1 Basics

1 Introduction to Mechanical Engineering Design

Chapter Outline

Mechanical design is a complex undertaking, requiring many skills. Extensive relationships need to be subdivided into a series of simple tasks. The complexity of the subject requires a sequence in which ideas are introduced and iterated.

We first address the nature of design in general, and then mechanical engineering design in particular. Design is an iterative process with many interactive phases. Many resources exist to support the designer, including many sources of information and an abundance of computational design tools. The design engineer needs not only to develop competence in their field but must also cultivate a strong sense of responsibility and professional work ethic.

There are roles to be played by codes and standards, ever-present economics, safety, and considerations of product liability. The survival of a mechanical component is often related through stress and strength. Matters of uncertainty are ever-present in engineering design and are typically addressed by the design factor and factor of safety, either in the form of a deterministic (absolute) or statistical sense. The latter, statistical approach, deals with a design's *reliability* and requires good statistical data.

In mechanical design, other considerations include dimensions and tolerances, units, and calculations.

The book consists of four parts. Part 1, *Basics,* begins by explaining some differences between design and analysis and introducing some fundamental notions and approaches to design. It continues with three chapters reviewing material properties, stress analysis, and stiffness and deflection analysis, which are the key principles necessary for the remainder of the book.

Part 2, *Failure Prevention,* consists of two chapters on the prevention of failure of mechanical parts. Why machine parts fail and how they can be designed to prevent failure are difficult questions, and so we take two chapters to answer them, one on preventing failure due to static loads, and the other on preventing fatigue failure due to time-varying, cyclic loads.

In Part 3, *Design of Mechanical Elements,* the material of Parts 1 and 2 is applied to the analysis, selection, and design of specific mechanical elements such as shafts, fasteners, weldments, springs, rolling contact bearings, film bearings, gears, belts, chains, and wire ropes.

Part 4, *Analysis Tools,* provides introductions to two important methods used in mechanical design, finite element analysis and statistical analysis. This is optional study material, but some sections and examples in Parts 1 to 3 demonstrate the use of these tools.

There are two appendixes at the end of the book. Appendix A contains many useful tables referenced throughout the book. Appendix B contains answers to selected end-of-chapter problems.

1–1 Design

To design is either to formulate a plan for the satisfaction of a specified need or to solve a problem. If the plan results in the creation of something having a physical reality, then the product must be functional, safe, reliable, competitive, usable, manufacturable, and marketable.

Design is an innovative and highly iterative process. It is also a decision-making process. Decisions sometimes have to be made with too little information, occasionally with just the right amount of information, or with an excess of partially contradictory information. Decisions are sometimes made tentatively, with the right reserved to adjust as more becomes known. The point is that the engineering designer has to be personally comfortable with a decision-making, problem-solving role.

Design is a communication-intensive activity in which both words and pictures are used, and written and oral forms are employed. Engineers have to communicate effectively and work with people of many disciplines. These are important skills, and an engineer's success depends on them.

A designer's personal resources of creativeness, communicative ability, and problem-solving skill are intertwined with knowledge of technology and first principles. Engineering tools (such as mathematics, statistics, computers, graphics, and languages) are combined to produce a plan that, when carried out, produces a product that is *functional, safe, reliable, competitive, usable, manufacturable, and marketable,* regardless of who builds it or who uses it.

1–2 Mechanical Engineering Design

Mechanical engineers are associated with the production and processing of energy and with providing the means of production, the tools of transportation, and the techniques of automation. The skill and knowledge base are extensive. Among the disciplinary bases are mechanics of solids and fluids, mass and momentum transport, manufacturing processes, and electrical and information theory. Mechanical engineering design involves all the disciplines of mechanical engineering.

Real problems resist compartmentalization. A simple journal bearing involves fluid flow, heat transfer, friction, energy transport, material selection, thermomechanical treatments, statistical descriptions, and so on. A building is environmentally controlled. The heating, ventilation, and air-conditioning considerations are sufficiently specialized that some speak of heating, ventilating, and air-conditioning design as if it is separate and distinct from mechanical engineering design. Similarly, internal-combustion engine design, turbomachinery design, and jet-engine design are sometimes considered discrete entities. Here, the leading string of words preceding the word design is merely a product descriptor. Similarly, there are phrases such as machine design, machine-element design, machine-component design, systems design, and fluid-power design. All of these phrases are somewhat more focused *examples* of mechanical engineering design. They all draw on the same bodies of knowledge, are similarly organized, and require similar skills.

1–3 Phases and Interactions of the Design Process

What is the design process? How does it begin? Does the engineer simply sit down at a desk with a blank sheet of paper and jot down some ideas? What happens next? What factors influence or control the decisions that have to be made? Finally, how does the design process end?

The complete design process, from start to finish, is often outlined as in Fig. 1–1. The process begins with an identification of a need and a decision to do something about it. After many iterations, the process ends with the presentation of the plans for satisfying the need. Depending on the nature of the design task, several design phases may be repeated throughout the life of the product, from inception to termination. In the next several subsections, we shall examine these steps in the design process in detail.

Identification of need generally starts the design process. Recognition of the need and phrasing the need often constitute a highly creative act, because the need may be only a vague discontent, a feeling of uneasiness, or a sensing that something is not right. The need is often not evident at all; recognition is usually triggered by a particular

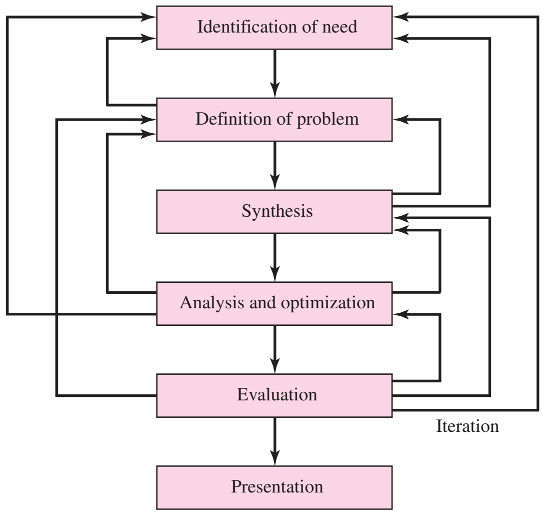


Figure 1–1

The phases in design, acknowledging the many feedbacks and iterations.

adverse circumstance or a set of random circumstances that arises almost simultaneously. For example, the need to do something about a food-packaging machine may be indicated by the noise level, by a variation in package weight, and by slight but perceptible variations in the quality of the packaging or wrap.

There is a distinct difference between the statement of the need and the definition of the problem. The *definition of problem* is more specific and must include all the specifications for the object that is to be designed. The specifications are the input and output quantities, the characteristics and dimensions of the space the object must occupy, and all the limitations on these quantities. We can regard the object to be designed as something in a black box. In this case we must specify the inputs and outputs of the box, together with their characteristics and limitations. The specifications define the cost, the number to be manufactured, the expected life, the range, the operating temperature, and the reliability. Specified characteristics can include the speeds, feeds, temperature limitations, maximum range, expected variations in the variables, dimensional and weight limitations, etc.

There are many implied specifications that result either from the designer's particular environment or from the nature of the problem itself. The manufacturing processes that are available, together with the facilities of a certain plant, constitute restrictions on a designer's freedom, and hence are a part of the implied specifications. It may be that a small plant, for instance, does not own cold-working machinery. Knowing this, the designer might select other metal-processing methods that can be performed in the plant. The labor skills available and the competitive situation also constitute implied constraints. Anything that limits the designer's freedom of choice is a constraint. Many materials and sizes are listed in supplier's catalogs, for instance, but these are not all easily available and shortages frequently occur. Furthermore, inventory economics requires that a manufacturer stock a minimum number of materials and sizes. An example of a specification is given in Sec. 1–16. This example is for a case study of a power transmission that is presented throughout this text.

The synthesis of a scheme connecting possible system elements is sometimes called the *invention of the concept* or *concept design*. This is the first and most important step in the synthesis task. Various schemes must be proposed, investigated, and quantified in terms of established metrics.[1] As the fleshing out of the scheme progresses, analyses must be performed to assess whether the system performance is satisfactory or better, and, if satisfactory, just how well it will perform. System schemes that do not survive analysis are revised, improved, or discarded. Those with potential are optimized to determine the best performance of which the scheme is capable. Competing schemes are compared so that the path leading to the most competitive product can be chosen. Figure 1–1 shows that *synthesis* and *analysis and optimization* are intimately and iteratively related.

We have noted, and we emphasize, that design is an iterative process in which we proceed through several steps, evaluate the results, and then return to an earlier phase of the procedure. Thus, we may synthesize several components of a system, analyze and optimize them, and return to synthesis to see what effect this has on the remaining parts of the system. For example, the design of a system to transmit power requires attention to the design and selection of individual components (e.g., gears, bearings, shaft). However, as is often the case in design, these components are not independent. In order to design the shaft for stress and deflection, it is necessary to know the applied forces. If the forces are transmitted through gears, it is necessary to know the gear specifications in order to determine the forces that will be transmitted to the shaft. But stock gears come with certain bore sizes, requiring knowledge of the necessary shaft diameter. Clearly, rough estimates will need to be made in order to proceed through the process, refining and iterating until a final design is obtained that is satisfactory for each individual component as well as for the overall design specifications. Throughout the text we will elaborate on this process for the case study of a power transmission design.

Both analysis and optimization require that we construct or devise abstract models of the system that will admit some form of mathematical analysis. We call these models mathematical models. In creating them it is our hope that we can find one that will simulate the real physical system very well. As indicated in Fig. 1–1, *evaluation* is a significant phase of the total design process. Evaluation is the final proof of a successful design and usually involves the testing of a prototype in the laboratory. Here we wish to discover if the design really satisfies the needs. Is it reliable? Will it compete successfully with similar products? Is it economical to manufacture and to use? Is it easily maintained and adjusted? Can a profit be made from its sale or use? How likely is it to result in product-liability lawsuits? And is insurance easily and cheaply obtained? Is it likely that recalls will be needed to replace defective parts or systems?

Communicating the design to others is the final, vital *presentation* step in the design process. Undoubtedly, many great designs, inventions, and creative works have been lost to posterity simply because the originators were unable or unwilling to explain their accomplishments to others. Presentation is a selling job. The engineer, when presenting a new solution to administrative, management, or supervisory persons, is attempting to sell or to prove to them that this solution is a better one. Unless this can be done successfully, the time and effort spent on obtaining the solution have been largely wasted. When designers sell a new idea, they also sell themselves. If they are repeatedly successful in selling ideas, designs, and new solutions to management, they begin to receive salary increases and promotions; in fact, this is how anyone succeeds in his or her profession.

[1]An excellent reference for this topic is presented by Stuart Pugh, *Total Design—Integrated Methods for Successful Product Engineering,* Addison-Wesley, 1991. A description of the *Pugh method* is also provided in Chap. 8, David G. Ullman, *The Mechanical Design Process,* 3rd ed., McGraw-Hill, 2003.

Design Considerations

Sometimes the strength required of an element in a system is an important factor in the determination of the geometry and the dimensions of the element. In such a situation we say that strength is an important design consideration. When we use the expression design consideration, we are referring to some characteristic that influences the design of the element or, perhaps, the entire system. Usually quite a number of such characteristics must be considered and prioritized in a given design situation. Many of the important ones are as follows (not necessarily in order of importance):

1. Functionality
2. Strength/stress
3. Distortion/deflection/stiffness
4. Wear
5. Corrosion
6. Safety
7. Reliability
8. Manufacturability
9. Utility
10. Cost
11. Friction
12. Weight
13. Life
14. Noise
15. Styling
16. Shape
17. Size
18. Control
19. Thermal properties
20. Surface
21. Lubrication
22. Marketability
23. Maintenance
24. Volume
25. Liability
26. Remanufacturing/resource recovery

Some of these characteristics have to do directly with the dimensions, the material, the processing, and the joining of the elements of the system. Several characteristics may be interrelated, which affects the configuration of the total system.

1–4 Design Tools and Resources

Today, the engineer has a great variety of tools and resources available to assist in the solution of design problems. Inexpensive microcomputers and robust computer software packages provide tools of immense capability for the design, analysis, and simulation of mechanical components. In addition to these tools, the engineer always needs technical information, either in the form of basic science/engineering behavior or the characteristics of specific off-the-shelf components. Here, the resources can range from science/engineering textbooks to manufacturers' brochures or catalogs. Here too, the computer can play a major role in gathering information.[2]

Computational Tools

Computer-aided design (CAD) software allows the development of three-dimensional (3-D) designs from which conventional two-dimensional orthographic views with automatic dimensioning can be produced. Manufacturing tool paths can be generated from the 3-D models, and in some cases, parts can be created directly from a 3-D database by using a rapid prototyping and manufacturing method (stereolithography)—*paperless manufacturing!* Another advantage of a 3-D database is that it allows rapid and accurate calculations of mass properties such as mass, location of the center of gravity, and mass moments of inertia. Other geometric properties such as areas and distances between points are likewise easily obtained. There are a great many CAD software packages available such

[2]An excellent and comprehensive discussion of the process of "gathering information" can be found in Chap. 4, George E. Dieter, *Engineering Design, A Materials and Processing Approach,* 3rd ed., McGraw-Hill, New York, 2000.

as Aries, AutoCAD, CadKey, I-Deas, Unigraphics, Solid Works, and ProEngineer, to name a few.

The term *computer-aided engineering* (CAE) generally applies to all computer-related engineering applications. With this definition, CAD can be considered as a subset of CAE. Some computer software packages perform specific engineering analysis and/or simulation tasks that assist the designer, but they are not considered a tool for the creation of the design that CAD is. Such software fits into two categories: engineering-based and non-engineering-specific. Some examples of engineering-based software for mechanical engineering applications—software that might also be integrated within a CAD system—include finite-element analysis (FEA) programs for analysis of stress and deflection (see Chap. 19), vibration, and heat transfer (e.g., Algor, ANSYS, and MSC/NASTRAN); computational fluid dynamics (CFD) programs for fluid-flow analysis and simulation (e.g., CFD++, FIDAP, and Fluent); and programs for simulation of dynamic force and motion in mechanisms (e.g., ADAMS, DADS, and Working Model).

Examples of non-engineering-specific computer-aided applications include software for word processing, spreadsheet software (e.g., Excel, Lotus, and Quattro-Pro), and mathematical solvers (e.g., Maple, MathCad, Matlab, Mathematica, and TKsolver).

Your instructor is the best source of information about programs that may be available to you and can recommend those that are useful for specific tasks. One caution, however: Computer software is no substitute for the human thought process. *You* are the driver here; the computer is the vehicle to assist you on your journey to a solution. Numbers generated by a computer can be far from the truth if you entered incorrect input, if you misinterpreted the application or the output of the program, if the program contained bugs, etc. It is your responsibility to assure the validity of the results, so be careful to check the application and results carefully, perform benchmark testing by submitting problems with known solutions, and monitor the software company and user-group newsletters.

Acquiring Technical Information

We currently live in what is referred to as the *information age,* where information is generated at an astounding pace. It is difficult, but extremely important, to keep abreast of past and current developments in one's field of study and occupation. The reference in Footnote 2 provides an excellent description of the informational resources available and is highly recommended reading for the serious design engineer. Some sources of information are:

- *Libraries (community, university, and private).* Engineering dictionaries and encyclopedias, textbooks, monographs, handbooks, indexing and abstract services, journals, translations, technical reports, patents, and business sources/brochures/catalogs.
- *Government sources.* Departments of Defense, Commerce, Energy, and Transportation; NASA; Government Printing Office; U.S. Patent and Trademark Office; National Technical Information Service; and National Institute for Standards and Technology.
- *Professional societies.* American Society of Mechanical Engineers, Society of Manufacturing Engineers, Society of Automotive Engineers, American Society for Testing and Materials, and American Welding Society.
- *Commercial vendors.* Catalogs, technical literature, test data, samples, and cost information.
- *Internet.* The computer network gateway to websites associated with most of the categories listed above.[3]

[3]Some helpful Web resources, to name a few, include www.globalspec.com, www.engnetglobal.com, www.efunda.com, www.thomasnet.com, and www.uspto.gov.

This list is not complete. The reader is urged to explore the various sources of information on a regular basis and keep records of the knowledge gained.

1–5 The Design Engineer's Professional Responsibilities

In general, the design engineer is required to satisfy the needs of customers (management, clients, consumers, etc.) and is expected to do so in a competent, responsible, ethical, and professional manner. Much of engineering course work and practical experience focuses on competence, but when does one begin to develop engineering responsibility and professionalism? To start on the road to success, you should start to develop these characteristics early in your educational program. You need to cultivate your professional work ethic and process skills before graduation, so that when you begin your formal engineering career, you will be prepared to meet the challenges.

It is not obvious to some students, but communication skills play a large role here, and it is the wise student who continuously works to improve these skills—*even if it is not a direct requirement of a course assignment!* Success in engineering (achievements, promotions, raises, etc.) may in large part be due to competence but if you cannot communicate your ideas clearly and concisely, your technical proficiency may be compromised.

You can start to develop your communication skills by keeping a neat and clear journal/logbook of your activities, entering dated entries frequently. (Many companies require their engineers to keep a journal for patent and liability concerns.) Separate journals should be used for each design project (or course subject). When starting a project or problem, in the definition stage, make journal entries quite frequently. Others, as well as yourself, may later question why you made certain decisions. Good chronological records will make it easier to explain your decisions at a later date.

Many engineering students see themselves after graduation as practicing engineers designing, developing, and analyzing products and processes and consider the need of good communication skills, either oral or writing, as secondary. This is far from the truth. Most practicing engineers spend a good deal of time communicating with others, writing proposals and technical reports, and giving presentations and interacting with engineering and nonengineering support personnel. You have the time now to sharpen your communication skills. When given an assignment to write or make any presentation, technical *or* nontechnical, accept it enthusiastically, and work on improving your communication skills. It will be time well spent to learn the skills now rather than on the job.

When you are working on a design problem, it is important that you develop a systematic approach. Careful attention to the following action steps will help you to organize your solution processing technique.

- *Understand the problem.* Problem definition is probably the most significant step in the engineering design process. Carefully read, understand, and refine the problem statement.
- *Identify the known.* From the refined problem statement, describe concisely what information is known and relevant.
- *Identify the unknown and formulate the solution strategy.* State what must be determined, in what order, so as to arrive at a solution to the problem. Sketch the component or system under investigation, identifying known and unknown parameters. Create a flowchart of the steps necessary to reach the final solution. The steps may require the use of free-body diagrams; material properties from tables; equations

from first principles, textbooks, or handbooks relating the known and unknown parameters; experimentally or numerically based charts; specific computational tools as discussed in Sec. 1–4; etc.

- *State all assumptions and decisions.* Real design problems generally do not have unique, ideal, closed-form solutions. Selections, such as choice of materials, and heat treatments, require decisions. Analyses require assumptions related to the modeling of the real components or system. All assumptions and decisions should be identified and recorded.
- *Analyze the problem.* Using your solution strategy in conjunction with your decisions and assumptions, execute the analysis of the problem. Reference the sources of all equations, tables, charts, software results, etc. Check the credibility of your results. Check the order of magnitude, dimensionality, trends, signs, etc.
- *Evaluate your solution.* Evaluate each step in the solution, noting how changes in strategy, decisions, assumptions, and execution might change the results, in positive or negative ways. If possible, incorporate the positive changes in your final solution.
- *Present your solution.* Here is where your communication skills are important. At this point, you are selling yourself and your technical abilities. If you cannot skillfully explain what you have done, some or all of your work may be misunderstood and unaccepted. Know your audience.

As stated earlier, all design processes are interactive and iterative. Thus, it may be necessary to repeat some or all of the above steps more than once if less than satisfactory results are obtained.

In order to be effective, all professionals must keep current in their fields of endeavor. The design engineer can satisfy this in a number of ways by: being an active member of a professional society such as the American Society of Mechanical Engineers (ASME), the Society of Automotive Engineers (SAE), and the Society of Manufacturing Engineers (SME); attending meetings, conferences, and seminars of societies, manufacturers, universities, etc.; taking specific graduate courses or programs at universities; regularly reading technical and professional journals; etc. An engineer's education does not end at graduation.

The design engineer's professional obligations include conducting activities in an ethical manner. Reproduced here is the *Engineers' Creed* from the National Society of Professional Engineers (NSPE)[4]:

> *As a Professional Engineer I dedicate my professional knowledge and skill to the advancement and betterment of human welfare.*
> *I pledge:*
> *To give the utmost of performance;*
> *To participate in none but honest enterprise;*
> *To live and work according to the laws of man and the highest standards of professional conduct;*
> *To place service before profit, the honor and standing of the profession before personal advantage, and the public welfare above all other considerations.*
> *In humility and with need for Divine Guidance, I make this pledge.*

[4]Adopted by the National Society of Professional Engineers, June 1954. "The Engineer's Creed." Reprinted by permission of the National Society of Professional Engineers. This has been expanded and revised by NSPE. For the current revision, January 2006, see the website www.nspe.org/ethics/ehl-code.asp, or the pdf file, www.nspe.org/ethics/code-2006-Jan.pdf.

1–6 Standards and Codes

A *standard* is a set of specifications for parts, materials, or processes intended to achieve uniformity, efficiency, and a specified quality. One of the important purposes of a standard is to place a limit on the number of items in the specifications so as to provide a reasonable inventory of tooling, sizes, shapes, and varieties.

A *code* is a set of specifications for the analysis, design, manufacture, and construction of something. The purpose of a code is to achieve a specified degree of safety, efficiency, and performance or quality. It is important to observe that safety codes *do not* imply *absolute safety.* In fact, absolute safety is impossible to obtain. Sometimes the unexpected event really does happen. Designing a building to withstand a 120 mi/h wind does not mean that the designers think a 140 mi/h wind is impossible; it simply means that they think it is highly improbable.

All of the organizations and societies listed below have established specifications for standards and safety or design codes. The name of the organization provides a clue to the nature of the standard or code. Some of the standards and codes, as well as addresses, can be obtained in most technical libraries. The organizations of interest to mechanical engineers are:

Aluminum Association (AA)
American Gear Manufacturers Association (AGMA)
American Institute of Steel Construction (AISC)
American Iron and Steel Institute (AISI)
American National Standards Institute (ANSI)[5]
ASM International[6]
American Society of Mechanical Engineers (ASME)
American Society of Testing and Materials (ASTM)
American Welding Society (AWS)
American Bearing Manufacturers Association (ABMA)[7]
British Standards Institution (BSI)
Industrial Fasteners Institute (IFI)
Institution of Mechanical Engineers (I. Mech. E.)
International Bureau of Weights and Measures (BIPM)
International Standards Organization (ISO)
National Institute for Standards and Technology (NIST)[8]
Society of Automotive Engineers (SAE)

1–7 Economics

The consideration of cost plays such an important role in the design decision process that we could easily spend as much time in studying the cost factor as in the study of the entire subject of design. Here we introduce only a few general concepts and simple rules.

[5]In 1966 the American Standards Association (ASA) changed its name to the United States of America Standards Institute (USAS). Then, in 1969, the name was again changed, to American National Standards Institute, as shown above and as it is today. This means that you may occasionally find ANSI standards designated as ASA or USAS.

[6]Formally American Society for Metals (ASM). Currently the acronym ASM is undefined.

[7]In 1993 the Anti-Friction Bearing Manufacturers Association (AFBMA) changed its name to the American Bearing Manufacturers Association (ABMA).

[8]Former National Bureau of Standards (NBS).

First, observe that nothing can be said in an absolute sense concerning costs. Materials and labor usually show an increasing cost from year to year. But the costs of processing the materials can be expected to exhibit a decreasing trend because of the use of automated machine tools and robots. The cost of manufacturing a single product will vary from city to city and from one plant to another because of overhead, labor, taxes, and freight differentials and the inevitable slight manufacturing variations.

Standard Sizes

The use of standard or stock sizes is a first principle of cost reduction. An engineer who specifies an AISI 1020 bar of hot-rolled steel 53 mm square has added cost to the product, provided that a bar 50 or 60 mm square, both of which are preferred sizes, would do equally well. The 53-mm size can be obtained by special order or by rolling or machining a 60-mm square, but these approaches add cost to the product. To ensure that standard or preferred sizes are specified, designers must have access to stock lists of the materials they employ.

A further word of caution regarding the selection of preferred sizes is necessary. Although a great many sizes are usually listed in catalogs, they are not all readily available. Some sizes are used so infrequently that they are not stocked. A rush order for such sizes may mean more on expense and delay. Thus you should also have access to a list such as those in Table A–17 for preferred inch and millimeter sizes.

There are many purchased parts, such as motors, pumps, bearings, and fasteners, that are specified by designers. In the case of these, too, you should make a special effort to specify parts that are readily available. Parts that are made and sold in large quantities usually cost somewhat less than the odd sizes. The cost of rolling bearings, for example, depends more on the quantity of production by the bearing manufacturer than on the size of the bearing.

Large Tolerances

Among the effects of design specifications on costs, tolerances are perhaps most significant. Tolerances, manufacturing processes, and surface finish are interrelated and influence the producibility of the end product in many ways. Close tolerances may necessitate additional steps in processing and inspection or even render a part completely impractical to produce economically. Tolerances cover dimensional variation and surface-roughness range and also the variation in mechanical properties resulting from heat treatment and other processing operations.

Since parts having large tolerances can often be produced by machines with higher production rates, costs will be significantly smaller. Also, fewer such parts will be rejected in the inspection process, and they are usually easier to assemble. A plot of cost versus tolerance/machining process is shown in Fig. 1–2, and illustrates the drastic increase in manufacturing cost as tolerance diminishes with finer machining processing.

Breakeven Points

Sometimes it happens that, when two or more design approaches are compared for cost, the choice between the two depends on a set of conditions such as the quantity of production, the speed of the assembly lines, or some other condition. There then occurs a point corresponding to equal cost, which is called the *breakeven point.*

Figure 1–2

Cost versus tolerance/machining process.
(From David G. Ullman, The Mechanical Design Process, *3rd ed., McGraw-Hill, New York, 2003.)*

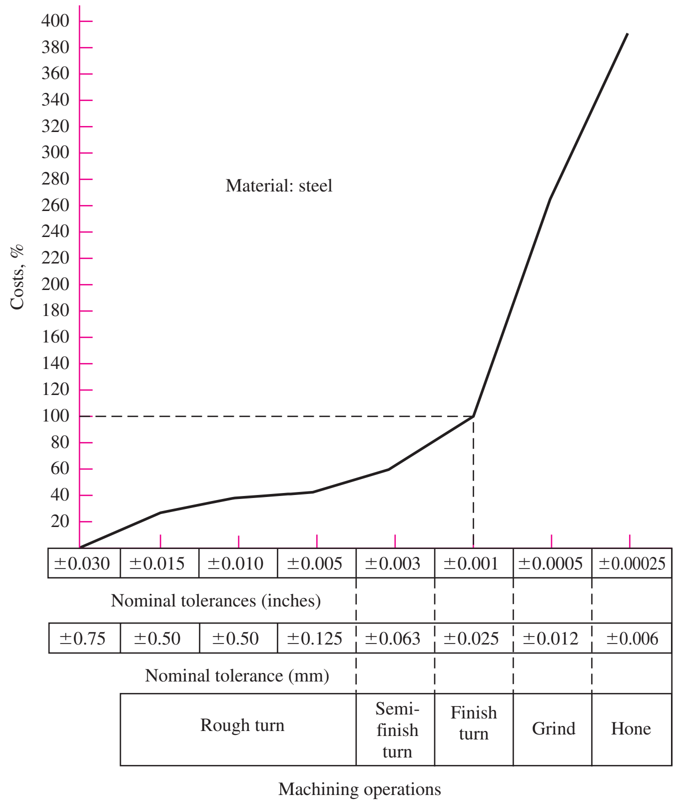
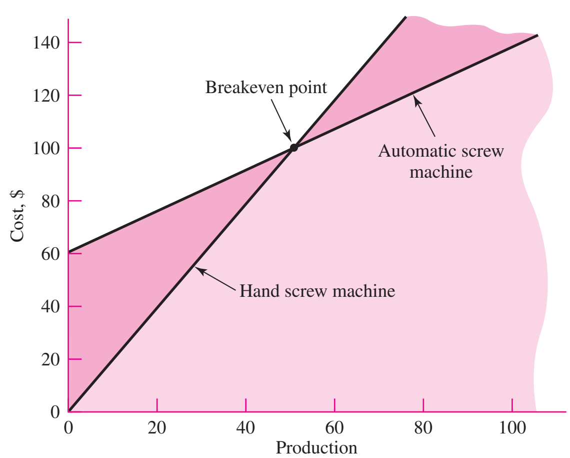


Figure 1–3

A breakeven point.



As an example, consider a situation in which a certain part can be manufactured at the rate of 25 parts per hour on an automatic screw machine or 10 parts per hour on a hand screw machine. Let us suppose, too, that the setup time for the automatic is 3 h and that the labor cost for either machine is \$20 per hour, including overhead. Figure 1–3 is a graph of cost versus production by the two methods. The breakeven point for this example corresponds to 50 parts. If the desired production is greater than 50 parts, the automatic machine should be used.

Cost Estimates

There are many ways of obtaining relative cost figures so that two or more designs can be roughly compared. A certain amount of judgment may be required in some instances. For example, we can compare the relative value of two automobiles by comparing the dollar cost per pound of weight. Another way to compare the cost of one design with another is simply to count the number of parts. The design having the smaller number of parts is likely to cost less. Many other cost estimators can be used, depending upon the application, such as area, volume, horsepower, torque, capacity, speed, and various performance ratios.[9]

1–8 Safety and Product Liability

The *strict liability* concept of product liability generally prevails in the United States. This concept states that the manufacturer of an article is liable for any damage or harm that results because of a defect. And it doesn't matter whether the manufacturer knew about the defect, or even could have known about it. For example, suppose an article was manufactured, say, 10 years ago. And suppose at that time the article could not have been considered defective on the basis of all technological knowledge then available. Ten years later, according to the concept of strict liability, the manufacturer is still liable. Thus, under this concept, the plaintiff needs only to prove that the article was defective and that the defect caused some damage or harm. Negligence of the manufacturer need not be proved.

The best approaches to the prevention of product liability are good engineering in analysis and design, quality control, and comprehensive testing procedures. Advertising managers often make glowing promises in the warranties and sales literature for a product. These statements should be reviewed carefully by the engineering staff to eliminate excessive promises and to insert adequate warnings and instructions for use.

1–9 Stress and Strength

The survival of many products depends on how the designer adjusts the maximum stresses in a component to be less than the component's strength at specific locations of interest. The designer must allow the maximum stress to be less than the strength by a sufficient margin so that despite the uncertainties, failure is rare.

In focusing on the stress-strength comparison at a critical (controlling) location, we often look for "strength in the geometry and condition of use." Strengths are the magnitudes of stresses at which something of interest occurs, such as the proportional limit, 0.2 percent-offset yielding, or fracture. In many cases, such events represent the stress level at which loss of function occurs.

Strength is a property of a material or of a mechanical element. The strength of an element depends on the choice, the treatment, and the processing of the material. Consider, for example, a shipment of springs. We can associate a strength with a specific spring. When this spring is incorporated into a machine, external forces are applied that result in load-induced stresses in the spring, the magnitudes of which depend on its geometry and are independent of the material and its processing. If the spring is removed from the machine unharmed, the stress due to the external forces will return

[9]For an overview of estimating manufacturing costs, see Chap. 11, Karl T. Ulrich and Steven D. Eppinger, *Product Design and Development,* 3rd ed., McGraw-Hill, New York, 2004.

to zero. But the strength remains as one of the properties of the spring. Remember, then, that *strength is an inherent property of a part,* a property built into the part because of the use of a particular material and process.

Various metalworking and heat-treating processes, such as forging, rolling, and cold forming, cause variations in the strength from point to point throughout a part. The spring cited above is quite likely to have a strength on the outside of the coils different from its strength on the inside because the spring has been formed by a cold winding process, and the two sides may not have been deformed by the same amount. Remember, too, therefore, that a strength value given for a part may apply to only a particular point or set of points on the part.

In this book we shall use the capital letter S to denote strength, with appropriate subscripts to denote the type of strength. Thus, S_s is a shear strength, S_y a yield strength, and S_u an ultimate strength.

In accordance with accepted engineering practice, we shall employ the Greek letters σ (sigma) and τ (tau) to designate normal and shear stresses, respectively. Again, various subscripts will indicate some special characteristic. For example, σ_1 is a principal stress, σ_y a stress component in the y direction, and σ_r a stress component in the radial direction.

Stress is a state property at a *specific* point within a body, which is a function of load, geometry, temperature, and manufacturing processing. In an elementary course in mechanics of materials, stress related to load and geometry is emphasized with some discussion of thermal stresses. However, stresses due to heat treatments, molding, assembly, etc. are also important and are sometimes neglected. A review of stress analysis for basic load states and geometry is given in Chap. 3.

1–10 Uncertainty

Uncertainties in machinery design abound. Examples of uncertainties concerning stress and strength include

- Composition of material and the effect of variation on properties.
- Variations in properties from place to place within a bar of stock.
- Effect of processing locally, or nearby, on properties.
- Effect of nearby assemblies such as weldments and shrink fits on stress conditions.
- Effect of thermomechanical treatment on properties.
- Intensity and distribution of loading.
- Validity of mathematical models used to represent reality.
- Intensity of stress concentrations.
- Influence of time on strength and geometry.
- Effect of corrosion.
- Effect of wear.
- Uncertainty as to the length of any list of uncertainties.

Engineers must accommodate uncertainty. Uncertainty always accompanies change. Material properties, load variability, fabrication fidelity, and validity of mathematical models are among concerns to designers.

There are mathematical methods to address uncertainties. The primary techniques are the deterministic and stochastic methods. The deterministic method establishes a *design factor* based on the absolute uncertainties of a loss-of-function parameter and a maximum allowable parameter. Here the parameter can be load, stress, deflection, etc. Thus, the design factor n_d is defined as

$$n_d = \frac{\text{loss-of-function parameter}}{\text{maximum allowable parameter}} \tag{1–1}$$

If the parameter is load, then the maximum allowable load can be found from

$$\text{Maximum allowable load} = \frac{\text{loss-of-function load}}{n_d} \tag{1–2}$$

EXAMPLE 1–1 Consider that the maximum load on a structure is known with an uncertainty of ±20 percent, and the load causing failure is known within ±15 percent. If the load causing failure is *nominally* 2000 lbf, determine the design factor and the maximum allowable load that will offset the absolute uncertainties.

Solution To account for its uncertainty, the loss-of-function load must increase to 1/0.85, whereas the maximum allowable load must decrease to 1/1.2. Thus to offset the absolute uncertainties the design factor should be

Answer
$$n_d = \frac{1/0.85}{1/1.2} = 1.4$$

From Eq. (1–2), the maximum allowable load is found to be

Answer
$$\text{Maximum allowable load} = \frac{2000}{1.4} = 1400 \text{ lbf}$$

Stochastic methods (see Chap. 20) are based on the statistical nature of the design parameters and focus on the probability of survival of the design's function (that is, on reliability). Sections 5–13 and 6–17 demonstrate how this is accomplished.

1–11 Design Factor and Factor of Safety

A general approach to the allowable load versus loss-of-function load problem is the deterministic design factor method, and sometimes called the classical method of design. The fundamental equation is Eq. (1–1) where n_d is called the *design factor.* All loss-of-function modes must be analyzed, and the mode leading to the smallest design factor governs. After the design is completed, the *actual* design factor may change as a result of changes such as rounding up to a standard size for a cross section or using off-the-shelf components with higher ratings instead of employing what is calculated by using the design factor. The factor is then referred to as the *factor of safety, n.* The factor of safety has the same definition as the design factor, but it generally differs numerically.

Since stress may not vary linearly with load (see Sec. 3–19), using load as the loss-of-function parameter may not be acceptable. It is more common then to express the design factor in terms of a stress and a relevant strength. Thus Eq. (1–1) can be rewritten as

$$n_d = \frac{\text{loss-of-function strength}}{\text{allowable stress}} = \frac{S}{\sigma(\text{or } \tau)} \tag{1–3}$$

The stress and strength terms in Eq. (1–3) must be of the same type and units. Also, the stress and strength must apply to the same critical location in the part.

EXAMPLE 1–2

A rod with a cross-sectional area of A and loaded in tension with an axial force of $P =$ 2000 lbf undergoes a stress of $\sigma = P/A$. Using a material strength of 24 kpsi and a *design factor* of 3.0, determine the minimum diameter of a solid circular rod. Using Table A–17, select a preferred fractional diameter and determine the rod's *factor of safety*.

Solution

Since $A = \pi d^2/4$, and $\sigma = S/n_d$, then

$$\sigma = \frac{S}{n_d} = \frac{24\ 000}{3} = \frac{P}{A} = \frac{2\ 000}{\pi d^2/4}$$

or,

Answer

$$d = \left(\frac{4Pn_d}{\pi S}\right)^{1/2} = \left(\frac{4(2000)3}{\pi(24\ 000)}\right)^{1/2} = 0.564 \text{ in}$$

From Table A–17, the next higher preferred size is $\frac{5}{8}$ in $= 0.625$ in. Thus, according to the same equation developed earlier, the factor of safety n is

Answer

$$n = \frac{\pi S d^2}{4P} = \frac{\pi(24\ 000)0.625^2}{4(2000)} = 3.68$$

Thus rounding the diameter has increased the actual design factor.

1–12 Reliability

In these days of greatly increasing numbers of liability lawsuits and the need to conform to regulations issued by governmental agencies such as EPA and OSHA, it is very important for the designer and the manufacturer to know the reliability of their product. The reliability method of design is one in which we obtain the distribution of stresses and the distribution of strengths and then relate these two in order to achieve an acceptable success rate.

The statistical measure of the probability that a mechanical element will not fail in use is called the *reliability* of that element. The reliability R can be expressed by a number having the range $0 \le R \le 1$. A reliability of $R = 0.90$ means that there is a 90 percent chance that the part will perform its proper function without failure. The failure of 6 parts out of every 1000 manufactured might be considered an acceptable failure rate for a certain class of products. This represents a reliability of

$$R = 1 - \frac{6}{1000} = 0.994$$

or 99.4 percent.

In the *reliability method of design,* the designer's task is to make a judicious selection of materials, processes, and geometry (size) so as to achieve a specific reliability goal. Thus, if the objective reliability is to be 99.4 percent, as above, what combination of materials, processing, and dimensions is needed to meet this goal?

Analyses that lead to an assessment of reliability address uncertainties, or their estimates, in parameters that describe the situation. Stochastic variables such as stress, strength, load, or size are described in terms of their means, standard deviations, and distributions. If bearing balls are produced by a manufacturing process in which a diameter distribution is created, we can say upon choosing a ball that there is uncertainty as to size. If we wish to consider weight or moment of inertia in rolling, this size uncertainty can be considered to be *propagated* to our knowledge of weight or inertia. There are ways of estimating the statistical parameters describing weight and inertia from those describing size and density. These methods are variously called *propagation of error, propagation of uncertainty,* or *propagation of dispersion.* These methods are integral parts of analysis or synthesis tasks when probability of failure is involved.

It is important to note that good statistical data and estimates are essential to perform an acceptable reliability analysis. This requires a good deal of testing and validation of the data. In many cases, this is not practical and a deterministic approach to the design must be undertaken.

1–13 Dimensions and Tolerances

The following terms are used generally in dimensioning:

- *Nominal size.* The size we use in speaking of an element. For example, we may specify a $1\frac{1}{2}$-in pipe or a $\frac{1}{2}$-in bolt. Either the theoretical size or the actual measured size may be quite different. The theoretical size of a $1\frac{1}{2}$-in pipe is 1.900 in for the outside diameter. And the diameter of the $\frac{1}{2}$-in bolt, say, may actually measure 0.492 in.
- *Limits.* The stated maximum and minimum dimensions.
- *Tolerance.* The difference between the two limits.
- *Bilateral tolerance.* The variation in both directions from the basic dimension. That is, the basic size is between the two limits, for example, 1.005 ± 0.002 in. The two parts of the tolerance need not be equal.
- *Unilateral tolerance.* The basic dimension is taken as one of the limits, and variation is permitted in only one direction, for example,

$$1.005\ {}^{+0.004}_{-0.000}\ \text{in}$$

- *Clearance.* A general term that refers to the mating of cylindrical parts such as a bolt and a hole. The word clearance is used only when the internal member is smaller than the external member. The *diametral clearance* is the measured difference in the two diameters. The *radial clearance* is the difference in the two radii.
- *Interference.* The opposite of clearance, for mating cylindrical parts in which the internal member is larger than the external member.
- *Allowance.* The minimum stated clearance or the maximum stated interference for mating parts.

When several parts are assembled, the gap (or interference) depends on the dimensions and tolerances of the individual parts.

EXAMPLE 1–3

A shouldered screw contains three hollow right circular cylindrical parts on the screw before a nut is tightened against the shoulder. To sustain the function, the gap w must equal or exceed 0.003 in. The parts in the assembly depicted in Fig. 1–4 have dimensions and tolerances as follows:

$$a = 1.750 \pm 0.003 \text{ in} \qquad b = 0.750 \pm 0.001 \text{ in}$$

$$c = 0.120 \pm 0.005 \text{ in} \qquad d = 0.875 \pm 0.001 \text{ in}$$

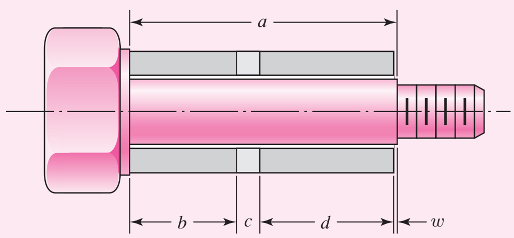


Figure 1–4

An assembly of three cylindrical sleeves of lengths *a*, *b*, and *c* on a shoulder bolt shank of length *a*. The gap *w* is of interest.

All parts except the part with the dimension d are supplied by vendors. The part containing the dimension d is made in-house.
(a) Estimate the mean and tolerance on the gap w.
(b) What basic value of d will assure that $w \geq 0.003$ in?

Solution (a) The mean value of w is given by

Answer

$$\bar{w} = \bar{a} - \bar{b} - \bar{c} - \bar{d} = 1.750 - 0.750 - 0.120 - 0.875 = 0.005 \text{ in}$$

For equal bilateral tolerances, the tolerance of the gap is

Answer

$$t_w = \sum_{\text{all}} t = 0.003 + 0.001 + 0.005 + 0.001 = 0.010 \text{ in}$$

Then, $w = 0.005 \pm 0.010$, and

$$w_{\max} = \bar{w} + t_w = 0.005 + 0.010 = 0.015 \text{ in}$$

$$w_{\min} = \bar{w} - t_w = 0.005 - 0.010 = -0.005 \text{ in}$$

Thus, both clearance and interference are possible.
(b) If $w_{\min}$ is to be 0.003 in, then, $\bar{w} = w_{\min} + t_w = 0.003 + 0.010 = 0.013$ in. Thus,

Answer

$$\bar{d} = \bar{a} - \bar{b} - \bar{c} - \bar{w} = 1.750 - 0.750 - 0.120 - 0.013 = 0.867 \text{ in}$$

The previous example represented an *absolute tolerance system.* Statistically, gap dimensions near the gap limits are rare events. Using a *statistical tolerance system,* the probability that the gap falls within a given limit is determined.[10] This probability deals with the statistical distributions of the individual dimensions. For example, if the distributions of the dimensions in the previous example were normal and the tolerances, t, were

[10] See Chapter 20 for a description of the statistical terminology.

given in terms of standard deviations of the dimension distribution, the standard deviation of the gap $\bar{w}$ would be $t_w = \sqrt{\sum_{\text{all}} t^2}$. However, this assumes a normal distribution for the individual dimensions, a rare occurrence. To find the distribution of w and/or the probability of observing values of w within certain limits requires a computer simulation in most cases. *Monte Carlo* computer simulations are used to determine the distribution of w by the following approach:

1. Generate an instance for each dimension in the problem by selecting the value of each dimension based on its probability distribution.
2. Calculate w using the values of the dimensions obtained in step 1.
3. Repeat steps 1 and 2 N times to generate the distribution of w. As the number of trials increases, the reliability of the distribution increases.

1–14 Units

In the symbolic units equation for Newton's second law, $F = ma$,

$$F = MLT^{-2} \tag{1–4}$$

F stands for force, M for mass, L for length, and T for time. Units chosen for *any* three of these quantities are called *base* units. The first three having been chosen, the fourth unit is called a *derived* unit. When force, length, and time are chosen as base units, the mass is the derived unit and the system that results is called a *gravitational system of units*. When mass, length, and time are chosen as base units, force is the derived unit and the system that results is called an *absolute system of units*.

In some English-speaking countries, the *U.S. customary foot-pound-second system* (fps) and the *inch-pound-second system* (ips) are the two standard gravitational systems most used by engineers. In the fps system the unit of mass is

$$M = \frac{FT^2}{L} = \frac{(\text{pound-force})(\text{second})^2}{\text{foot}} = \text{lbf} \cdot \text{s}^2/\text{ft} = \text{slug} \tag{1–5}$$

Thus, length, time, and force are the three base units in the fps gravitational system.

The unit of force in the fps system is the pound, more properly the *pound-force.* We shall often abbreviate this unit as lbf; the abbreviation lb is permissible however, since we shall be dealing only with the U.S. customary gravitational system. In some branches of engineering it is useful to represent 1000 lbf as a kilopound and to abbreviate it as kip. *Note:* In Eq. (1–5) the derived unit of mass in the fps gravitational system is the lbf · s^2/ft and is called a *slug;* there is no abbreviation for slug.

The unit of mass in the ips gravitational system is

$$M = \frac{FT^2}{L} = \frac{(\text{pound-force})(\text{second})^2}{\text{inch}} = \text{lbf} \cdot \text{s}^2/\text{in} \tag{1–6}$$

The mass unit lbf · s^2/in has no official name.

The *International System of Units* (SI) is an absolute system. The base units are the meter, the kilogram (for mass), and the second. The unit of force is derived by using Newton's second law and is called the *newton.* The units constituting the newton (N) are

$$F = \frac{ML}{T^2} = \frac{(\text{kilogram})(\text{meter})}{(\text{second})^2} = \text{kg} \cdot \text{m/s}^2 = \text{N} \tag{1–7}$$

The weight of an object is the force exerted upon it by gravity. Designating the weight as W and the acceleration due to gravity as g, we have

$$W = mg \tag{1–8}$$

In the fps system, standard gravity is $g = 32.1740$ ft/s^2. For most cases this is rounded off to 32.2. Thus the weight of a mass of 1 slug in the fps system is

$$W = mg = (1\ \text{slug})(32.2\ \text{ft/s}^2) = 32.2\ \text{lbf}$$

In the ips system, standard gravity is 386.088 or about 386 in/s^2. Thus, in this system, a unit mass weighs

$$W = (1\ \text{lbf} \cdot \text{s}^2/\text{in})(386\ \text{in/s}^2) = 386\ \text{lbf}$$

With SI units, standard gravity is 9.806 or about 9.81 m/s. Thus, the weight of a 1-kg mass is

$$W = (1\ \text{kg})(9.81\ \text{m/s}^2) = 9.81\ \text{N}$$

A series of names and symbols to form multiples and submultiples of SI units has been established to provide an alternative to the writing of powers of 10. Table A–1 includes these prefixes and symbols.

Numbers having four or more digits are placed in groups of three and separated by a space instead of a comma. However, the space may be omitted for the special case of numbers having four digits. A period is used as a decimal point. These recommendations avoid the confusion caused by certain European countries in which a comma is used as a decimal point, and by the English use of a centered period. Examples of correct and incorrect usage are as follows:

1924 or 1 924 but not 1,924
0.1924 or 0.192 4 but not 0.192,4
192 423.618 50 but not 192,423.61850

The decimal point should always be preceded by a zero for numbers less than unity.

1–15 Calculations and Significant Figures

The discussion in this section applies to real numbers, not integers. The accuracy of a real number depends on the number of significant figures describing the number. Usually, but not always, three or four significant figures are necessary for engineering accuracy. Unless otherwise stated, *no less* than three significant figures should be used in your calculations. The number of significant figures is usually inferred by the number of figures given (except for leading zeros). For example, 706, 3.14, and 0.002 19 are assumed to be numbers with three significant figures. For trailing zeros, a little more clarification is necessary. To display 706 to four significant figures insert a trailing zero and display either 706.0, 7.060×10^2, or 0.7060×10^3. Also, consider a number such as 91 600. Scientific notation should be used to clarify the accuracy. For three significant figures express the number as 91.6×10^3. For four significant figures express it as 91.60×10^3.

Computers and calculators display calculations to many significant figures. However, you should never report a number of significant figures of a calculation any greater than the smallest number of significant figures of the numbers used for the calculation. Of course, you should use the greatest accuracy possible when performing a calculation. For example, determine the circumference of a solid shaft with a diameter of $d = 0.40$ in. The circumference is given by $C = \pi d$. Since d is given with two significant figures, C should be reported with only two significant figures. Now if we used only two significant figures for π our calculator would give $C = 3.1\ (0.40) = 1.24$ in. This rounds off to two significant figures as $C = 1.2$ in. However, using $\pi = 3.141\ 592\ 654$ as programmed in the calculator, $C = 3.141\ 592\ 654\ (0.40) = 1.256\ 637\ 061$ in. This rounds off to $C = 1.3$ in, which is 8.3 percent higher than the first calculation. Note, however, since d is given with two significant figures, it is implied that the range of d is 0.40 ± 0.005. This means that the calculation of C is only accurate to within $\pm 0.005/0.40 = \pm 0.0125 = \pm 1.25\%$. The calculation could also be one in a series of calculations, and rounding each calculation separately may lead to an accumulation of greater inaccuracy. Thus, it is considered good engineering practice to make all calculations to the greatest accuracy possible and report the results within the accuracy of the given input.

1–16 Power Transmission Case Study Specifications

A case study incorporating the many facets of the design process for a power transmission speed reducer will be considered throughout this textbook. The problem will be introduced here with the definition and specification for the product to be designed. Further details and component analysis will be presented in subsequent chapters. Chapter 18 provides an overview of the entire process, focusing on the design sequence, the interaction between the component designs, and other details pertinent to transmission of power. It also contains a complete case study of the power transmission speed reducer introduced here.

Many industrial applications require machinery to be powered by engines or electric motors. The power source usually runs most efficiently at a narrow range of rotational speed. When the application requires power to be delivered at a slower speed than supplied by the motor, a speed reducer is introduced. The speed reducer should transmit the power from the motor to the application with as little energy loss as practical, while reducing the speed and consequently increasing the torque. For example, assume that a company wishes to provide off-the-shelf speed reducers in various capacities and speed ratios to sell to a wide variety of target applications. The marketing team has determined a need for one of these speed reducers to satisfy the following customer requirements.

Design Requirements

Power to be delivered: 20 hp
Input speed: 1750 rev/min
Output speed: 85 rev/min
Targeted for uniformly loaded applications, such as conveyor belts, blowers, and generators
Output shaft and input shaft in-line
Base mounted with 4 bolts
Continuous operation
6-year life, with 8 hours/day, 5 days/wk
Low maintenance
Competitive cost
Nominal operating conditions of industrialized locations
Input and output shafts standard size for typical couplings

In reality, the company would likely design for a whole range of speed ratios for each power capacity, obtainable by interchanging gear sizes within the same overall design. For simplicity, in this case study only one speed ratio will be considered.

Notice that the list of customer requirements includes some numerical specifics, but also includes some generalized requirements, e.g., low maintenance and competitive cost. These general requirements give some guidance on what needs to be considered in the design process, but are difficult to achieve with any certainty. In order to pin down these nebulous requirements, it is best to further develop the customer requirements into a set of product specifications that are measurable. This task is usually achieved through the work of a team including engineering, marketing, management, and customers. Various tools may be used (see Footnote 1) to prioritize the requirements, determine suitable metrics to be achieved, and to establish target values for each metric. The goal of this process is to obtain a product specification that identifies precisely what the product must satisfy. The following product specifications provide an appropriate framework for this design task.

Design Specifications

Power to be delivered: 20 hp
Power efficiency: >95%
Steady state input speed: 1750 rev/min
Maximum input speed: 2400 rev/min
Steady-state output speed: 82–88 rev/min
Usually low shock levels, occasional moderate shock
Input and output shaft diameter tolerance: ±0.001 in
Output shaft and input shaft in-line: concentricity ±0.005 in, alignment ±0.001 rad
Maximum allowable loads on input shaft: axial, 50 lbf; transverse, 100 lbf
Maximum allowable loads on output shaft: axial, 50 lbf; transverse, 500 lbf
Base mounted with 4 bolts
Mounting orientation only with base on bottom
100% duty cycle
Maintenance schedule: lubrication check every 2000 hours; change of lubrication every 8000 hours of operation; gears and bearing life >12,000 hours; infinite shaft life; gears, bearings, and shafts replaceable
Access to check, drain, and refill lubrication without disassembly or opening of gasketed joints.
Manufacturing cost per unit: <$300
Production: 10,000 units per year
Operating temperature range: −10° to 120°F
Sealed against water and dust from typical weather
Noise: <85 dB from 1 meter

PROBLEMS

1–1 Select a mechanical component from Part 3 of this book (roller bearings, springs, etc.), go to your university's library or the appropriate internet website, and, using the *Thomas Register of American Manufacturers,* report on the information obtained on five manufacturers or suppliers.

1–2 Select a mechanical component from Part 3 of this book (roller bearings, springs, etc.), go to the Internet, and, using a search engine, report on the information obtained on five manufacturers or suppliers.

1–3 Select an organization listed in Sec. 1–6, go to the Internet, and list what information is available on the organization.

1–4 Go to the Internet and connect to the NSPE website (www.nspe.org). Read the full version of the NSPE Code of Ethics for Engineers and briefly discuss your reading.

1–5 Highway tunnel traffic (two parallel lanes in the same direction) experience indicates the average spacing between vehicles increases with speed. Data from a New York tunnel show that between 15 and 35 mi/h, the space x between vehicles (in miles) is $x = 0.324/(42.1 - v)$ where v is the vehicle's speed in miles per hour.

(*a*) Ignoring the length of individual vehicles, what speed will give the tunnel the largest volume in vehicles per hour?

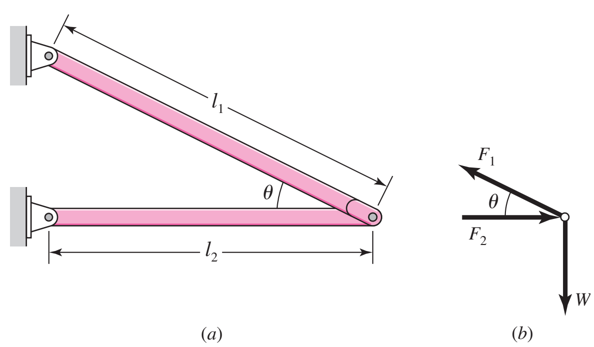
(*b*) Does including the length of the vehicles cut the tunnel capacity prediction significantly? Assume the average vehicle length is 10 ft.

(*c*) For part (*b*), does the optimal speed change much?

1–6 The engineering designer must create (invent) the concept and connectivity of the elements that constitute a design, and not lose sight of the need to develop ideas with optimality in mind. A useful design attribute can be cost, which can be related to the amount of material used (volume or weight). When you think about it, the weight is a function of the geometry and density. When the design is solidified, finding the weight is a straightforward, sometimes tedious task. The figure depicts a simple bracket frame that has supports that project from a wall column. The bracket supports a chain-fall hoist. Pinned joints are used to avoid bending. The cost of a link can be approximated by $\$ = ¢ A l \gamma$, where ¢ is the cost of the link per unit weight, A is the cross-sectional area of the prismatic link, l is the pin-to-pin link length, and γ is the specific weight of the material used. To be sure, this is approximate because no decisions have been made concerning the geometric form of the links or their fittings. By investigating cost now in this approximate way, one can detect whether a particular set of proportions of the bracket (indexed by angle θ) is advantageous. Is there a preferable angle θ? Show that the cost can be expressed as

$$\$ = \frac{\gamma ¢ W l_2}{S}\left(\frac{1+\cos^2\theta}{\sin\theta\cos\theta}\right)$$

where W is the weight of the hoist and load, and S is the allowable tensile or compressive stress in the link material (assume $S = |F_i/A|$ and no column buckling action). What is the desirable angle θ corresponding to the minimal cost?



Problem 1–6
(a) A chain-hoist bracket frame. *(b)* Free body of pin.

1–7 When one knows the true values x_1 and x_2 and has approximations X_1 and X_2 at hand, one can see where errors may arise. By viewing error as something to be added to an approximation to attain a true value, it follows that the error e_i, is related to X_i, and x_i as $x_i = X_i + e_i$

(*a*) Show that the error in a sum $X_1 + X_2$ is

$$(x_1 + x_2) - (X_1 + X_2) = e_1 + e_2$$

(*b*) Show that the error in a difference $X_1 - X_2$ is

$$(x_1 - x_2) - (X_1 - X_2) = e_1 - e_2$$

(*c*) Show that the error in a product $X_1 X_2$ is

$$x_1 x_2 - X_1 X_2 = X_1 X_2\left(\frac{e_1}{X_1} + \frac{e_2}{X_2}\right)$$

(*d*) Show that in a quotient X_1/X_2 the error is

$$\frac{x_1}{x_2} - \frac{X_1}{X_2} = \frac{X_1}{X_2}\left(\frac{e_1}{X_1} - \frac{e_2}{X_2}\right)$$

1–8 Use the true values $x_1 = \sqrt{5}$ and $x_2 = \sqrt{6}$

(*a*) Demonstrate the correctness of the error equation from Prob. 1–7 for addition if three correct digits are used for X_1 and X_2.

(*b*) Demonstrate the correctness of the error equation for addition using three-digit significant numbers for X_1 and X_2.

1–9 Convert the following to appropriate SI units:

(*a*) A stress of 20 000 psi.
(*b*) A force of 350 lbf.
(*c*) A moment of 1200 lbf · in.
(*d*) An area of 2.4 in^2.
(*e*) A second moment of area of 17.4 in^4.
(*f*) An area of 3.6 mi^2.
(*g*) A modulus of elasticity of 21 Mpsi.
(*h*) A speed of 45 mi/h.
(*i*) A volume of 60 in^3.

1–10 Convert the following to appropriate ips units:

(*a*) A length of 1.5 m.
(*b*) A stress of 600 MPa.
(*c*) A pressure of 160 kPa.
(*d*) A section modulus of 1.84 (10^5) mm^3.
(*e*) A unit weight of 38.1 N/m.
(*f*) A deflection of 0.05 mm.
(*g*) A velocity of 6.12 m/s.
(*h*) A unit strain of 0.0021 m/m.
(*i*) A volume of 30 L.

1–11 Generally, final design results are rounded to or fixed to three digits because the given data cannot justify a greater display. In addition, prefixes should be selected so as to limit number strings to no more than four digits to the left of the decimal point. Using these rules, as well as those for the choice of prefixes, solve the following relations:

(*a*) $\sigma = M/Z$, where $M = 200$ N · m and $Z = 15.3 \times 10^3$ mm^3.
(*b*) $\sigma = F/A$, where $F = 42$ kN and $A = 600$ mm^2.
(*c*) $y = Fl^3/3EI$, where $F = 1200$ N, $l = 800$ mm, $E = 207$ GPa, and $I = 64 \times 10^3$ mm^4.
(*d*) $\theta = Tl/GJ$, where $J = \pi d^4/32$, $T = 1100$ N · m, $l = 250$ mm, $G = 79.3$ GPa, and $d =$ 25 mm. Convert results to degrees of angle.

1–12 Repeat Prob. 1–11 for the following:

(*a*) $\sigma = F/wt$, where $F = 600$ N, $w = 20$ mm, and $t = 6$ mm.
(*b*) $I = bh^3/12$, where $b = 8$ mm and $h = 24$ mm.
(*c*) $I = \pi d^4/64$, where $d = 32$ mm.
(*d*) $\tau = 16T/\pi d^3$, where $T = 16$ N · m and $d = 25$ mm.

1–13 Repeat Prob. 1–11 for:

(*a*) $\tau = F/A$, where $A = \pi d^2/4$, $F = 120$ kN, and $d = 20$ mm.
(*b*) $\sigma = 32\,Fa/\pi d^3$, where $F = 800$ N, $a = 800$ mm, and $d = 32$ mm.
(*c*) $Z = (\pi/32d)(d^4 - d_i^4)$ for $d = 36$ mm and $d_i = 26$ mm.
(*d*) $k = (d^4G)/(8D^3N)$, where $d = 1.6$ mm, $G = 79.3$ GPa, $D = 19.2$ mm, and $N = 32$ (a dimensionless number).

2 Materials

Chapter Outline

The selection of a material for a machine part or a structural member is one of the most important decisions the designer is called on to make. The decision is usually made before the dimensions of the part are established. After choosing the process of creating the desired geometry and the material (the two cannot be divorced), the designer can proportion the member so that loss of function can be avoided or the chance of loss of function can be held to an acceptable risk.

In Chaps. 3 and 4, methods for estimating stresses and deflections of machine members are presented. These estimates are based on the properties of the material from which the member will be made. For deflections and stability evaluations, for example, the elastic (stiffness) properties of the material are required, and evaluations of stress at a critical location in a machine member require a comparison with the strength of the material at that location in the geometry and condition of use. This strength is a material property found by testing and is adjusted to the geometry and condition of use as necessary.

As important as stress and deflection are in the design of mechanical parts, the selection of a material is not always based on these factors. Many parts carry no loads on them whatever. Parts may be designed merely to fill up space or for aesthetic qualities. Members must frequently be designed to also resist corrosion. Sometimes temperature effects are more important in design than stress and strain. So many other factors besides stress and strain may govern the design of parts that the designer must have the versatility that comes only with a broad background in materials and processes.

2–1 Material Strength and Stiffness

The standard tensile test is used to obtain a variety of material characteristics and strengths that are used in design. Figure 2–1 illustrates a typical tension-test specimen and its characteristic dimensions.[1] The original diameter d_0 and the gauge length l_0, used to measure the deflections, are recorded before the test is begun. The specimen is then mounted in the test machine and slowly loaded in tension while the load P and deflection are observed. The load is converted to stress by the calculation

$$\sigma = \frac{P}{A_0} \tag{2–1}$$

where $A_0 = \frac{1}{4}\pi d_0^2$ is the original area of the specimen.

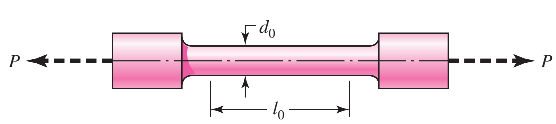


Figure 2–1

A typical tension-test specimen. Some of the standard dimensions used for d_0 are 2.5, 6.25, and 12.5 mm and 0.505 in, but other sections and sizes are in use. Common gauge lengths l_0 used are 10, 25, and 50 mm and 1 and 2 in.

[1]See ASTM standards E8 and E-8 m for standard dimensions.

The deflection, or extension of the gage length, is given by $l - l_0$ where l is the gauge length corresponding to the load P. The normal strain is calculated from

$$\epsilon = \frac{l - l_0}{l_0} \tag{2–2}$$

At the conclusion of, or during, the test, the results are plotted as a *stress-strain diagram*. Figure 2–2 depicts typical stress-strain diagrams for ductile and brittle materials. Ductile materials deform much more than brittle materials.

Point pl in Fig. 2–2*a* is called the *proportional limit.* This is the point at which the curve first begins to deviate from a straight line. No permanent set will be observable in the specimen if the load is removed at this point. In the linear range, the uniaxial stress-strain relation is given by *Hooke's law* as

$$\sigma = E\epsilon \tag{2–3}$$

where the constant of proportionality E, the slope of the linear part of the stress-strain curve, is called *Young's modulus* or the *modulus of elasticity*. E is a measure of the stiffness of a material, and since strain is dimensionless, the units of E are the same as stress. Steel, for example, has a modulus of elasticity of about 30 Mpsi (207 GPa) *regardless of heat treatment, carbon content, or alloying.* Stainless steel is about 27.5 Mpsi (190 GPa).

Point el in Fig. 2–2 is called the *elastic limit*. If the specimen is loaded beyond this point, the deformation is said to be plastic and the material will take on a permanent set when the load is removed. Between pl and el the diagram is not a perfectly straight line, even though the specimen is elastic.

During the tension test, many materials reach a point at which the strain begins to increase very rapidly without a corresponding increase in stress. This point is called the *yield point.* Not all materials have an obvious yield point, especially for brittle materials. For this reason, *yield strength* S_y is often defined by an *offset method* as shown in Fig. 2–2, where line *ay* is drawn at slope E. Point a corresponds to a definite or stated amount of permanent set, usually 0.2 percent of the original gauge length ($\epsilon = 0.002$), although 0.01, 0.1, and 0.5 percent are sometimes used.

The *ultimate,* or *tensile, strength* S_u or S_{ut} corresponds to point u in Fig. 2–2 and is the maximum stress reached on the stress-strain diagram.[2] As shown in Fig. 2–2*a*,

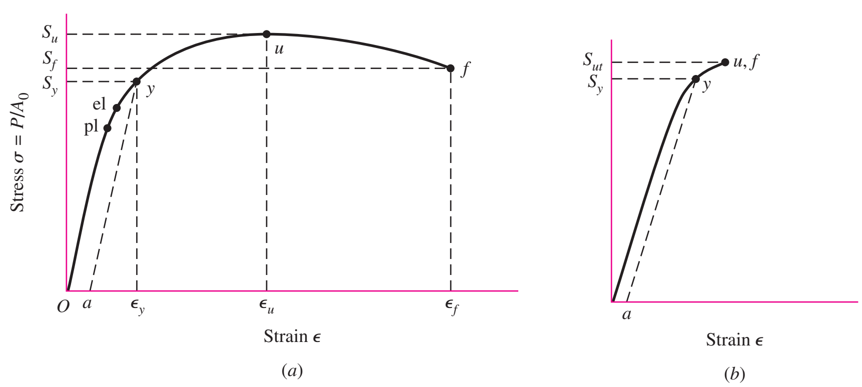


Figure 2–2

Stress-strain diagram obtained from the standard tensile test (*a*) Ductile material; (*b*) brittle material.
pl marks the proportional limit; el, the elastic limit; *y*, the offset-yield strength as defined by offset strain *a*; *u*, the maximum or ultimate strength; and *f*, the fracture strength.

[2] Usage varies. For a long time engineers used the term *ultimate strength,* hence the subscript u in S_u or S_{ut}. However, in material science and metallurgy the term *tensile strength* is used.

some materials exhibit a downward trend after the maximum stress is reached and fracture at point *f* on the diagram. Others, such as some of the cast irons and high-strength steels, fracture while the stress-strain trace is still rising, as shown in Fig. 2–2*b*, where points *u* and *f* are identical.

As noted in Sec. 1–9, *strength,* as used in this book, is a built-in property of a material, or of a mechanical element, because of the selection of a particular material or process or both. The strength of a connecting rod at the critical location in the geometry and condition of use, for example, is the same no matter whether it is already an element in an operating machine or whether it is lying on a workbench awaiting assembly with other parts. On the other hand, *stress* is something that occurs in a part, usually as a result of its being assembled into a machine and loaded. However, stresses may be built into a part by processing or handling. For example, shot peening produces a compressive *stress* in the outer surface of a part, and also improves the fatigue strength of the part. Thus, in this book we will be very careful in distinguishing between *strength,* designated by S, and *stress,* designated by σ or τ.

The diagrams in Fig. 2–2 are called *engineering* stress-strain diagrams because the stresses and strains calculated in Eqs. (2–1) and (2–2) are not *true* values. The stress calculated in Eq. (2–1) is based on the original area *before* the load is applied. In reality, as the load is applied the area reduces so that the *actual* or *true stress* is larger than the *engineering stress*. To obtain the true stress for the diagram the load and the cross-sectional area must be measured simultaneously during the test. Figure 2–2*a* represents a ductile material where the stress appears to decrease from points *u* to *f*. Typically, beyond point *u* the specimen begins to "neck" at a location of weakness where the area reduces dramatically, as shown in Fig. 2–3. For this reason, the true stress is much higher than the engineering stress at the necked section.

The engineering strain given by Eq. (2–2) is based on net change in length from the *original* length. In plotting the *true stress-strain diagram,* it is customary to use a term called *true strain* or, sometimes, *logarithmic strain.* True strain is the sum of the incremental elongations divided by the *current* gauge length at load P, or

$$\varepsilon = \int_{l_0}^{l} \frac{dl}{l} = \ln \frac{l}{l_0} \tag{2–4}$$

where the symbol ε is used to represent true strain. The most important characteristic of a true stress-strain diagram (Fig. 2–4) is that the true stress continually increases all the way to fracture. Thus, as shown in Fig. 2–4, the true fracture stress σ_f is greater than the true ultimate stress σ_u. Contrast this with Fig. 2–2*a*, where the engineering fracture strength S_f is less than the engineering ultimate strength S_u.

Compression tests are more difficult to conduct, and the geometry of the test specimens differs from the geometry of those used in tension tests. The reason for this is that the specimen may buckle during testing or it may be difficult to distribute the stresses evenly. Other difficulties occur because ductile materials will bulge after yielding. However, the results can be plotted on a stress-strain diagram also, and the same strength definitions can be applied as used in tensile testing. For most ductile materials the compressive strengths are about the same as the tensile strengths. When substantial differences occur between tensile and compressive strengths, however, as is the case with

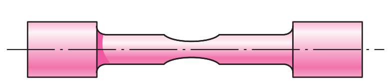


Figure 2–3

Tension specimen after necking.

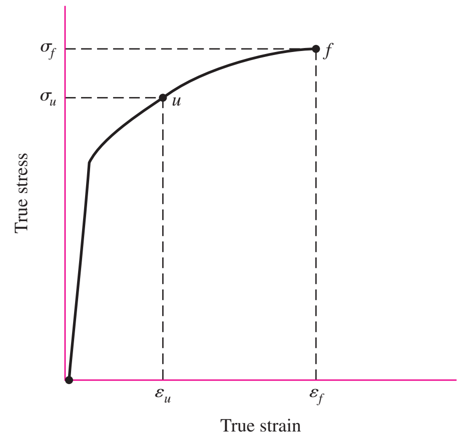


Figure 2–4

True stress-strain diagram plotted in Cartesian coordinates.

the cast irons, the tensile and compressive strengths should be stated separately, S_{ut}, S_{uc}, where S_{uc} is reported as a *positive* quantity.

Torsional strengths are found by twisting solid circular bars and recording the torque and the twist angle. The results are then plotted as a *torque-twist diagram.* The shear stresses in the specimen are linear with respect to radial location, being zero at the center of the specimen and maximum at the outer radius r (see Chap. 3). The maximum shear stress $\tau_{\max}$ is related to the angle of twist θ by

$$\tau_{\max} = \frac{Gr}{l_0}\theta \tag{2–5}$$

where θ is in radians, r is the radius of the specimen, l_0 is the gauge length, and G is the material stiffness property called the *shear modulus* or the *modulus of rigidity*. The maximum shear stress is also related to the applied torque T as

$$\tau_{\max} = \frac{Tr}{J} \tag{2–6}$$

where $J = \frac{1}{2}\pi r^4$ is the polar second moment of area of the cross section.

The torque-twist diagram will be similar to Fig. 2–2, and, using Eqs. (2–5) and (2–6), the modulus of rigidity can be found as well as the elastic limit and the *torsional yield strength* S_{sy}. The maximum point on a torque-twist diagram, corresponding to point u on Fig. 2–2, is T_u. The equation

$$S_{su} = \frac{T_u r}{J} \tag{2–7}$$

defines the *modulus of rupture* for the torsion test. Note that it is incorrect to call S_{su} the ultimate torsional strength, as the outermost region of the bar is in a plastic state at the torque T_u and the stress distribution is no longer linear.

All of the stresses and strengths defined by the stress-strain diagram of Fig. 2–2 and similar diagrams are specifically known as *engineering stresses* and *strengths* or *nominal stresses* and *strengths.* These are the values normally used in all engineering design calculations. The adjectives *engineering* and *nominal* are used here to emphasize that the stresses are computed by using the *original* or *unstressed cross-sectional area* of the specimen. In this book we shall use these modifiers only when we specifically wish to call attention to this distinction.

2–2 The Statistical Significance of Material Properties

There is some subtlety in the ideas presented in the previous section that should be pondered carefully before continuing. Figure 2–2 depicts the result of a *single* tension test (*one* specimen, now fractured). It is common for engineers to consider these important *stress* values (at points pl, el, y, u, and f) as properties and to denote them as strengths with a special notation, uppercase S, in lieu of lowercase sigma σ, with subscripts added: S_{pl} for proportional limit, S_y for yield strength, S_u for ultimate tensile strength (S_{ut} or S_{uc}, if tensile or compressive sense is important).

If there were 1000 nominally identical specimens, the values of strength obtained would be distributed between some minimum and maximum values. It follows that the description of strength, a material property, is distributional and thus is statistical in nature. Chapter 20 provides more detail on statistical considerations in design. Here we will simply describe the results of one example, Ex. 20-4. Consider the following table, which is a histographic report containing the maximum stresses of 1000 tensile tests on a 1020 steel from a single heat. Here we are seeking the ultimate tensile strength S_{ut}. The class frequency is the number of occurrences within a 1 kpsi range given by the class midpoint. Thus, 18 maximum stress values occurred in the range of 57 to 58 kpsi.

Class Frequency f_i	2	18	23	31	83	109	138	151	139	130	82	49	28	11	4	2
Class Midpoint x_i, kpsi	56.5	57.5	58.5	59.5	60.5	61.5	62.5	63.5	64.5	65.5	66.5	67.5	68.5	69.5	70.5	71.5

The *probability density* is defined as the number of occurrences divided by the total sample number. The bar chart in Fig. 2–5 depicts the histogram of the probability density. If the data is in the form of a *Gaussian* or *normal distribution*, the *probability density function* determined in Ex. 20-4 is

$$f(x) = \frac{1}{2.594\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x - 63.62}{2.594}\right)^2\right]$$

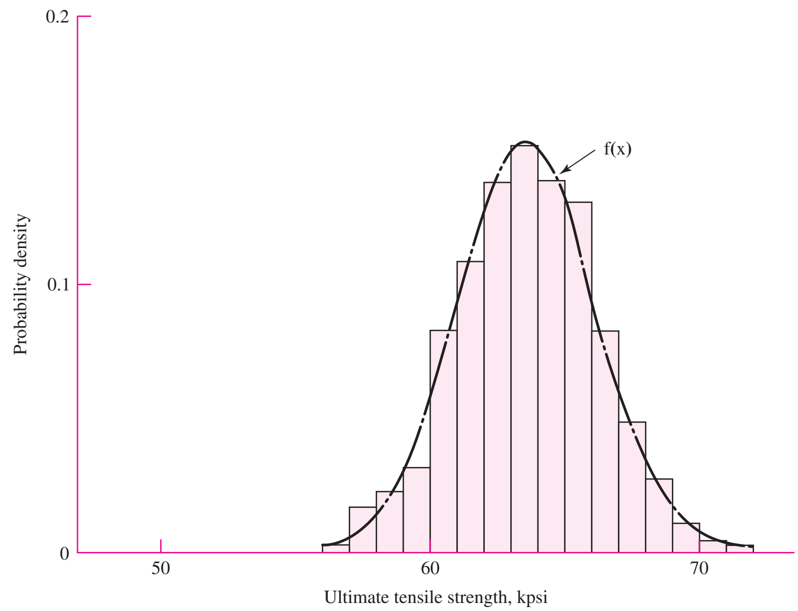
where the mean stress is 63.62 kpsi and the standard deviation is 2.594 kpsi. A plot of $f(x)$ is included in Fig. 2–5. The description of the strength S_{ut} is then expressed in terms of its statistical parameters and its distribution type. In this case $\mathbf{S}_{ut} = \mathbf{N}(63.62, 2.594)$ kpsi.

Note that the test program has described 1020 property $\mathbf{S}_{ut}$, for only one heat of one supplier. Testing is an involved and expensive process. Tables of properties are often prepared to be helpful to other persons. A statistical quantity is described by its mean, standard deviation, and distribution type. Many tables display a single number, which is often the mean, minimum, or some percentile, such as the 99th percentile. Always read the foonotes to the table. If no qualification is made in a single-entry table, the table is subject to serious doubt.

Since it is no surprise that useful descriptions of a property are statistical in nature, engineers, when ordering property tests, should couch the instructions so the data generated are enough for them to observe the statistical parameters and to identify the distributional characteristic. The tensile test program on 1000 specimens of 1020 steel is a large one. If you were faced with putting something in a table of ultimate tensile strengths and constrained to a single number, what would it be and just how would your footnote read?

Figure 2–5

Histogram for 1000 tensile tests on a 1020 steel from a single heat.



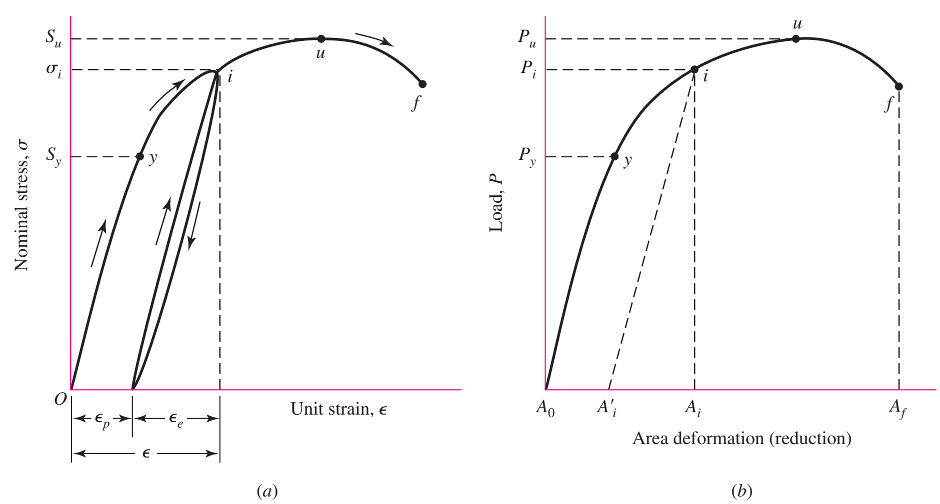
2–3 Strength and Cold Work

Cold working is the process of plastic straining below the recrystallization temperature in the plastic region of the stress-strain diagram. Materials can be deformed plastically by the application of heat, as in blacksmithing or hot rolling, but the resulting mechanical properties are quite different from those obtained by cold working. The purpose of this section is to explain what happens to the significant mechanical properties of a material when that material is cold-worked.

Consider the stress-strain diagram of Fig. 2–6*a*. Here a material has been stressed beyond the yield strength at y to some point i, in the plastic region, and then the load removed. At this point the material has a permanent plastic deformation ϵ_p. If the load corresponding to point i is now reapplied, the material will be elastically deformed by

Figure 2–6

(*a*) Stress-strain diagram showing unloading and reloading at point *i* in the plastic region; (*b*) analogous load-deformation diagram.



the amount ϵ_e. Thus at point i the total unit strain consists of the two components ϵ_p and ϵ_e and is given by the equation

$$\epsilon = \epsilon_p + \epsilon_e \tag{a}$$

This material can be unloaded and reloaded any number of times from and to point i, and it is found that the action always occurs along the straight line that is approximately parallel to the initial elastic line Oy. Thus

$$\epsilon_e = \frac{\sigma_i}{E} \tag{b}$$

The material now has a higher yield point, is less ductile as a result of a reduction in strain capacity, and is said to be *strain-hardened*. If the process is continued, increasing ϵ_p, the material can become brittle and exhibit sudden fracture.

It is possible to construct a similar diagram, as in Fig. 2–6*b*, where the abscissa is the area deformation and the ordinate is the applied load. The *reduction in area* corresponding to the load P_f, at fracture, is defined as

$$R = \frac{A_0 - A_f}{A_0} = 1 - \frac{A_f}{A_0} \tag{2–8}$$

where A_0 is the original area. The quantity R in Eq. (2–8) is usually expressed in percent and tabulated in lists of mechanical properties as a measure of *ductility*. See Appendix Table A–20, for example. Ductility is an important property because it measures the ability of a material to absorb overloads and to be cold-worked. Thus such operations as bending, drawing, heading, and stretch forming are metal-processing operations that require ductile materials.

Figure 2–6*b* can also be used to define the quantity of cold work. The *cold-work factor* W is defined as

$$W = \frac{A_0 - A_i'}{A_0} \approx \frac{A_0 - A_i}{A_0} \tag{2–9}$$

where A_i' corresponds to the area after the load P_i has been released. The approximation in Eq. (2–9) results because of the difficulty of measuring the small diametral changes in the elastic region. If the amount of cold work is known, then Eq. (2–9) can be solved for the area A_i'. The result is

$$A_i' = A_0(1 - W) \tag{2–10}$$

Cold working a material produces a new set of values for the strengths, as can be seen from stress-strain diagrams. Datsko[3] describes the plastic region of the true stress–true strain diagram by the equation

$$\sigma = \sigma_0 \varepsilon^m \tag{2–11}$$

[3]Joseph Datsko, "Solid Materials," Chap. 32 in Joseph E. Shigley, Charles R. Mischke, and Thomas H. Brown, Jr. (eds.), *Standard Handbook of Machine Design,* 3rd ed., McGraw-Hill, New York, 2004. See also Joseph Datsko, "New Look at Material Strength," *Machine Design,* vol. 58, no. 3, Feb. 6, 1986, pp. 81–85.

where σ = true stress

σ_0 = a strength coefficient, or strain-strengthening coefficient

ε = true plastic strain

m = strain-strengthening exponent

It can be shown[4] that

$$m = \varepsilon_u \tag{2–12}$$

provided that the load-deformation curve exhibits a stationary point (a place of zero slope).

Difficulties arise when using the gauge length to evaluate the true strain in the plastic range, since necking causes the strain to be nonuniform. A more satisfactory relation can be obtained by using the area at the neck. Assuming that the change in volume of the material is small, $Al = A_0 l_0$. Thus, $l/l_0 = A_0/A$, and the true strain is given by

$$\varepsilon = \ln \frac{l}{l_0} = \ln \frac{A_0}{A} \tag{2–13}$$

Returning to Fig. 2–6*b*, if point *i* is to the left of point *u*, that is, $P_i < P_u$, then the new yield strength is

$$S'_y = \frac{P_i}{A'_i} = \sigma_0 \varepsilon_i^m \qquad P_i \le P_u \tag{2–14}$$

Because of the reduced area, that is, because $A'_i < A_0$, the ultimate strength also changes, and is

$$S'_u = \frac{P_u}{A'_i} \tag{c}$$

Since $P_u = S_u A_0$, we find, with Eq. (2–10), that

$$S'_u = \frac{S_u A_0}{A_0(1 - W)} = \frac{S_u}{1 - W} \qquad \varepsilon_i \le \varepsilon_u \tag{2–15}$$

which is valid only when point *i* is to the left of point *u*.

For points to the right of *u*, the yield strength is approaching the ultimate strength, and, with small loss in accuracy,

$$S'_u \doteq S'_y \doteq \sigma_0 \varepsilon_i^m \qquad \varepsilon_i \le \varepsilon_u \tag{2–16}$$

A little thought will reveal that a bar will have the same ultimate load in tension after being strain-strengthened in tension as it had before. The new strength is of interest to us not because the static ultimate load increases, but—since fatigue strengths are correlated with the local ultimate strengths—because the fatigue strength improves. Also the yield strength increases, giving a larger range of sustainable *elastic* loading.

[4]See Sec. 5–2, J. E. Shigley and C. R. Mischke, *Mechanical Engineering Design,* 6th ed., McGraw-Hill, New York, 2001.

EXAMPLE 2–1 An annealed AISI 1018 steel (see Table A–22) has $S_y = 32.0$ kpsi, $S_u = 49.5$ kpsi, $\sigma_f = 91.1$ kpsi, $\sigma_0 = 90$ kpsi, $m = 0.25$, and $\varepsilon_f = 1.05$ in/in. Find the new values of the strengths if the material is given 15 percent cold work.

Solution From Eq. (2–12), we find the true strain corresponding to the ultimate strength to be

$$\varepsilon_u = m = 0.25$$

The ratio A_0/A_i is, from Eq. (2–9),

$$\frac{A_0}{A_i} = \frac{1}{1-W} = \frac{1}{1-0.15} = 1.176$$

The true strain corresponding to 15 percent cold work is obtained from Eq. (2–13). Thus

$$\varepsilon_i = \ln\frac{A_0}{A_i} = \ln 1.176 = 0.1625$$

Since $\varepsilon_i < \varepsilon_u$, Eqs. (2–14) and (2–15) apply. Therefore,

Answer

$$S'_y = \sigma_0 \varepsilon_i^m = 90(0.1625)^{0.25} = 57.1 \text{ kpsi}$$

Answer

$$S'_u = \frac{S_u}{1-W} = \frac{49.5}{1-0.15} = 58.2 \text{ kpsi}$$

2–4 Hardness

The resistance of a material to penetration by a pointed tool is called *hardness*. Though there are many hardness-measuring systems, we shall consider here only the two in greatest use.

Rockwell hardness tests are described by ASTM standard hardness method E–18 and measurements are quickly and easily made, they have good reproducibility, and the test machine for them is easy to use. In fact, the hardness number is read directly from a dial. Rockwell hardness scales are designated as $A, B, C, \ldots$, etc. The indenters are described as a diamond, a $\frac{1}{16}$-in-diameter ball, and a diamond for scales A, B, and C, respectively, where the load applied is either 60, 100, or 150 kg. Thus the Rockwell B scale, designated R_B, uses a 100-kg load and a No. 2 indenter, which is a $\frac{1}{16}$-in-diameter ball. The Rockwell C scale R_C uses a diamond cone, which is the No. 1 indenter, and a load of 150 kg. Hardness numbers so obtained are relative. Therefore a hardness $R_C = 50$ has meaning only in relation to another hardness number using the same scale.

The *Brinell hardness* is another test in very general use. In testing, the indenting tool through which force is applied is a ball and the hardness number H_B is found as a number equal to the applied load divided by the spherical surface area of the indentation. Thus the units of H_B are the same as those of stress, though they are seldom used. Brinell hardness testing takes more time, since H_B must be computed from the test data. The primary advantage of both methods is that they are nondestructive in most cases. Both are empirically and directly related to the ultimate strength of the material tested. This means that the strength of parts could, if desired, be tested part by part during manufacture.

For *steels,* the relationship between the minimum ultimate strength and the Brinell hardness number for $200 \le H_B \le 450$ is found to be

$$S_u = \begin{cases} 0.495 H_B & \text{kpsi} \\ 3.41 H_B & \text{MPa} \end{cases} \tag{2–17}$$

Similar relationships for *cast iron* can be derived from data supplied by Krause.[5] Data from 72 tests of gray iron produced by one foundry and poured in two sizes of test bars are reported in graph form. The minimum strength, as defined by the ASTM, is found from these data to be

$$S_u = \begin{cases} 0.23 H_B - 12.5 \text{ kpsi} \\ 1.58 H_B - 86 \text{ MPa} \end{cases} \tag{2–18}$$

Walton[6] shows a chart from which the SAE minimum strength can be obtained. The result is

$$S_u = 0.2375 H_B - 16 \text{ kpsi} \tag{2–19}$$

which is even more conservative than the values obtained from Eq. (2–18).

EXAMPLE 2–2

It is necessary to ensure that a certain part supplied by a foundry always meets or exceeds ASTM No. 20 specifications for cast iron (see Table A–24). What hardness should be specified?

Solution

From Eq. (2–18), with $(S_u)_{\min} = 20$ kpsi, we have

Answer

$$H_B = \frac{S_u + 12.5}{0.23} = \frac{20 + 12.5}{0.23} = 141$$

If the foundry can control the hardness within 20 points, routinely, then specify $145 < H_B < 165$. This imposes no hardship on the foundry and assures the designer that ASTM grade 20 will always be supplied at a predictable cost.

2–5 Impact Properties

An external force applied to a structure or part is called an *impact load* if the time of application is less than one-third the lowest natural period of vibration of the part or structure. Otherwise it is called simply a *static load*.

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[5]D. E. Krause, "Gray Iron—A Unique Engineering Material," ASTM Special Publication 455, 1969, pp. 3–29, as reported in Charles F. Walton (ed.), *Iron Castings Handbook,* Iron Founders Society, Inc., Cleveland, 1971, pp. 204, 205.

[6]Ibid.

The *Charpy* (commonly used) and *Izod* (rarely used) *notched-bar tests* utilize bars of specified geometries to determine brittleness and impact strength. These tests are helpful in comparing several materials and in the determination of low-temperature brittleness. In both tests the specimen is struck by a pendulum released from a fixed height, and the energy absorbed by the specimen, called the *impact value,* can be computed from the height of swing after fracture, but is read from a dial that essentially "computes" the result.

The effect of temperature on impact values is shown in Fig. 2–7 for a material showing a ductile-brittle transition. Not all materials show this transition. Notice the narrow region of critical temperatures where the impact value increases very rapidly. In the low-temperature region the fracture appears as a brittle, shattering type, whereas the appearance is a tough, tearing type above the critical-temperature region. The critical temperature seems to be dependent on both the material and the geometry of the notch. For this reason designers should not rely too heavily on the results of notched-bar tests.

The average strain rate used in obtaining the stress-strain diagram is about 0.001 in/(in · s) or less. When the strain rate is increased, as it is under impact conditions, the strengths increase, as shown in Fig. 2–8. In fact, at very high strain rates the yield strength seems to approach the ultimate strength as a limit. But note that the curves show little change in the elongation. This means that the ductility remains about the same. Also, in view of the sharp increase in yield strength, a mild steel could be expected to behave elastically throughout practically its entire strength range under impact conditions.

Figure 2–7

A mean trace shows the effect of temperature on impact values. The result of interest is the brittle-ductile transition temperature, often defined as the temperature at which the mean trace passes through the 15 ft · lbf level. The critical temperature *is* dependent on the geometry of the notch, which is why the Charpy V notch is closely defined.

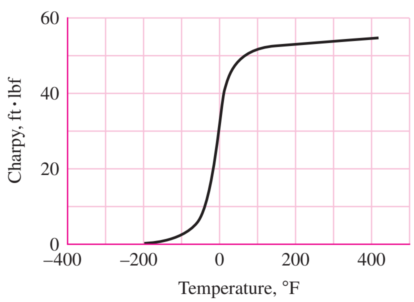
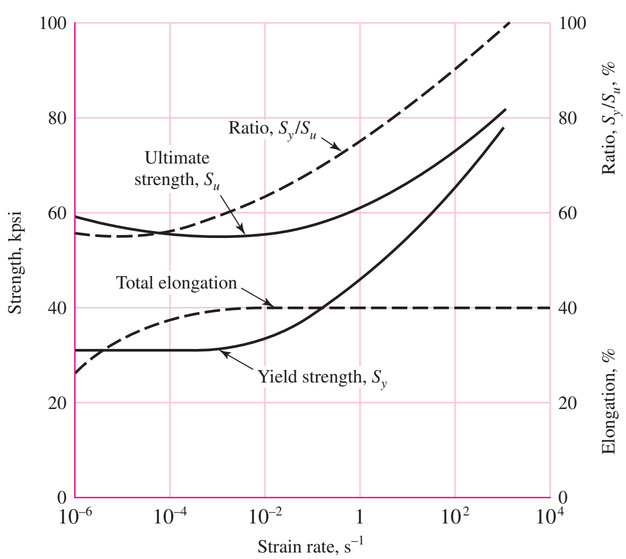


Figure 2–8

Influence of strain rate on tensile properties.



The Charpy and Izod tests really provide toughness data under dynamic, rather than static, conditions. It may well be that impact data obtained from these tests are as dependent on the notch geometry as they are on the strain rate. For these reasons it may be better to use the concepts of notch sensitivity, fracture toughness, and fracture mechanics, discussed in Chaps. 5 and 6, to assess the possibility of cracking or fracture.

2–6 Temperature Effects

Strength and ductility, or brittleness, are properties affected by the temperature of the operating environment.

The effect of temperature on the static properties of steels is typified by the strength versus temperature chart of Fig. 2–9. Note that the tensile strength changes only a small amount until a certain temperature is reached. At that point it falls off rapidly. The yield strength, however, decreases continuously as the environmental temperature is increased. There is a substantial increase in ductility, as might be expected, at the higher temperatures.

Many tests have been made of ferrous metals subjected to constant loads for long periods of time at elevated temperatures. The specimens were found to be permanently deformed during the tests, even though at times the actual stresses were less than the yield strength of the material obtained from short-time tests made at the same temperature. This continuous deformation under load is called *creep*.

One of the most useful tests to have been devised is the long-time creep test under constant load. Figure 2–10 illustrates a curve that is typical of this kind of test. The curve is obtained at a constant stated temperature. A number of tests are usually run simultaneously at different stress intensities. The curve exhibits three distinct regions. In the first stage are included both the elastic and the plastic deformation. This stage shows a decreasing creep rate, which is due to the strain hardening. The second stage shows a constant minimum creep rate caused by the annealing effect. In the third stage the specimen shows a considerable reduction in area, the true stress is increased, and a higher creep eventually leads to fracture.

When the operating temperatures are lower than the transition temperature (Fig. 2–7), the possibility arises that a part could fail by a brittle fracture. This subject will be discussed in Chap. 5.

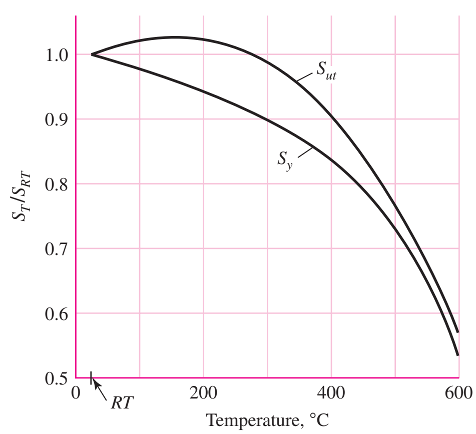


Figure 2–9

A plot of the results of 145 tests of 21 carbon and alloy steels showing the effect of operating temperature on the yield strength S_y and the ultimate strength S_{ut}. The ordinate is the ratio of the strength at the operating temperature to the strength at room temperature. The standard deviations were $0.0442 \le \hat{\sigma}_{Sy} \le 0.152$ for S_y and $0.099 \le \hat{\sigma}_{Sut} \le 0.11$ for S_{ut}. *(Data source: E. A. Brandes (ed.),* Smithells Metal Reference Book, 6th ed., *Butterworth, London, 1983 pp. 22–128 to 22–131.)*

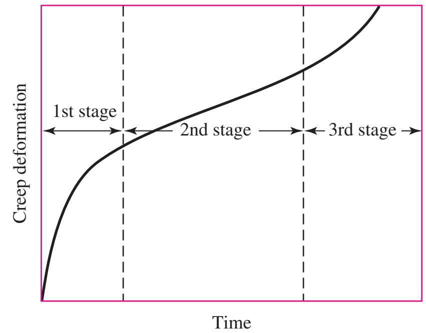


Figure 2–10
Creep-time curve.

Of course, heat treatment, as will be shown, is used to make substantial changes in the mechanical properties of a material.

Heating due to electric and gas welding also changes the mechanical properties. Such changes may be due to clamping during the welding process, as well as heating; the resulting stresses then remain when the parts have cooled and the clamps have been removed. Hardness tests can be used to learn whether the strength has been changed by welding, but such tests will not reveal the presence of residual stresses.

2–7 Numbering Systems

The Society of Automotive Engineers (SAE) was the first to recognize the need, and to adopt a system, for the numbering of steels. Later the American Iron and Steel Institute (AISI) adopted a similar system. In 1975 the SAE published the Unified Numbering System for Metals and Alloys (UNS); this system also contains cross-reference numbers for other material specifications.[7] The UNS uses a letter prefix to designate the material, as, for example, G for the carbon and alloy steels, A for the aluminum alloys, C for the copper-base alloys, and S for the stainless or corrosion-resistant steels. For some materials, not enough agreement has as yet developed in the industry to warrant the establishment of a designation.

For the steels, the first two numbers following the letter prefix indicate the composition, excluding the carbon content. The various compositions used are as follows:

G10	Plain carbon
G11	Free-cutting carbon steel with more sulfur or phosphorus
G13	Manganese
G23	Nickel
G25	Nickel
G31	Nickel-chromium
G33	Nickel-chromium
G40	Molybdenum
G41	Chromium-molybdenum
G43	Nickel-chromium-molybdenum
G46	Nickel-molybdenum
G48	Nickel-molybdenum
G50	Chromium
G51	Chromium
G52	Chromium
G61	Chromium-vanadium
G86	Chromium-nickel-molybdenum
G87	Chromium-nickel-molybdenum
G92	Manganese-silicon
G94	Nickel-chromium-molybdenum

[7]Many of the materials discussed in the balance of this chapter are listed in the Appendix tables. Be sure to review these.

Table 2–1

Aluminum Alloy Designations

Aluminum 99.00% pure and greater	Ax1xxx
Copper alloys	Ax2xxx
Manganese alloys	Ax3xxx
Silicon alloys	Ax4xxx
Magnesium alloys	Ax5xxx
Magnesium-silicon alloys	Ax6xxx
Zinc alloys	Ax7xxx

The second number pair refers to the approximate carbon content. Thus, G10400 is a plain carbon steel with a nominal carbon content of 0.40 percent (0.37 to 0.44 percent). The fifth number following the prefix is used for special situations. For example, the old designation AISI 52100 represents a chromium alloy with about 100 points of carbon. The UNS designation is G52986.

The UNS designations for the stainless steels, prefix S, utilize the older AISI designations for the first three numbers following the prefix. The next two numbers are reserved for special purposes. The first number of the group indicates the approximate composition. Thus 2 is a chromium-nickel-manganese steel, 3 is a chromium-nickel steel, and 4 is a chromium alloy steel. Sometimes stainless steels are referred to by their alloy content. Thus S30200 is often called an 18-8 stainless steel, meaning 18 percent chromium and 8 percent nickel.

The prefix for the aluminum group is the letter A. The first number following the prefix indicates the processing. For example, A9 is a wrought aluminum, while A0 is a casting alloy. The second number designates the main alloy group as shown in Table 2–1. The third number in the group is used to modify the original alloy or to designate the impurity limits. The last two numbers refer to other alloys used with the basic group.

The American Society for Testing and Materials (ASTM) numbering system for cast iron is in widespread use. This system is based on the tensile strength. Thus ASTM A18 speaks of classes; e.g., 30 cast iron has a minimum tensile strength of 30 kpsi. Note from Appendix A-24, however, that the *typical* tensile strength is 31 kpsi. You should be careful to designate which of the two values is used in design and problem work because of the significance of factor of safety.

2–8 Sand Casting

Sand casting is a basic low-cost process, and it lends itself to economical production in large quantities with practically no limit to the size, shape, or complexity of the part produced.

In sand casting, the casting is made by pouring molten metal into sand molds. A pattern, constructed of metal or wood, is used to form the cavity into which the molten metal is poured. Recesses or holes in the casting are produced by sand cores introduced into the mold. The designer should make an effort to visualize the pattern and casting in the mold. In this way the problems of core setting, pattern removal, draft, and solidification can be studied. Castings to be used as test bars of cast iron are cast separately and properties may vary.

Steel castings are the most difficult of all to produce, because steel has the highest melting temperature of all materials normally used for casting. This high temperature aggravates all casting problems.

The following rules will be found quite useful in the design of any sand casting:

1. All sections should be designed with a uniform thickness.
2. The casting should be designed so as to produce a gradual change from section to section where this is necessary.
3. Adjoining sections should be designed with generous fillets or radii.
4. A complicated part should be designed as two or more simple castings to be assembled by fasteners or by welding.

Steel, gray iron, brass, bronze, and aluminum are most often used in castings. The minimum wall thickness for any of these materials is about 5 mm, though with particular care, thinner sections can be obtained with some materials.

2–9 Shell Molding

The shell-molding process employs a heated metal pattern, usually made of cast iron, aluminum, or brass, which is placed in a shell-molding machine containing a mixture of dry sand and thermosetting resin. The hot pattern melts the plastic, which, together with the sand, forms a shell about 5 to 10 mm thick around the pattern. The shell is then baked at from 400 to 700°F for a short time while still on the pattern. It is then stripped from the pattern and placed in storage for use in casting.

In the next step the shells are assembled by clamping, bolting, or pasting; they are placed in a backup material, such as steel shot; and the molten metal is poured into the cavity. The thin shell permits the heat to be conducted away so that solidification takes place rapidly. As solidification takes place, the plastic bond is burned and the mold collapses. The permeability of the backup material allows the gases to escape and the casting to air-cool. All this aids in obtaining a fine-grain, stress-free casting.

Shell-mold castings feature a smooth surface, a draft that is quite small, and close tolerances. In general, the rules governing sand casting also apply to shell-mold casting.

2–10 Investment Casting

Investment casting uses a pattern that may be made from wax, plastic, or other material. After the mold is made, the pattern is melted out. Thus a mechanized method of casting a great many patterns is necessary. The mold material is dependent upon the melting point of the cast metal. Thus a plaster mold can be used for some materials while others would require a ceramic mold. After the pattern is melted out, the mold is baked or fired; when firing is completed, the molten metal may be poured into the hot mold and allowed to cool.

If a number of castings are to be made, then metal or permanent molds may be suitable. Such molds have the advantage that the surfaces are smooth, bright, and accurate, so that little, if any, machining is required. *Metal-mold castings* are also known as *die castings* and *centrifugal castings*.

2–11 Powder-Metallurgy Process

The powder-metallurgy process is a quantity-production process that uses powders from a single metal, several metals, or a mixture of metals and nonmetals. It consists essentially of mechanically mixing the powders, compacting them in dies at high pressures, and heating the compacted part at a temperature less than the melting point of the major ingredient. The particles are united into a single strong part similar to what would be obtained by melting the same ingredients together. The advantages are (1) the elimination of scrap or waste material, (2) the elimination of machining operations, (3) the low unit cost when mass-produced, and (4) the exact control of composition. Some of the disadvantages are (1) the high cost of dies, (2) the lower physical properties, (3) the higher cost of materials, (4) the limitations on the design, and (5) the limited range of materials that can be used. Parts commonly made by this process are oil-impregnated bearings, incandescent lamp filaments, cemented-carbide tips for tools, and permanent magnets. Some products can be made only by powder metallurgy: surgical implants, for example. The structure is different from what can be obtained by melting the same ingredients.

2–12 Hot-Working Processes

By *hot working* are meant such processes as rolling, forging, hot extrusion, and hot pressing, in which the metal is heated above its recrystallation temperature.

Hot rolling is usually used to create a bar of material of a particular shape and dimension. Figure 2–11 shows some of the various shapes that are commonly produced by the hot-rolling process. All of them are available in many different sizes as well as in different materials. The materials most available in the hot-rolled bar sizes are steel, aluminum, magnesium, and copper alloys.

Tubing can be manufactured by hot-rolling strip or plate. The edges of the strip are rolled together, creating seams that are either butt-welded or lap-welded. Seamless tubing is manufactured by roll-piercing a solid heated rod with a piercing mandrel.

Extrusion is the process by which great pressure is applied to a heated metal billet or blank, causing it to flow through a restricted orifice. This process is more common with materials of low melting point, such as aluminum, copper, magnesium, lead, tin, and zinc. Stainless steel extrusions are available on a more limited basis.

Forging is the hot working of metal by hammers, presses, or forging machines. In common with other hot-working processes, forging produces a refined grain structure that results in increased strength and ductility. Compared with castings, forgings have greater strength for the same weight. In addition, drop forgings can be made smoother and more accurate than sand castings, so that less machining is necessary. However, the initial cost of the forging dies is usually greater than the cost of patterns for castings, although the greater unit strength rather than the cost is usually the deciding factor between these two processes.

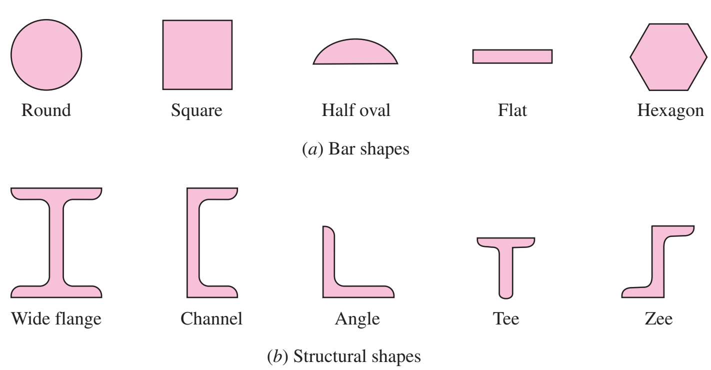


Figure 2–11

Common shapes available through hot rolling.

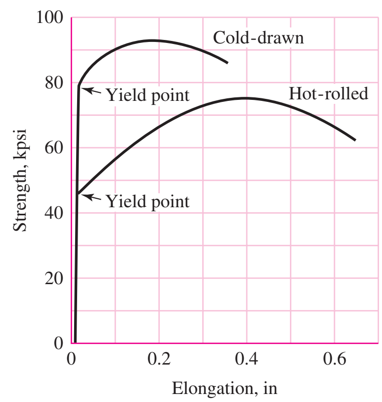


Figure 2–12
Stress-strain diagram for hot-rolled and cold-drawn UNS G10350 steel.

2–13 Cold-Working Processes

By *cold working* is meant the forming of the metal while at a low temperature (usually room temperature). In contrast to parts produced by hot working, cold-worked parts have a bright new finish, are more accurate, and require less machining.

Cold-finished bars and shafts are produced by rolling, drawing, turning, grinding, and polishing. Of these methods, by far the largest percentage of products are made by the cold-rolling and cold-drawing processes. Cold rolling is now used mostly for the production of wide flats and sheets. Practically all cold-finished bars are made by cold drawing but even so are sometimes mistakenly called "cold-rolled bars." In the drawing process, the hot-rolled bars are first cleaned of scale and then drawn by pulling them through a die that reduces the size about $\frac{1}{32}$ to $\frac{1}{16}$ in. This process does not remove material from the bar but reduces, or "draws" down, the size. Many different shapes of hot-rolled bars may be used for cold drawing.

Cold rolling and cold drawing have the same effect upon the mechanical properties. The cold-working process does not change the grain size but merely distorts it. Cold working results in a large increase in yield strength, an increase in ultimate strength and hardness, and a decrease in ductility. In Fig. 2–12 the properties of a cold-drawn bar are compared with those of a hot-rolled bar of the same material.

Heading is a cold-working process in which the metal is gathered, or upset. This operation is commonly used to make screw and rivet heads and is capable of producing a wide variety of shapes. *Roll threading* is the process of rolling threads by squeezing and rolling a blank between two serrated dies. *Spinning* is the operation of working sheet material around a rotating form into a circular shape. *Stamping* is the term used to describe punch-press operations such as *blanking, coining, forming,* and *shallow drawing*.

2–14 The Heat Treatment of Steel

Heat treatment of steel refers to time- and temperature-controlled processes that relieve residual stresses and/or modifies material properties such as hardness (strength), ductility, and toughness. Other mechanical or chemical operations are sometimes grouped under the heading of heat treatment. The common heat-treating operations are annealing, quenching, tempering, and case hardening.

Annealing

When a material is cold- or hot-worked, residual stresses are built in, and, in addition, the material usually has a higher hardness as a result of these working operations. These operations change the structure of the material so that it is no longer represented by the equilibrium diagram. Full annealing and normalizing is a heating operation that permits the material to transform according to the equilibrium diagram. The material to be annealed is heated to a temperature that is approximately 100°F above the critical temperature. It is held at this temperature for a time that is sufficient for the carbon to become dissolved and diffused through the material. The object being treated is then allowed to cool slowly, usually in the furnace in which it was treated. If the transformation is complete, then it is said to have a full anneal. Annealing is used to soften a material and make it more ductile, to relieve residual stresses, and to refine the grain structure.

The term *annealing* includes the process called *normalizing*. Parts to be normalized may be heated to a slightly higher temperature than in full annealing. This produces a coarser grain structure, which is more easily machined if the material is a low-carbon steel. In the normalizing process the part is cooled in still air at room temperature. Since this cooling is more rapid than the slow cooling used in full annealing, less time is available for equilibrium, and the material is harder than fully annealed steel. Normalizing is often used as the final treating operation for steel. The cooling in still air amounts to a slow quench.

Quenching

Eutectoid steel that is fully annealed consists entirely of pearlite, which is obtained from austenite under conditions of equilibrium. A fully annealed hypoeutectoid steel would consist of pearlite plus ferrite, while hypereutectoid steel in the fully annealed condition would consist of pearlite plus cementite. The hardness of steel of a given carbon content depends upon the structure that replaces the pearlite when full annealing is not carried out.

The absence of full annealing indicates a more rapid rate of cooling. The rate of cooling is the factor that determines the hardness. A controlled cooling rate is called *quenching*. A mild quench is obtained by cooling in still air, which, as we have seen, is obtained by the normalizing process. The two most widely used media for quenching are water and oil. The oil quench is quite slow but prevents quenching cracks caused by rapid expansion of the object being treated. Quenching in water is used for carbon steels and for medium-carbon, low-alloy steels.

The effectiveness of quenching depends upon the fact that when austenite is cooled it does not transform into pearlite instantaneously but requires time to initiate and complete the process. Since the transformation ceases at about 800°F, it can be prevented by rapidly cooling the material to a lower temperature. When the material is cooled rapidly to 400°F or less, the austenite is transformed into a structure called *martensite*. Martensite is a supersaturated solid solution of carbon in ferrite and is the hardest and strongest form of steel.

If steel is rapidly cooled to a temperature between 400 and 800°F and held there for a sufficient length of time, the austenite is transformed into a material that is generally called *bainite*. Bainite is a structure intermediate between pearlite and martensite. Although there are several structures that can be identified between the temperatures given, depending upon the temperature used, they are collectively known as bainite. By the choice of this transformation temperature, almost any variation of structure may be obtained. These range all the way from coarse pearlite to fine martensite.

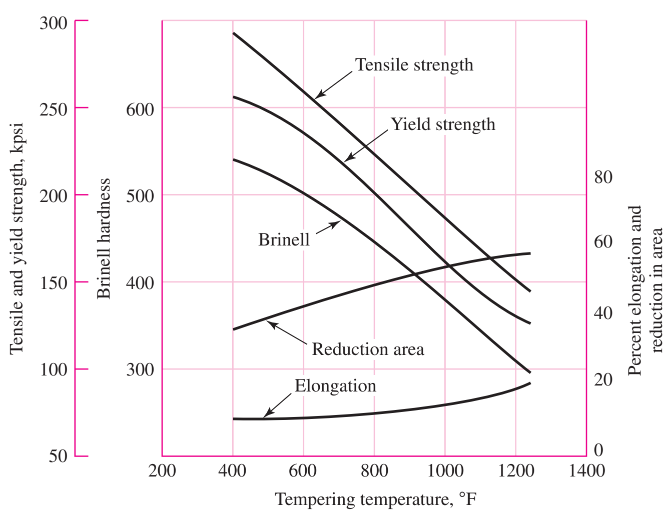
Tempering

When a steel specimen has been fully hardened, it is very hard and brittle and has high residual stresses. The steel is unstable and tends to contract on aging. This tendency is increased when the specimen is subjected to externally applied loads, because the resultant stresses contribute still more to the instability. These internal stresses can be relieved by a modest heating process called *stress relieving,* or a combination of stress relieving and softening called *tempering* or *drawing*. After the specimen has been fully hardened by being quenched from above the critical temperature, it is reheated to some temperature below the critical temperature for a certain period of time and then allowed to cool in still air. The temperature to which it is reheated depends upon the composition and the degree of hardness or toughness desired.[8] This reheating operation releases the carbon held in the martensite, forming carbide crystals. The structure obtained is called *tempered martensite*. It is now essentially a superfine dispersion of iron carbide(s) in fine-grained ferrite.

The effect of heat-treating operations upon the various mechanical properties of a low alloy steel is shown graphically in Fig. 2–13.

Figure 2–13

The effect of thermal-mechanical history on the mechanical properties of AISI 4340 steel. *(Prepared by the International Nickel Company.)*



Condition	Tensile strength, kpsi	Yield strength, kpsi	Reduction in area, %	Elongation in 2 in, %	Brinell hardness, Bhn
Normalized	200	147	20	10	410
As rolled	190	144	18	9	380
Annealed	120	99	43	18	228

[8]For the quantitative aspects of tempering in plain carbon and low-alloy steels, see Charles R. Mischke, "The Strength of Cold-Worked and Heat-Treated Steels," Chap. 33 in Joseph E. Shigley, Charles R. Mischke, and Thomas H. Brown, Jr. (eds.), *Standard Handbook of Machine Design,* 3rd ed., McGraw-Hill, New York, 2004.

Case Hardening

The purpose of case hardening is to produce a hard outer surface on a specimen of low-carbon steel while at the same time retaining the ductility and toughness in the core. This is done by increasing the carbon content at the surface. Either solid, liquid, or gaseous carburizing materials may be used. The process consists of introducing the part to be carburized into the carburizing material for a stated time and at a stated temperature, depending upon the depth of case desired and the composition of the part. The part may then be quenched directly from the carburization temperature and tempered, or in some cases it must undergo a double heat treatment in order to ensure that both the core and the case are in proper condition. Some of the more useful case-hardening processes are pack carburizing, gas carburizing, nitriding, cyaniding, induction hardening, and flame hardening. In the last two cases carbon is not added to the steel in question, generally a medium carbon steel, for example SAE/AISI 1144.

Quantitative Estimation of Properties of Heat-Treated Steels

Courses in metallurgy (or material science) for mechanical engineers usually present the addition method of Crafts and Lamont for the prediction of heat-treated properties from the Jominy test for plain carbon steels.[9] If this has not been in your prerequisite experience, then refer to the *Standard Handbook of Machine Design,* where the addition method is covered with examples.[10] If this book is a textbook for a machine elements course, it is a good class project (many hands make light work) to study the method and report to the class.

For low-alloy steels, the multiplication method of Grossman[11] and Field[12] is explained in the *Standard Handbook of Machine Design* (Secs. 29.6 and 33.6).

Modern Steels and Their Properties Handbook explains how to predict the Jominy curve by the method of Grossman and Field from a ladle analysis and grain size.[13] Bethlehem Steel has developed a circular plastic slide rule that is convenient to the purpose.

2–15 Alloy Steels

Although a plain carbon steel is an alloy of iron and carbon with small amounts of manganese, silicon, sulfur, and phosphorus, the term *alloy steel* is applied when one or more elements other than carbon are introduced in sufficient quantities to modify its properties substantially. The alloy steels not only possess more desirable physical properties but also permit a greater latitude in the heat-treating process.

Chromium

The addition of chromium results in the formation of various carbides of chromium that are very hard, yet the resulting steel is more ductile than a steel of the same hardness produced by a simple increase in carbon content. Chromium also refines the grain structure so that these two combined effects result in both increased toughness and increased hardness. The addition of chromium increases the critical range of temperatures and moves the eutectoid point to the left. Chromium is thus a very useful alloying element.

[9] W. Crafts and J. L. Lamont, Hardenability and Steel Selection, Pitman and Sons, London, 1949.

[10] Charles R. Mischke, Chap. 33 in Joseph E. Shigley, Charles R. Mischke, and Thomas H. Brown, Jr. (eds.), *Standard Handbook of Machine Design,* 3rd ed., McGraw-Hill, New York, 2004, p. 33.9.

[11] M. A. Grossman, *AIME,* February 1942.

[12] J. Field, *Metals Progress,* March 1943.

[13] *Modern Steels and Their Properties,* 7th ed., Handbook 2757, Bethlehem Steel, 1972, pp. 46–50.

Nickel

The addition of nickel to steel also causes the eutectoid point to move to the left and increases the critical range of temperatures. Nickel is soluble in ferrite and does not form carbides or oxides. This increases the strength without decreasing the ductility. Case hardening of nickel steels results in a better core than can be obtained with plain carbon steels. Chromium is frequently used in combination with nickel to obtain the toughness and ductility provided by the nickel and the wear resistance and hardness contributed by the chromium.

Manganese

Manganese is added to all steels as a deoxidizing and desulfurizing agent, but if the sulfur content is low and the manganese content is over 1 percent, the steel is classified as a manganese alloy. Manganese dissolves in the ferrite and also forms carbides. It causes the eutectoid point to move to the left and lowers the critical range of temperatures. It increases the time required for transformation so that oil quenching becomes practicable.

Silicon

Silicon is added to all steels as a deoxidizing agent. When added to very-low-carbon steels, it produces a brittle material with a low hysteresis loss and a high magnetic permeability. The principal use of silicon is with other alloying elements, such as manganese, chromium, and vanadium, to stabilize the carbides.

Molybdenum

While molybdenum is used alone in a few steels, it finds its greatest use when combined with other alloying elements, such as nickel, chromium, or both. Molybdenum forms carbides and also dissolves in ferrite to some extent, so that it adds both hardness and toughness. Molybdenum increases the critical range of temperatures and substantially lowers the transformation point. Because of this lowering of the transformation point, molybdenum is most effective in producing desirable oil-hardening and air-hardening properties. Except for carbon, it has the greatest hardening effect, and because it also contributes to a fine grain size, this results in the retention of a great deal of toughness.

Vanadium

Vanadium has a very strong tendency to form carbides; hence it is used only in small amounts. It is a strong deoxidizing agent and promotes a fine grain size. Since some vanadium is dissolved in the ferrite, it also toughens the steel. Vanadium gives a wide hardening range to steel, and the alloy can be hardened from a higher temperature. It is very difficult to soften vanadium steel by tempering; hence, it is widely used in tool steels.

Tungsten

Tungsten is widely used in tool steels because the tool will maintain its hardness even at red heat. Tungsten produces a fine, dense structure and adds both toughness and hardness. Its effect is similar to that of molybdenum, except that it must be added in greater quantities.

2–16 Corrosion-Resistant Steels

Iron-base alloys containing at least 12 percent chromium are called *stainless steels*. The most important characteristic of these steels is their resistance to many, but not all, corrosive conditions. The four types available are the ferritic chromium steels, the austenitic chromium-nickel steels, and the martensitic and precipitation-hardenable stainless steels.

The ferritic chromium steels have a chromium content ranging from 12 to 27 percent. Their corrosion resistance is a function of the chromium content, so that alloys containing less than 12 percent still exhibit some corrosion resistance, although they may rust. The quench-hardenability of these steels is a function of both the chromium and the carbon content. The very high carbon steels have good quench hardenability up to about 18 percent chromium, while in the lower carbon ranges it ceases at about 13 percent. If a little nickel is added, these steels retain some degree of hardenability up to 20 percent chromium. If the chromium content exceeds 18 percent, they become difficult to weld, and at the very high chromium levels the hardness becomes so great that very careful attention must be paid to the service conditions. Since chromium is expensive, the designer will choose the lowest chromium content consistent with the corrosive conditions.

The chromium-nickel stainless steels retain the austenitic structure at room temperature; hence, they are not amenable to heat treatment. The strength of these steels can be greatly improved by cold working. They are not magnetic unless cold-worked. Their work hardenability properties also cause them to be difficult to machine. All the chromium-nickel steels may be welded. They have greater corrosion-resistant properties than the plain chromium steels. When more chromium is added for greater corrosion resistance, more nickel must also be added if the austenitic properties are to be retained.

2–17 Casting Materials

Gray Cast Iron

Of all the cast materials, gray cast iron is the most widely used. This is because it has a very low cost, is easily cast in large quantities, and is easy to machine. The principal objections to the use of gray cast iron are that it is brittle and that it is weak in tension. In addition to a high carbon content (over 1.7 percent and usually greater than 2 percent), cast iron also has a high silicon content, with low percentages of sulfur, manganese, and phosphorus. The resultant alloy is composed of pearlite, ferrite, and graphite, and under certain conditions the pearlite may decompose into graphite and ferrite. The resulting product then contains all ferrite and graphite. The graphite, in the form of thin flakes distributed evenly throughout the structure, darkens it; hence, the name *gray cast iron*.

Gray cast iron is not readily welded, because it may crack, but this tendency may be reduced if the part is carefully preheated. Although the castings are generally used in the as-cast condition, a mild anneal reduces cooling stresses and improves the machinability. The tensile strength of gray cast iron varies from 100 to 400 MPa (15 to 60 kpsi), and the compressive strengths are 3 to 4 times the tensile strengths. The modulus of elasticity varies widely, with values extending all the way from 75 to 150 GPa (11 to 22 Mpsi).

Ductile and Nodular Cast Iron

Because of the lengthy heat treatment required to produce malleable cast iron, engineers have long desired a cast iron that would combine the ductile properties of malleable iron with the ease of casting and machining of gray iron and at the same time would possess these properties in the as-cast conditions. A process for producing such a material using magnesium-containing material seems to fulfill these requirements.

Ductile cast iron, or *nodular cast iron,* as it is sometimes called, is essentially the same as malleable cast iron, because both contain graphite in the form of spheroids. However, ductile cast iron in the as-cast condition exhibits properties very close to those of malleable iron, and if a simple 1-h anneal is given and is followed by a slow cool, it exhibits even more ductility than the malleable product. Ductile iron is made by adding MgFeSi to the melt; since magnesium boils at this temperature, it is necessary to alloy it with other elements before it is introduced.

Ductile iron has a high modulus of elasticity (172 GPa or 25 Mpsi) as compared with gray cast iron, and it is elastic in the sense that a portion of the stress-strain curve is a straight line. Gray cast iron, on the other hand, does not obey Hooke's law, because the modulus of elasticity steadily decreases with increase in stress. Like gray cast iron, however, nodular iron has a compressive strength that is higher than the tensile strength, although the difference is not as great. In 40 years it has become extensively used.

White Cast Iron

If all the carbon in cast iron is in the form of cementite and pearlite, with no graphite present, the resulting structure is white and is known as *white cast iron*. This may be produced in two ways. The composition may be adjusted by keeping the carbon and silicon content low, or the gray-cast-iron composition may be cast against chills in order to promote rapid cooling. By either method, a casting with large amounts of cementite is produced, and as a result the product is very brittle and hard to machine but also very resistant to wear. A chill is usually used in the production of gray-iron castings in order to provide a very hard surface within a particular area of the casting, while at the same time retaining the more desirable gray structure within the remaining portion. This produces a relatively tough casting with a wear-resistant area.

Malleable Cast Iron

If white cast iron within a certain composition range is annealed, a product called *malleable cast iron* is formed. The annealing process frees the carbon so that it is present as graphite, just as in gray cast iron but in a different form. In gray cast iron the graphite is present in a thin flake form, while in malleable cast iron it has a nodular form and is known as *temper carbon*. A good grade of malleable cast iron may have a tensile strength of over 350 MPa (50 kpsi), with an elongation of as much as 18 percent. The percentage elongation of a gray cast iron, on the other hand, is seldom over 1 percent. Because of the time required for annealing (up to 6 days for large and heavy castings), malleable iron is necessarily somewhat more expensive than gray cast iron.

Alloy Cast Irons

Nickel, chromium, and molybdenum are the most common alloying elements used in cast iron. Nickel is a general-purpose alloying element, usually added in amounts up to 5 percent. Nickel increases the strength and density, improves the wearing qualities, and raises the machinability. If the nickel content is raised to 10 to 18 percent, an austenitic structure with valuable heat- and corrosion-resistant properties results. Chromium increases the hardness and wear resistance and, when used with a chill, increases the tendency to form white iron. When chromium and nickel are both added, the hardness and strength are improved without a reduction in the machinability rating. Molybdenum added in quantities up to 1.25 percent increases the stiffness, hardness, tensile strength, and impact resistance. It is a widely used alloying element.

Cast Steels

The advantage of the casting process is that parts having complex shapes can be manufactured at costs less than fabrication by other means, such as welding. Thus the choice of steel castings is logical when the part is complex and when it must also have a high strength. The higher melting temperatures for steels do aggravate the casting problems and require closer attention to such details as core design, section thicknesses, fillets, and the progress of cooling. The same alloying elements used for the wrought steels can be used for cast steels to improve the strength and other mechanical properties. Cast-steel parts can also be heat-treated to alter the mechanical properties, and, unlike the cast irons, they can be welded.

2–18 Nonferrous Metals

Aluminum

The outstanding characteristics of aluminum and its alloys are their strength-weight ratio, their resistance to corrosion, and their high thermal and electrical conductivity. The density of aluminum is about 2770 kg/m^3 (0.10 lbf/in^3), compared with 7750 kg/m^3 (0.28 lbf/in^3) for steel. Pure aluminum has a tensile strength of about 90 MPa (13 kpsi), but this can be improved considerably by cold working and also by alloying with other materials. The modulus of elasticity of aluminum, as well as of its alloys, is 71.7 GPa (10.4 Mpsi), which means that it has about one-third the stiffness of steel.

Considering the cost and strength of aluminum and its alloys, they are among the most versatile materials from the standpoint of fabrication. Aluminum can be processed by sand casting, die casting, hot or cold working, or extruding. Its alloys can be machined, press-worked, soldered, brazed, or welded. Pure aluminum melts at 660°C (1215°F), which makes it very desirable for the production of either permanent or sand-mold castings. It is commercially available in the form of plate, bar, sheet, foil, rod, and tube and in structural and extruded shapes. Certain precautions must be taken in joining aluminum by soldering, brazing, or welding; these joining methods are not recommended for all alloys.

The corrosion resistance of the aluminum alloys depends upon the formation of a thin oxide coating. This film forms spontaneously because aluminum is inherently very reactive. Constant erosion or abrasion removes this film and allows corrosion to take place. An extra-heavy oxide film may be produced by the process called *anodizing*. In this process the specimen is made to become the anode in an electrolyte, which may be chromic acid, oxalic acid, or sulfuric acid. It is possible in this process to control the color of the resulting film very accurately.

The most useful alloying elements for aluminum are copper, silicon, manganese, magnesium, and zinc. Aluminum alloys are classified as *casting alloys* or *wrought alloys*. The casting alloys have greater percentages of alloying elements to facilitate casting, but this makes cold working difficult. Many of the casting alloys, and some of the wrought alloys, cannot be hardened by heat treatment. The alloys that are heat-treatable use an alloying element that dissolves in the aluminum. The heat treatment consists of heating the specimen to a temperature that permits the alloying element to pass into solution, then quenching so rapidly that the alloying element is not precipitated. The aging process may be accelerated by heating slightly, which results in even greater hardness and strength. One of the better-known heat-treatable alloys is duraluminum, or 2017 (4 percent Cu, 0.5 percent Mg, 0.5 percent Mn). This alloy hardens in 4 days at room temperature. Because of this rapid aging, the alloy must be stored under refrigeration after quenching and before forming, or it must be formed immediately after quenching. Other alloys (such as 5053) have been developed that age-harden much more slowly, so that only mild refrigeration is required before forming. After forming, they are artificially aged in a furnace and possess approximately the same strength and hardness as the 2024 alloys. Those alloys of aluminum that cannot be heat-treated can be hardened only by cold working. Both work hardening and the hardening produced by heat treatment may be removed by an annealing process.

Magnesium

The density of magnesium is about 1800 kg/m^3 (0.065 lb/in^3), which is two-thirds that of aluminum and one-fourth that of steel. Since it is the lightest of all commercial metals, its greatest use is in the aircraft and automotive industries, but other uses are now being found for it. Although the magnesium alloys do not have great strength, because of their light weight the strength-weight ratio compares favorably with the stronger aluminum and steel alloys. Even so, magnesium alloys find their greatest use in applications where strength is not an important consideration. Magnesium will not withstand elevated temperatures; the yield point is definitely reduced when the temperature is raised to that of boiling water.

Magnesium and its alloys have a modulus of elasticity of 45 GPa (6.5 Mpsi) in tension and in compression, although some alloys are not as strong in compression as in tension. Curiously enough, cold working reduces the modulus of elasticity. A range of cast magnesium alloys are also available.

Titanium

Titanium and its alloys are similar in strength to moderate-strength steel but weigh half as much as steel. The material exhibits very good resistence to corrosion, has low thermal conductivity, is nonmagnetic, and has high-temperature strength. Its modulus of elasticity is between those of steel and aluminum at 16.5 Mpsi (114 GPa). Because of its many advantages over steel and aluminum, applications include: aerospace and military aircraft structures and components, marine hardware, chemical tanks and processing equipment, fluid handling systems, and human internal replacement devices. The disadvantages of titanium are its high cost compared to steel and aluminum and the difficulty of machining it.

Copper-Base Alloys

When copper is alloyed with zinc, it is usually called *brass*. If it is alloyed with another element, it is often called *bronze*. Sometimes the other element is specified too, as, for example, *tin bronze* or *phosphor bronze*. There are hundreds of variations in each category.

Brass with 5 to 15 Percent Zinc

The low-zinc brasses are easy to cold work, especially those with the higher zinc content. They are ductile but often hard to machine. The corrosion resistance is good. Alloys included in this group are *gilding brass* (5 percent Zn), *commercial bronze* (10 percent Zn), and *red brass* (15 percent Zn). Gilding brass is used mostly for jewelry and articles to be gold-plated; it has the same ductility as copper but greater strength, accompanied by poor machining characteristics. Commercial bronze is used for jewelry and for forgings and stampings, because of its ductility. Its machining properties are poor, but it has excellent cold-working properties. Red brass has good corrosion resistance as well as high-temperature strength. Because of this it is used a great deal in the form of tubing or piping to carry hot water in such applications as radiators or condensers.

Brass with 20 to 36 Percent Zinc

Included in the intermediate-zinc group are *low brass* (20 percent Zn), *cartridge brass* (30 percent Zn), and *yellow brass* (35 percent Zn). Since zinc is cheaper than copper, these alloys cost less than those with more copper and less zinc. They also have better machinability and slightly greater strength; this is offset, however, by poor corrosion resistance and the possibility of cracking at points of residual stresses. Low brass is very similar to red brass and is used for articles requiring deep-drawing operations. Of the copper-zinc alloys, cartridge brass has the best combination of ductility and strength. Cartridge cases were originally manufactured entirely by cold working; the process consisted of a series of deep draws, each draw being followed by an anneal to place the material in condition for the next draw, hence the name cartridge brass. Although the hot-working ability of yellow brass is poor, it can be used in practically any other fabricating process and is therefore employed in a large variety of products.

When small amounts of lead are added to the brasses, their machinability is greatly improved and there is some improvement in their abilities to be hot-worked. The addition of lead impairs both the cold-working and welding properties. In this group are *low-leaded brass* ($32\frac{1}{2}$ percent Zn, $\frac{1}{2}$ percent Pb), *high-leaded brass* (34 percent Zn, 2 percent Pb), and *free-cutting brass* ($35\frac{1}{2}$ percent Zn, 3 percent Pb). The low-leaded brass is not only easy to machine but has good cold-working properties. It is used for various screw-machine parts. High-leaded brass, sometimes called *engraver's brass,* is used for instrument, lock, and watch parts. Free-cutting brass is also used for screw-machine parts and has good corrosion resistance with excellent mechanical properties.

Admiralty metal (28 percent Zn) contains 1 percent tin, which imparts excellent corrosion resistance, especially to saltwater. It has good strength and ductility but only fair machining and working characteristics. Because of its corrosion resistance it is used in power-plant and chemical equipment. *Aluminum brass* (22 percent Zn) contains 2 percent aluminum and is used for the same purposes as admiralty metal, because it has nearly the same properties and characteristics. In the form of tubing or piping, it is favored over admiralty metal, because it has better resistance to erosion caused by high-velocity water.

Brass with 36 to 40 Percent Zinc

Brasses with more than 38 percent zinc are less ductile than cartridge brass and cannot be cold-worked as severely. They are frequently hot-worked and extruded. *Muntz metal* (40 percent Zn) is low in cost and mildly corrosion-resistant. *Naval brass* has the same composition as Muntz metal except for the addition of 0.75 percent tin, which contributes to the corrosion resistance.

Bronze

Silicon bronze, containing 3 percent silicon and 1 percent manganese in addition to the copper, has mechanical properties equal to those of mild steel, as well as good corrosion resistance. It can be hot- or cold-worked, machined, or welded. It is useful wherever corrosion resistance combined with strength is required.

Phosphor bronze, made with up to 11 percent tin and containing small amounts of phosphorus, is especially resistant to fatigue and corrosion. It has a high tensile strength and a high capacity to absorb energy, and it is also resistant to wear. These properties make it very useful as a spring material.

Aluminum bronze is a heat-treatable alloy containing up to 12 percent aluminum. This alloy has strength and corrosion-resistance properties that are better than those of brass, and in addition, its properties may be varied over a wide range by cold working, heat treating, or changing the composition. When iron is added in amounts up to 4 percent, the alloy has a high endurance limit, a high shock resistance, and excellent wear resistance.

Beryllium bronze is another heat-treatable alloy, containing about 2 percent beryllium. This alloy is very corrosion resistant and has high strength, hardness, and resistance to wear. Although it is expensive, it is used for springs and other parts subjected to fatigue loading where corrosion resistance is required.

With slight modification most copper-based alloys are available in cast form.

2–19 Plastics

The term *thermoplastics* is used to mean any plastic that flows or is moldable when heat is applied to it; the term is sometimes applied to plastics moldable under pressure. Such plastics can be remolded when heated.

A *thermoset* is a plastic for which the polymerization process is finished in a hot molding press where the plastic is liquefied under pressure. Thermoset plastics cannot be remolded.

Table 2–2 lists some of the most widely used thermoplastics, together with some of their characteristics and the range of their properties. Table 2–3, listing some of the

Table 2–2

The Thermoplastics *Source:* These data have been obtained from the *Machine Design Materials Reference Issue,* published by Penton/IPC, Cleveland. These reference issues are published about every 2 years and constitute an excellent source of data on a great variety of materials.

Name	S_u, kpsi	E, Mpsi	Hardness Rockwell	Elongation %	Dimensional Stability	Heat Resistance	Chemical Resistance	Processing
ABS group	2–8	0.10–0.37	60–110R	3–50	Good	*	Fair	EMST
Acetal group	8–10	0.41–0.52	80–94M	40–60	Excellent	Good	High	M
Acrylic	5–10	0.20–0.47	92–110M	3–75	High	*	Fair	EMS
Fluoroplastic group	0.50–7	. . .	50–80D	100–300	High	Excellent	Excellent	MPR†
Nylon	8–14	0.18–0.45	112–120R	10–200	Poor	Poor	Good	CEM
Phenylene oxide	7–18	0.35–0.92	115R, 106L	5–60	Excellent	Good	Fair	EFM
Polycarbonate	8–16	0.34–0.86	62–91M	10–125	Excellent	Excellent	Fair	EMS
Polyester	8–18	0.28–1.6	65–90M	1–300	Excellent	Poor	Excellent	CLMR
Polyimide	6–50	. . .	88–120M	Very low	Excellent	Excellent	Excellent†	CLMP
Polyphenylene sulfide	14–19	0.11	122R	1.0	Good	Excellent	Excellent	M
Polystyrene group	1.5–12	0.14–0.60	10–90M	0.5–60	. . .	Poor	Poor	EM
Polysulfone	10	0.36	120R	50–100	Excellent	Excellent	Excellent†	EFM
Polyvinyl chloride	1.5–7.5	0.35–0.60	65–85D	40–450	. . .	Poor	Poor	EFM

*Heat-resistant grades available.

†With exceptions.

C Coatings L Laminates R Resins E Extrusions M Moldings S Sheet F Foams P Press and sinter methods T Tubing

Table 2–3

The Thermosets *Source:* These data have been obtained from the *Machine Design Materials Reference Issue,* published by Penton/IPC, Cleveland. These reference issues are published about every 2 years and constitute an excellent source of data on a great variety of materials.

Name	S_u, kpsi	E, Mpsi	Hardness Rockwell	Elongation %	Dimensional Stability	Heat Resistance	Chemical Resistance	Processing
Alkyd	3–9	0.05–0.30	99M*	. . .	Excellent	Good	Fair	M
Allylic	4–10	. . .	105–120M	. . .	Excellent	Excellent	Excellent	CM
Amino group	5–8	0.13–0.24	110–120M	0.30–0.90	Good	Excellent*	Excellent*	LR
Epoxy	5–20	0.03–0.30*	80–120M	1–10	Excellent	Excellent	Excellent	CMR
Phenolics	5–9	0.10–0.25	70–95E	. . .	Excellent	Excellent	Good	EMR
Silicones	5–6	. . .	80–90M	. . .	. . .	Excellent	Excellent	CLMR

*With exceptions.

C Coatings L Laminates R Resins E Extrusions M Moldings S Sheet F Foams P Press and sinter methods T Tubing

thermosets, is similar. These tables are presented for information only and should not be used to make a final design decision. The range of properties and characteristics that can be obtained with plastics is very great. The influence of many factors, such as cost, moldability, coefficient of friction, weathering, impact strength, and the effect of fillers and reinforcements, must be considered. Manufacturers' catalogs will be found quite helpful in making possible selections.

2–20 Composite Materials[14]

Composite materials are formed from two or more dissimilar materials, each of which contributes to the final properties. Unlike metallic alloys, the materials in a composite remain distinct from each other at the macroscopic level.

Most engineering composites consist of two materials: a reinforcement called a *filler* and a *matrix*. The filler provides stiffness and strength; the matrix holds the material together and serves to transfer load among the discontinuous reinforcements. The most common reinforcements, illustrated in Fig. 2–14, are continuous fibers, either straight or woven, short chopped fibers, and particulates. The most common matrices are various plastic resins although other materials including metals are used.

Metals and other traditional engineering materials are uniform, or isotropic, in nature. This means that material properties, such as strength, stiffness, and thermal conductivity, are independent of both position within the material and the choice of coordinate system. The discontinuous nature of composite reinforcements, though, means that material properties can vary with both position and direction. For example, an

[14]For references see I. M. Daniel and O. Ishai, *Engineering Mechanics of Composite Materials,* Oxford University Press, 1994, and *ASM Engineered Materials Handbook: Composites,* ASM International, Materials Park, OH, 1988.

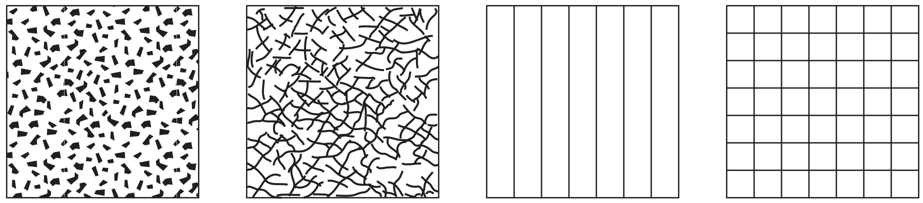


Figure 2–14

Composites categorized by type of reinforcement.

epoxy resin reinforced with continuous graphite fibers will have very high strength and stiffness in the direction of the fibers, but very low properties normal or transverse to the fibers. For this reason, structures of composite materials are normally constructed of multiple plies (laminates) where each ply is oriented to achieve optimal structural stiffness and strength performance.

High strength-to-weight ratios, up to 5 times greater than those of high-strength steels, can be achieved. High stiffness-to-weight ratios can also be obtained, as much as 8 times greater than those of structural metals. For this reason, composite materials are becoming very popular in automotive, aircraft, and spacecraft applications where weight is a premium.

The directionality of properties of composite materials increases the complexity of structural analyses. Isotropic materials are fully defined by two engineering constants: Young's modulus E and Poisson's ratio ν. A single ply of a composite material, however, requires four constants, defined with respect to the ply coordinate system. The constants are two Young's moduli (the longitudinal modulus in the direction of the fibers, E_1, and the transverse modulus normal to the fibers, E_2), one Poisson's ratio (ν_{12}, called the major Poisson's ratio), and one shear modulus (G_{12}). A fifth constant, the minor Poisson's ratio, ν_{21}, is determined through the reciprocity relation, $\nu_{21}/E_2 = \nu_{12}/E_1$. Combining this with multiple plies oriented at different angles makes structural analysis of complex structures unapproachable by manual techniques. For this reason, computer software is available to calculate the properties of a laminated composite construction.[15]

2–21 Materials Selection

As stated earlier, the selection of a material for a machine part or structural member is one of the most important decisions the designer is called on to make. Up to this point in this chapter we have discussed many important material physical properties, various characteristics of typical engineering materials, and various material production processes. The actual selection of a material for a particular design application can be an easy one, say, based on previous applications (1020 steel is always a good candidate because of its many positive attributes), or the selection process can be as involved and daunting as any design problem with the evaluation of the many material physical, economical, and processing parameters. There are systematic and optimizing approaches to material selection. Here, for illustration, we will only look at how to approach some material properties. One basic technique is to list all the important material properties associated with the design, e.g., strength, stiffness, and cost. This can be prioritized by using a weighting measure depending on what properties are more

[15]About Composite Materials Software listing, http://composite.about.com/cs/software/index.htm.

important than others. Next, for each property, list all available materials and rank them in order beginning with the best material; e.g., for strength, high-strength steel such as 4340 steel should be near the top of the list. For completeness of available materials, this might require a large source of material data. Once the lists are formed, select a manageable amount of materials from the top of each list. From each reduced list select the materials that are contained within every list for further review. The materials in the reduced lists can be graded within the list and then weighted according to the importance of each property.

M. F. Ashby has developed a powerful systematic method using *materials selection charts*.[16] This method has also been implemented in a software package called CES Edupack.[17] The charts display data of various properties for the families and classes of materials listed in Table 2–4. For example, considering material stiffness properties, a simple bar chart plotting Young's modulus E on the y axis is shown in Fig. 2–15. Each vertical line represents the range of values of E for a particular material. Only some of the materials are labeled. Now, more material information can be displayed if the x axis represents another material property, say density.

Table 2–4

Material Families and Classes

Family	Classes	Short Name
Metals (the metals and alloys of engineering)	Aluminum alloys	Al alloys
	Copper alloys	Cu alloys
	Lead alloys	Lead alloys
	Magnesium alloys	Mg alloys
	Nickel alloys	Ni alloys
	Carbon steels	Steels
	Stainless steels	Stainless steels
	Tin alloys	Tin alloys
	Titanium alloys	Ti alloys
	Tungsten alloys	W alloys
	Lead alloys	Pb alloys
	Zinc alloys	Zn alloys
Ceramics Technical ceramics (fine ceramics capable of load-bearing application)	Alumina	Al_2O_3
	Aluminum nitride	AlN
	Boron carbide	B_4C
	Silicon carbide	SiC
	Silicon nitride	Si_3N_4
	Tungsten carbide	WC
Nontechnical ceramics (porous ceramics of construction)	Brick	Brick
	Concrete	Concrete
	Stone	Stone

(continued)

[16] M. F. Ashby, *Materials Selection in Mechanical Design*, 3rd ed., Elsevier Butterworth-Heinemann, Oxford, 2005.

[17] Produced by Granta Design Limited. See www.grantadesign.com.

Table 2–4 *(continued)*

Family	Classes	Short Name
Glasses	Soda-lime glass	Soda-lime glass
	Borosilicate glass	Borosilicate glass
	Silica glass	Silica glass
	Glass ceramic	Glass ceramic
Polymers (the thermoplastics and thermosets of engineering)	Acrylonitrile butadiene styrene	ABS
	Cellulose polymers	CA
	Ionomers	Ionomers
	Epoxies	Epoxy
	Phenolics	Phenolics
	Polyamides (nylons)	PA
	Polycarbonate	PC
	Polyesters	Polyester
	Polyetheretherkeytone	PEEK
	Polyethylene	PE
	Polyethylene terephalate	PET or PETE
	Polymethylmethacrylate	PMMA
	Polyoxymethylene(Acetal)	POM
	Polypropylene	PP
	Polystyrene	PS
	Polytetrafluorethylene	PTFE
	Polyvinylchloride	PVC
Elastomers (engineering rubbers, natural and synthetic)	Butyl rubber	Butyl rubber
	EVA	EVA
	Isoprene	Isoprene
	Natural rubber	Natural rubber
	Polychloroprene (Neoprene)	Neoprene
	Polyurethane	PU
	Silicon elastomers	Silicones
Hybrids	Carbon-fiber reinforced polymers	CFRP
Composites	Glass-fiber reinforced polymers	GFRP
	SiC reinforced aluminum	Al-SiC
Foams	Flexible polymer foams	Flexible foams
	Rigid polymer foams	Rigid foams
Natural materials	Cork	Cork
	Bamboo	Bamboo
	Wood	Wood

From M. F. Ashby, *Materials Selection in Mechanical Design,* 3rd ed., Elsevier Butterworth-Heinemann, Oxford, 2005. Table 4–1, pp. 49–50.

Figure 2–15

Young's modulus E for various materials. *(Figure courtesy of Prof. Mike Ashby, Granta Design, Cambridge, U.K.)*

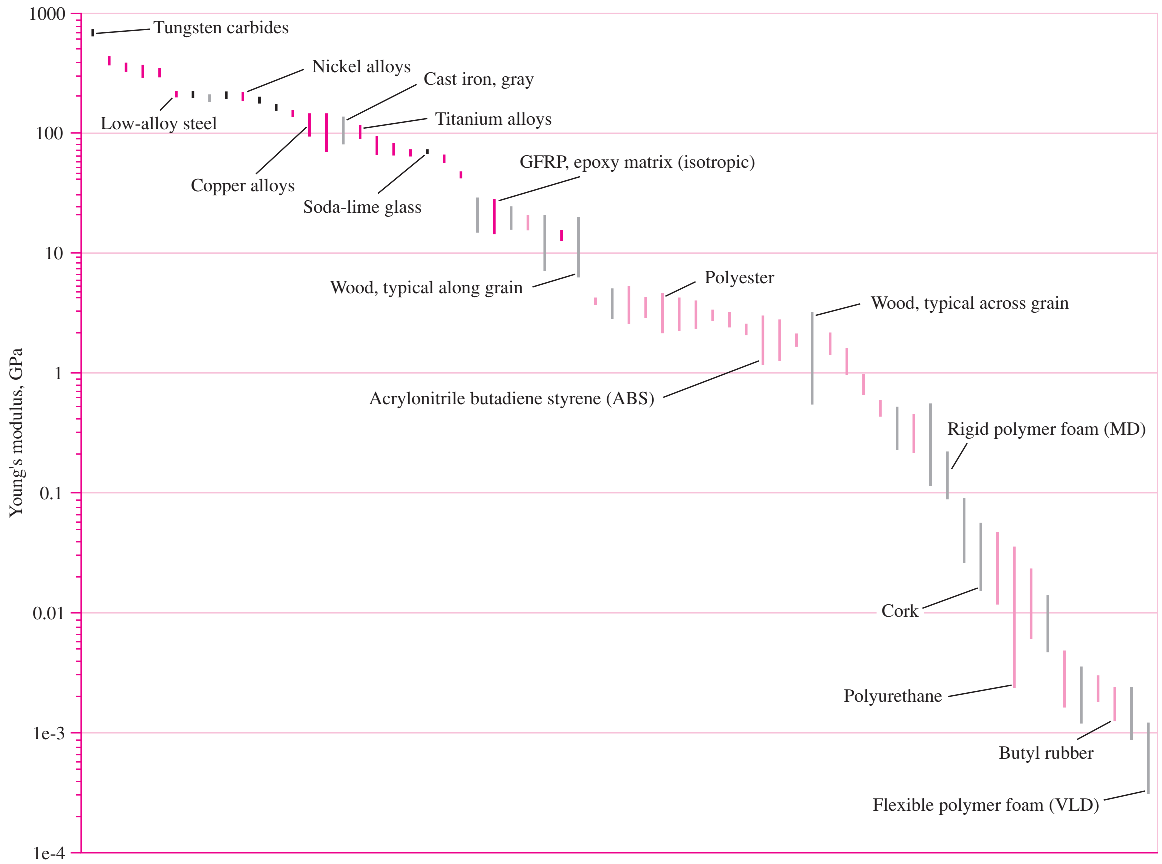


Figure 2–16, called a "bubble" chart, represents Young's modulus E plotted against density ρ. The line ranges for each material property plotted two-dimensionally now form ellipses, or bubbles. This plot is more useful than the two separate bar charts of each property. Now, we also see how stiffness/weight for various materials relate. Figure 2–16 also shows groups of bubbles outlined according to the material families of Table 2–4. In addition, dotted lines in the lower right corner of the chart indicate ratios of E^β/ρ, which assist in material selection for minimum mass design. Lines drawn parallel to these lines represent different values for E^β/ρ. For example, several parallel dotted lines are shown in Fig. 2–16 that represent different values of $E/\rho(\beta = 1)$. Since $(E/\rho)^{1/2}$ represents the speed of sound in a material, each dotted line, E/ρ, represents a different speed as indicated.

To see how β fits into the mix, consider the following. The performance metric P of a structural element depends on (1) the functional requirements, (2) the geometry, and (3) the material properties of the structure. That is,

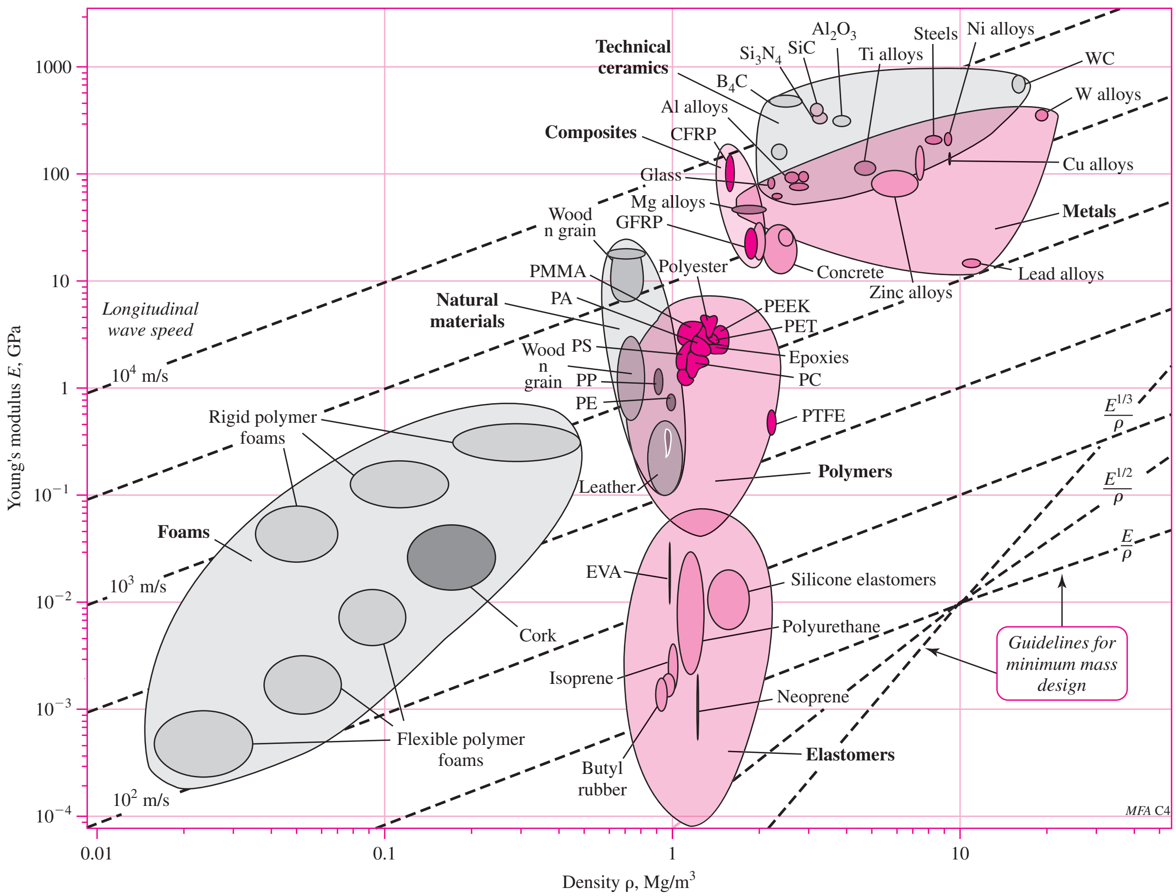
$$P = \left[\begin{pmatrix}\text{functional} \\ \text{requirements } F\end{pmatrix}, \begin{pmatrix}\text{geometric} \\ \text{parameters } G\end{pmatrix}, \begin{pmatrix}\text{material} \\ \text{properties } M\end{pmatrix}\right]$$

or, symbolically,

$$P = f(F, G, M) \tag{2–20}$$

Figure 2–16

Young's modulus E versus density ρ for various materials. *(Figure courtesy of Prof. Mike Ashby, Granta Design, Cambridge, U.K.)*



If the function is *separable,* which it often is, we can write Eq. (2–20) as

$$P = f_1(F) \cdot f_2(G) \cdot f_3(M) \tag{2–21}$$

For optimum design, we desire to maximize or minimize P. With regards to material properties alone, this is done by maximizing or minimizing $f_3(M)$, called the *material efficiency coefficient*.

For illustration, say we want to design a light, stiff, end-loaded cantilever beam with a circular cross section. For this we will use the mass m of the beam for the performance metric to minimize. The stiffness of the beam is related to its material and geometry. The stiffness of a beam is given by $k = F/\delta$, where F and δ are the end load and deflection, respectively (see Chap. 4). The end deflection of an end-loaded cantilever beam is given in Table A–9, beam 1, as $\delta = y_{\max} = (Fl^3)/(3EI)$, where E is Young's modulus, I the second moment of the area, and l the length of the beam. Thus, the stiffness is given by

$$k = \frac{F}{\delta} = \frac{3EI}{l^3} \tag{2–22}$$

From Table A-18, the second moment of the area of a circular cross section is

$$I = \frac{\pi D^4}{64} = \frac{A^2}{4\pi} \tag{2–23}$$

where D and A are the diameter and area of the cross section, respectively. Substituting Eq. (2–23) in (2–22) and solving for A, we obtain

$$A = \left(\frac{4\pi k l^3}{3E}\right)^{1/2} \tag{2–24}$$

The mass of the beam is given by

$$m = Al\rho \tag{2–25}$$

Substituting Eq. (2–24) into (2–25) and rearranging yields

$$m = 2\sqrt{\frac{\pi}{3}}(k^{1/2})(l^{5/2})\left(\frac{\rho}{E^{1/2}}\right) \tag{2–26}$$

Equation (2–26) is of the form of Eq. (2–21). The term $2\sqrt{\pi/3}$ is simply a constant and can be associated with any function, say $f_1(F)$. Thus, $f_1(F) = 2\sqrt{\pi/3}(k^{1/2})$ is the functional requirement, stiffness; $f_2(G) = (l^{5/2})$, the geometric parameter, length; and the material efficiency coefficient

$$f_3(M) = \frac{\rho}{E^{1/2}} \tag{2–27}$$

is the material property in terms of density and Young's modulus. To minimize m we want to minimize $f_3(M)$, or maximize

$$M = \frac{E^{1/2}}{\rho} \tag{2–28}$$

where M is called the *material index,* and $\beta = \frac{1}{2}$. Returning to Fig. 2–16, draw lines of various values of $E^{1/2}/\rho$ as shown in Fig. 2–17. Lines of increasing M move up and to the left as shown. Thus, we see that good candidates for a light, stiff, end-loaded cantilever beam with a circular cross section are certain woods, composites, and ceramics.

Other limits/constraints may warrant further investigation. Say, for further illustration, the design requirements indicate that we need a Young's modulus greater than 50 GPa. Figure 2–18 shows how this further restricts the search region. This eliminates woods as a possible material.

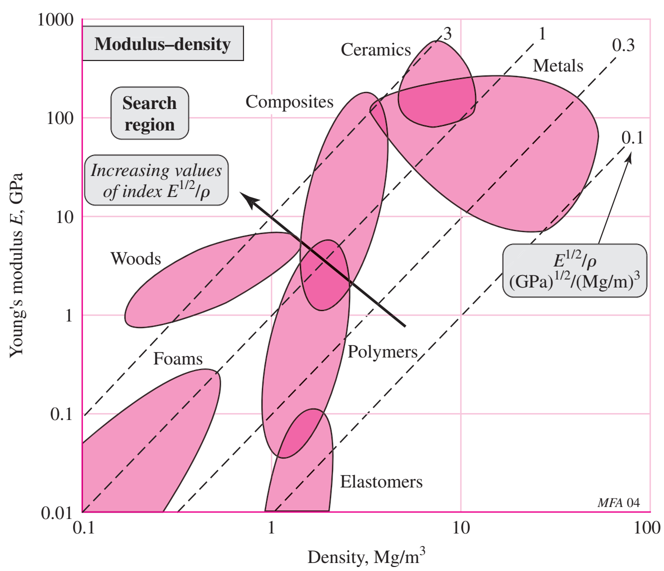


Figure 2–17

A schematic E versus ρ chart showing a grid of lines for various values the material index $M = E^{1/2}/\rho$. *(From M. F. Ashby, Materials Selection in Mechanical Design, 3rd ed., Elsevier Butterworth-Heinemann, Oxford, 2005.)*

Figure 2–18

The search region of Fig. 2–16 further reduced by restricting $E \geq 50$ GPa. *(From M. F. Ashby, Materials Selection in Mechanical Design, 3rd ed., Elsevier Butterworth-Heinemann, Oxford, 2005.)*

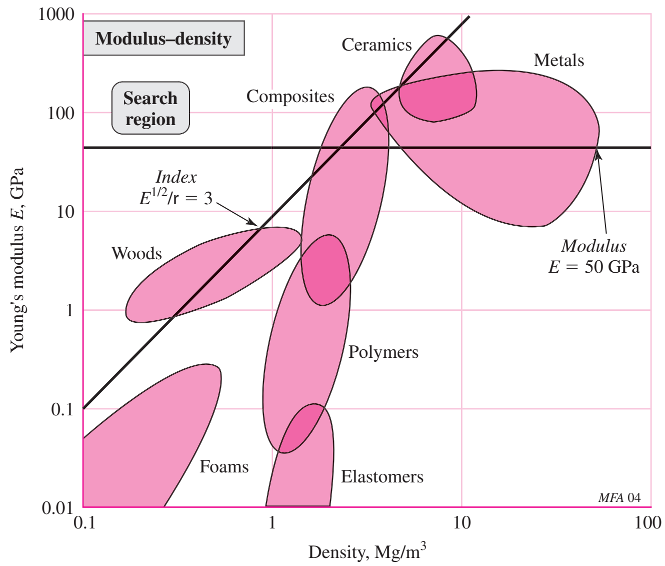
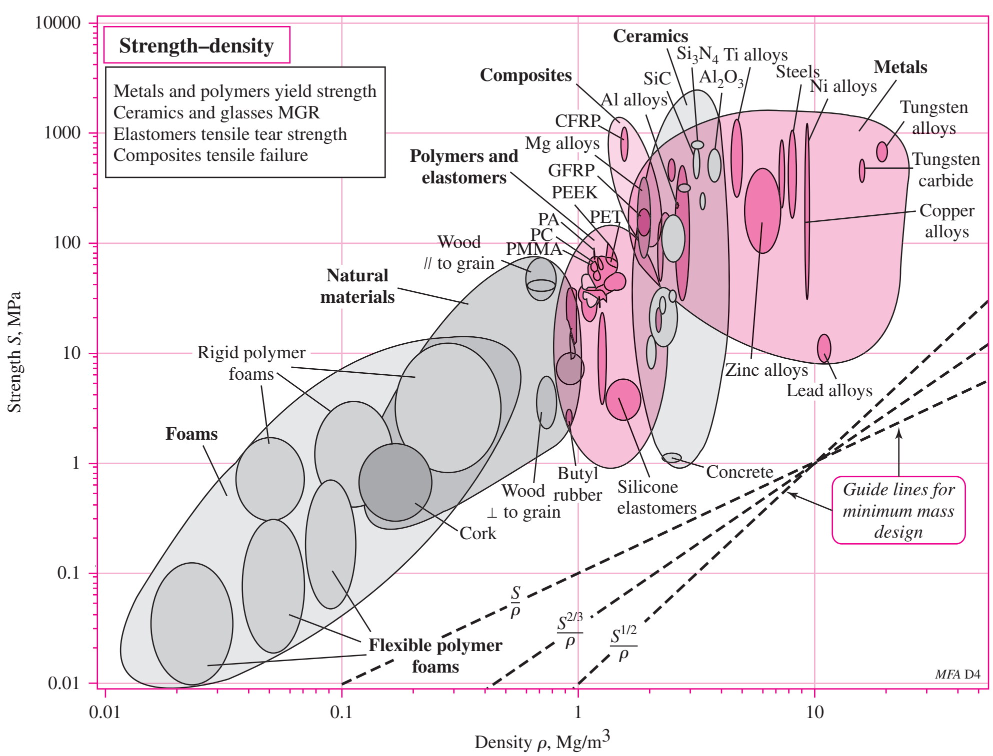


Figure 2–19

Strength S versus density ρ for various materials. For *metals*, S is the 0.2 percent offset yield strength. For *polymers*, S is the 1 percent yield strength. For *ceramics and glasses*, S is the compressive crushing strength. For *composites*, S is the tensile strength. For *elastomers*, S is the tear strength. *(Figure courtesy of Prof. Mike Ashby, Granta Design, Cambridge, U.K.)*



Certainly, in a given design exercise, there will be other considerations such as strength, environment, and cost, and other charts may be necessary to investigate. For example, Fig. 2–19 represents strength versus density for the material families. Also, we have not brought in the material process selection part of the picture. If done properly, material selection can result in a good deal of bookkeeping. This is where software packages such as CES Edupack become very effective.

PROBLEMS

2–1 Determine the minimum tensile and yield strengths for SAE 1020 cold-drawn steel.

2–2 Determine the minimum tensile and yield strengths for UNS G10500 hot-rolled steel.

2–3 For the materials in Probs. 2–1 and 2–2, compare the following properties: minimum tensile and yield strengths, ductility, and stiffness.

2–4 Assuming you were specifying an AISI 1040 steel for an application where you desired to maximize the yield strength, how would you specify it?

2–5 Assuming you were specifying an AISI 1040 steel for an application where you desired to maximize the ductility, how would you specify it?

2–6 Determine the yield strength-to-weight density ratios (called *specific strength*) in units of inches for UNS G10350 hot-rolled steel, 2024-T4 aluminum, Ti-6A1-4V titanium alloy, and ASTM No. 30 gray cast iron.

2–7 Determine the stiffness-to-weight density ratios (called *specific modulus*) in units of inches for UNS G10350 hot-rolled steel, 2024-T4 aluminum, Ti-6A1-4V titanium alloy, and ASTM No. 30 gray cast iron.

2–8 *Poisson's ratio* ν is a material property and is the ratio of the lateral strain and the longitudinal strain for a member in tension. For a homogeneous, isotropic material, the modulus of rigidity G is related to Young's modulus as

$$G = \frac{E}{2(1+\nu)}$$

Using the tabulated values of G and E, determine Poisson's ratio for steel, aluminum, beryllium copper, and gray cast iron.

2–9 A specimen of medium-carbon steel having an initial diameter of 0.503 in was tested in tension using a gauge length of 2 in. The following data were obtained for the elastic and plastic states:

Elastic State		Plastic State	
Load P, lbf	Elongation, in	Load P, lbf	Area A_i, in^2
1 000	0.0004	8 800	0.1984
2 000	0.0006	9 200	0.1978
3 000	0.0010	9 100	0.1963
4 000	0.0013	13 200	0.1924
7 000	0.0023	15 200	0.1875
8 400	0.0028	17 000	0.1563
8 800	0.0036	16 400	0.1307
9 200	0.0089	14 800	0.1077

Note that there is some overlap in the data. Plot the engineering or nominal stress-strain diagram using two scales for the unit strain ϵ, one from zero to about 0.02 in/in and the other from zero to maximum strain. From this diagram find the modulus of elasticity, the 0.2 percent offset yield strength, the ultimate strength, and the percent reduction in area.

2–10 Compute the true stress and the logarithmic strain using the data of Prob. 2–9 and plot the results on log-log paper. Then find the plastic strength coefficient σ_0 and the strain-strengthening exponent m. Find also the yield strength and the ultimate strength after the specimen has had 20 percent cold work.

2–11 The stress-strain data from a tensile test on a cast-iron specimen are

Engineering stress, kpsi	5	10	16	19	26	32	40	46	49	54
Engineering strain, $\epsilon \cdot 10^{-3}$ in/in	0.20	0.44	0.80	1.0	1.5	2.0	2.8	3.4	4.0	5.0

Plot the stress-strain locus and find the 0.1 percent offset yield strength, and the tangent modulus of elasticity at zero stress and at 20 kpsi.

2–12 A straight bar of arbitrary cross section and thickness h is cold-formed to an inner radius R about an anvil as shown in the figure. Some surface at distance N having an original length L_{AB} will remain unchanged in length after bending. This length is

$$L_{AB} = L_{AB'} = \frac{\pi(R+N)}{2}$$

The lengths of the outer and inner surfaces, after bending, are

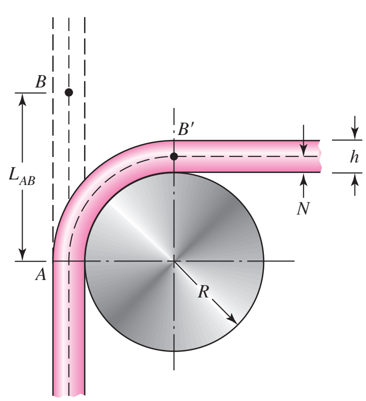
$$L_o = \frac{\pi}{2}(R+h) \qquad L_i = \frac{\pi}{2}R$$

Using Eq. (2–4), we then find the true strains to be

$$\varepsilon_o = \ln\frac{R+h}{R+N} \qquad \varepsilon_i = \ln\frac{R}{R+N}$$

Tests show that $|\varepsilon_o| = |\varepsilon_i|$. Show that

$$N = R\left[\left(1+\frac{h}{R}\right)^{1/2} - 1\right]$$



Problem 2–12

and

$$\varepsilon_o = \ln\left(1 + \frac{h}{R}\right)^{1/2}$$

2–13 A hot-rolled AISI 1212 steel is given 20 percent cold work. Determine the new values of the yield and ultimate strengths.

2–14 A steel member has a Brinell of $H_B = 250$. Estimate the ultimate strength of the steel in MPa.

2–15 Brinell hardness tests were made on a random sample of 10 steel parts during processing. The results were H_B values of 252 (2), 260, 254, 257 (2), 249 (3), and 251. Estimate the mean and standard deviation of the ultimate strength in kpsi.

2–16 Repeat Prob. 2–15 assuming the material to be cast iron.

2–17 *Toughness* is a term that relates to both strength and ductility. The fracture toughness, for example, is defined as the total area under the stress-strain curve to fracture, $u_T = \int_0^{\epsilon_f} \sigma \, d\epsilon$. This area, called the *modulus of toughness,* is the strain energy per unit volume required to cause the material to fracture. A similar term, but defined within the elastic limit of the material, is called the *modulus of resilience,* $u_R = \int_0^{\epsilon_y} \sigma d\epsilon$, where ϵ_y is the strain at yield. If the stress-strain is linear to $\sigma = S_y$, then it can be shown that $u_R = S_y^2/2E$.

For the material in Prob. 2–9: (*a*) Determine the modulus of resilience, and (*b*) Estimate the modulus of toughness, assuming that the last data point corresponds to fracture.

2–18 What is the material composition of AISI 4340 steel?

2–19 Search the website noted in Sec. 2–20 and report your findings.

2–20 Research the material Inconel, briefly described in Table A–5. Compare it to various carbon and alloy steels in stiffness, strength, ductility, and toughness. What makes this material so special?

2–21 Pick a specific material given in the tables (e.g., 2024-T4 aluminum, SAE 1040 steel), and consult a local or regional distributor (consulting either the Yellow Pages or the Thomas Register) to obtain as much information as you can about cost and availability of the material and in what form (bar, plate, etc.).

2–22 Consider a tie rod transmitting a tensile force F. The corresponding tensile stress is given by $\sigma = F/A$, where A is the area of the cross section. The deflection of the rod is given by Eq. (4–3), which is $\delta = (Fl)/(AE)$, where l is the length of the rod. Using the Ashby charts of Figs. 2–16 and 2–19, explore what ductile materials are best suited for a light, stiff, *and* strong tie rod. *Hints:* Consider stiffness and strength separately. For use of Fig. 2–16, prove that $\beta = 1$. For use of Fig. 2–19, relate the applied tensile stress to the material strength.

3 Load and Stress Analysis

Chapter Outline

One of the main objectives of this book is to describe how specific machine components function and how to design or specify them so that they function safely without failing structurally. Although earlier discussion has described structural strength in terms of load or stress versus strength, failure of function for structural reasons may arise from other factors such as excessive deformations or deflections.

Here it is assumed that the reader has completed basic courses in statics of rigid bodies and mechanics of materials and is quite familiar with the analysis of loads, and the stresses and deformations associated with the basic load states of simple prismatic elements. In this chapter and Chap. 4 we will review and extend these topics briefly. Complete derivations will not be presented here, and the reader is urged to return to basic textbooks and notes on these subjects.

This chapter begins with a review of equilibrium and free-body diagrams associated with load-carrying components. One must understand the nature of forces before attempting to perform an extensive stress or deflection analysis of a mechanical component. An extremely useful tool in handling discontinuous loading of structures employs *Macaulay* or *singularity functions*. Singularity functions are described in Sec. 3–3 as applied to the shear forces and bending moments in beams. In Chap. 4, the use of singularity functions will be expanded to show their real power in handling deflections of complex geometry and statically indeterminate problems.

Machine components transmit forces and motion from one point to another. The transmission of force can be envisioned as a flow or force distribution that can be further visualized by isolating internal surfaces within the component. Force distributed over a surface leads to the concept of stress, stress components, and stress transformations (Mohr's circle) for all possible surfaces at a point.

The remainder of the chapter is devoted to the stresses associated with the basic loading of prismatic elements, such as uniform loading, bending, and torsion, and topics with major design ramifications such as stress concentrations, thin- and thick-walled pressurized cylinders, rotating rings, press and shrink fits, thermal stresses, curved beams, and contact stresses.

3–1 Equilibrium and Free-Body Diagrams

Equilibrium

The word *system* will be used to denote any *isolated* part or portion of a machine or structure—including all of it if desired—that we wish to study. A system, under this definition, may consist of a particle, several particles, a part of a rigid body, an entire rigid body, or even several rigid bodies.

If we assume that the system to be studied is motionless or, at most, has constant velocity, then the system has zero acceleration. Under this condition the system is said to be in *equilibrium*. The phrase *static equilibrium* is also used to imply that the system is *at rest*. For equilibrium, the forces and moments acting on the system balance such that

$$\sum \mathbf{F} = 0 \tag{3–1}$$

$$\sum \mathbf{M} = 0 \tag{3–2}$$

which states that *the sum of all force* and the *sum of all moment vectors* acting upon a system in equilibrium is zero.

Free-Body Diagrams

We can greatly simplify the analysis of a very complex structure or machine by successively isolating each element and studying and analyzing it by the use of *free-body diagrams*. When all the members have been treated in this manner, the knowledge can be assembled to yield information concerning the behavior of the total system. Thus, free-body diagramming is essentially a means of breaking a complicated problem into manageable segments, analyzing these simple problems, and then, usually, putting the information together again.

Using free-body diagrams for force analysis serves the following important purposes:

- The diagram establishes the directions of reference axes, provides a place to record the dimensions of the subsystem and the magnitudes and directions of the known forces, and helps in assuming the directions of unknown forces.
- The diagram simplifies your thinking because it provides a place to store one thought while proceeding to the next.
- The diagram provides a means of communicating your thoughts clearly and unambiguously to other people.
- Careful and complete construction of the diagram clarifies fuzzy thinking by bringing out various points that are not always apparent in the statement or in the geometry of the total problem. Thus, the diagram aids in understanding all facets of the problem.
- The diagram helps in the planning of a logical attack on the problem and in setting up the mathematical relations.
- The diagram helps in recording progress in the solution and in illustrating the methods used.
- The diagram allows others to follow your reasoning, showing *all* forces.

EXAMPLE 3–1

Figure 3–1*a* shows a simplified rendition of a gear reducer where the input and output shafts AB and CD are rotating at constant speeds ω_i and ω_o, respectively. The input and output torques (torsional moments) are $T_i = 240$ lbf · in and T_o, respectively. The shafts are supported in the housing by bearings at A, B, C, and D. The pitch radii of gears G_1 and G_2 are $r_1 = 0.75$ in and $r_2 = 1.5$ in, respectively. Draw the free-body diagrams of each member and determine the net reaction forces and moments at all points.

Solution

First, we will list all simplifying assumptions.

1. Gears G_1 and G_2 are simple spur gears with a standard pressure angle $\phi = 20°$ (see Sec. 13–5).
2. The bearings are self-aligning and the shafts can be considered to be simply supported.
3. The weight of each member is negligible.
4. Friction is negligible.
5. The mounting bolts at E, F, H, and I are the same size.

The separate free-body diagrams of the members are shown in Figs. 3–1*b*–*d*. Note that Newton's third law, called *the law of action and reaction,* is used extensively where each member mates. The force transmitted between the spur gears is not tangential but at the pressure angle ϕ. Thus, $N = F \tan \phi$.

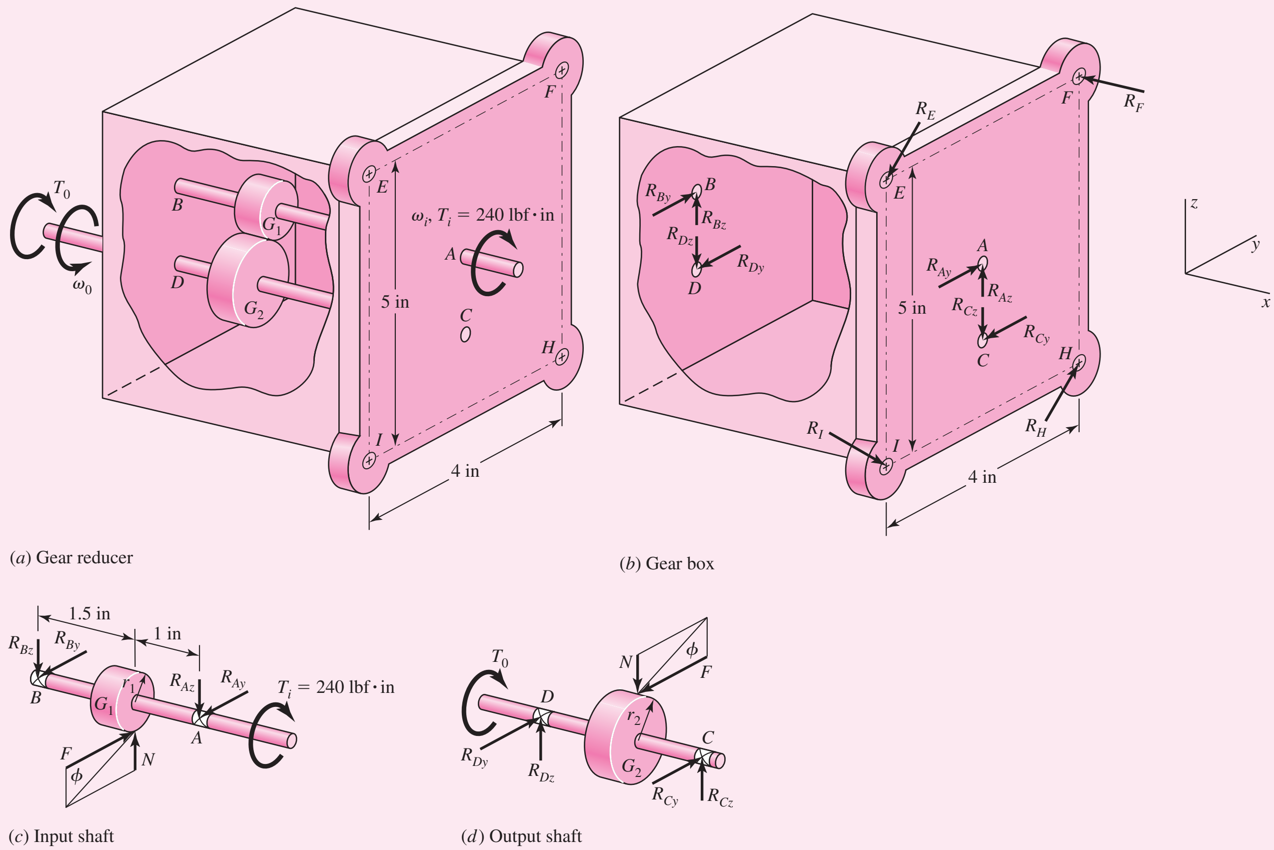


Figure 3–1

(*a*) Gear reducer; (*b–d*) free-body diagrams. Diagrams are not drawn to scale.

Summing moments about the x axis of shaft AB in Fig. 3–1d gives

$$\sum M_x = F(0.75) - 240 = 0$$
$$F = 320 \text{ lbf}$$

The normal force is $N = 320 \tan 20° = 116.5$ lbf.

Using the equilibrium equations for Figs. 3–1c and d, the reader should verify that: $R_{Ay} = 192$ lbf, $R_{Az} = 69.9$ lbf, $R_{By} = 128$ lbf, $R_{Bz} = 46.6$ lbf, $R_{Cy} = 192$ lbf, $R_{Cz} = 69.9$ lbf, $R_{Dy} = 128$ lbf, $R_{Dz} = 46.6$ lbf, and $T_o = 480$ lbf · in. The direction of the output torque T_o is opposite ω_o because it is the resistive load on the system opposing the motion ω_o.

Note in Fig. 3–1b the net force from the bearing reactions is zero whereas the net moment about the x axis is 2.25 (192) + 2.25 (128) = 720 lbf · in. This value is the same as $T_i + T_o = 240 + 480 = 720$ lbf · in, as shown in Fig. 3–1a. The reaction forces R_E, R_F, R_H, and R_I, from the mounting bolts cannot be determined from the equilibrium equations as there are too many unknowns. Only three equations are available, $\sum F_y = \sum F_z = \sum M_x = 0$. In case you were wondering about assumption 5, here is where we will use it (see Sec. 8–12). The gear box tends to rotate about the x axis because of a pure torsional moment of 720 lbf · in. The bolt forces must provide an equal but opposite torsional moment. The center of rotation relative to the bolts lies at the centroid of the bolt cross-sectional areas. Thus if the bolt areas are equal: the center of rotation is at the center of the four bolts, a distance of $\sqrt{(4/2)^2 + (5/2)^2} = 3.202$ in from each bolt; the bolt forces are equal ($R_E = R_F = R_H = R_I = R$), and each bolt force is perpendicular to the line from the bolt to the center of rotation. This gives a net torque from the four bolts of $4R(3.202) = 720$. Thus, $R_E = R_F = R_H = R_I = 56.22$ lbf.

3–2 Shear Force and Bending Moments in Beams

Figure 3–2*a shows a beam supported by* reactions R_1 and R_2 and loaded by the concentrated forces F_1, F_2, and F_3. If the beam is cut at some section located at $x = x_1$ and the left-hand portion is removed as a free body, an *internal shear force V* and *bending moment M* must act on the cut surface to ensure equilibrium (see Fig. 3–2*b*). The shear force is obtained by summing the forces on the isolated section. The bending moment is the sum of the moments of the forces to the left of the section taken about an axis through the isolated section. The sign conventions used for bending moment and shear force in this book are shown in Fig. 3–3. Shear force and bending moment are related by the equation

$$V = \frac{dM}{dx} \tag{3–3}$$

Sometimes the bending is caused by a distributed load $q(x)$, as shown in Fig. 3–4; $q(x)$ is called the *load intensity* with units of force per unit length and is positive in the

Figure 3–2

Free-body diagram of simply-supported beam with V and M shown in positive directions.

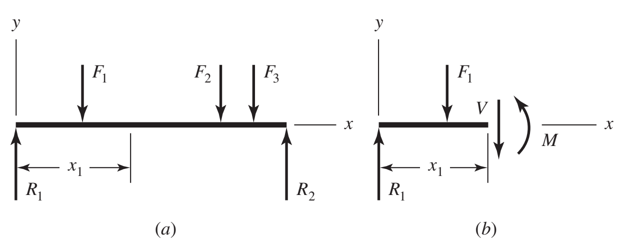


Figure 3–3

Sign conventions for bending and shear.

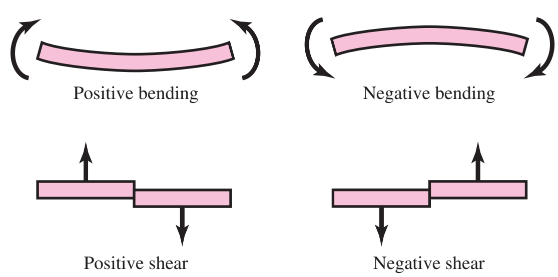
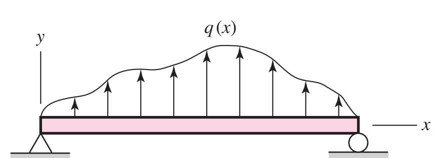


Figure 3–4

Distributed load on beam.



positive y direction. It can be shown that differentiating Eq. (3–3) results in

$$\frac{dV}{dx} = \frac{d^2M}{dx^2} = q \tag{3–4}$$

Normally the applied distributed load is directed downward and labeled w (e.g., see Fig. 3–6). In this case, $w = -q$.

Equations (3–3) and (3–4) reveal additional relations if they are integrated. Thus, if we integrate between, say, x_A and x_B, we obtain

$$\int_{V_A}^{V_B} dV = \int_{x_A}^{x_B} q\,dx = V_B - V_A \tag{3–5}$$

which states that *the change in shear force from A to B is equal to the area of the loading diagram between x_A and x_B*.

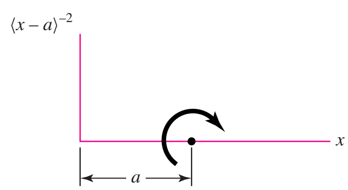
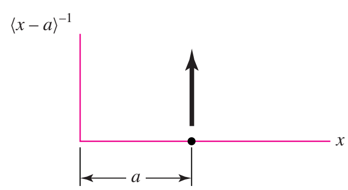
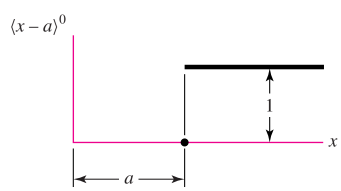
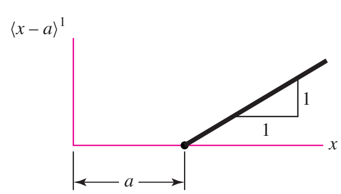
In a similar manner,

$$\int_{M_A}^{M_B} dM = \int_{x_A}^{x_B} V\,dx = M_B - M_A \tag{3–6}$$

which states that *the change in moment from A to B is equal to the area of the shear-force diagram between x_A and x_B*.

Table 3–1

Singularity (Macaulay†) Functions

Function	Graph of $f_n(x)$	Meaning
Concentrated moment (unit doublet)		$\langle x-a\rangle^{-2} = 0 \quad x \neq a$ $\langle x-a\rangle^{-2} = \pm\infty \quad x = a$ $\int \langle x-a\rangle^{-2}\,dx = \langle x-a\rangle^{-1}$
Concentrated force (unit impulse)		$\langle x-a\rangle^{-1} = 0 \quad x \neq a$ $\langle x-a\rangle^{-1} = +\infty \quad x = a$ $\int \langle x-a\rangle^{-1}\,dx = \langle x-a\rangle^{0}$
Unit step		$\langle x-a\rangle^{0} = \begin{cases} 0 & x < a \\ 1 & x \geq a \end{cases}$ $\int \langle x-a\rangle^{0}\,dx = \langle x-a\rangle^{1}$
Ramp		$\langle x-a\rangle^{1} = \begin{cases} 0 & x < a \\ x-a & x \geq a \end{cases}$ $\int \langle x-a\rangle^{1}\,dx = \frac{\langle x-a\rangle^{2}}{2}$

†W. H. Macaulay, "Note on the deflection of beams," *Messenger of Mathematics*, vol. 48, pp. 129–130, 1919.

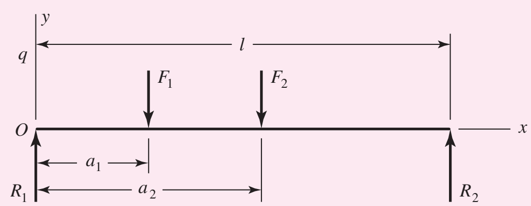
3–3 Singularity Functions

The four singularity functions defined in Table 3–1 constitute a useful and easy means of integrating across discontinuities. By their use, general expressions for shear force and bending moment in beams can be written when the beam is loaded by concentrated moments or forces. As shown in the table, the concentrated moment and force functions are zero for all values of x not equal to a. The functions are undefined for values of $x = a$. Note that the unit step and ramp functions are zero only for values of x that are less than a. The integration properties shown in the table constitute a part of the mathematical definition too. The first two integrations of $q(x)$ for $V(x)$ and $M(x)$ do not require constants of integration provided *all* loads on the beam are accounted for in $q(x)$. The examples that follow show how these functions are used.

EXAMPLE 3–2

Derive expressions for the loading, shear-force, and bending-moment diagrams for the beam of Fig. 3–5.

Figure 3–5



Solution

Using Table 3–1 and $q(x)$ for the loading function, we find

Answer

$$q = R_1\langle x\rangle^{-1} - F_1\langle x - a_1\rangle^{-1} - F_2\langle x - a_2\rangle^{-1} + R_2\langle x - l\rangle^{-1} \tag{1}$$

Next, we use Eq. (3–5) to get the shear force.

Answer

$$V = \int q\,dx = R_1\langle x\rangle^{0} - F_1\langle x - a_1\rangle^{0} - F_2\langle x - a_2\rangle^{0} + R_2\langle x - l\rangle^{0} \tag{2}$$

Note that $V = 0$ at $x = 0^-$.

A second integration, in accordance with Eq. (3–6), yields

Answer

$$M = \int V\,dx = R_1\langle x\rangle^{1} - F_1\langle x - a_1\rangle^{1} - F_2\langle x - a_2\rangle^{1} + R_2\langle x - l\rangle^{1} \tag{3}$$

The reactions R_1 and R_2 can be found by taking a summation of moments and forces as usual, *or* they can be found by noting that the shear force and bending moment must be zero everywhere except in the region $0 \le x \le l$. This means that Eq. (2) should give $V = 0$ at x slightly larger than l. Thus

$$R_1 - F_1 - F_2 + R_2 = 0 \tag{4}$$

Since the bending moment should also be zero in the same region, we have, from Eq. (3),

$$R_1 l - F_1(l - a_1) - F_2(l - a_2) = 0 \tag{5}$$

Equations (4) and (5) can now be solved for the reactions R_1 and R_2.

EXAMPLE 3–3

Figure 3–6*a* shows the loading diagram for a beam cantilevered at A with a uniform load of 20 lbf/in acting on the portion 3 in $\le x \le$ 7 in, and a concentrated counterclockwise moment of 240 lbf · in at $x = 10$ in. Derive the shear-force and bending-moment relations, and the support reactions M_1 and R_1.

Solution

Following the procedure of Example 3–2, we find the load intensity function to be

$$q = -M_1\langle x\rangle^{-2} + R_1\langle x\rangle^{-1} - 20\langle x-3\rangle^0 + 20\langle x-7\rangle^0 - 240\langle x-10\rangle^{-2} \qquad (1)$$

Note that the $20\langle x-7\rangle^0$ term was necessary to "turn off" the uniform load at C. Integrating successively gives

Answers

$$V = -M_1\langle x\rangle^{-1} + R_1\langle x\rangle^{0} - 20\langle x-3\rangle^1 + 20\langle x-7\rangle^1 - 240\langle x-10\rangle^{-1} \qquad (2)$$

$$M = -M_1\langle x\rangle^{0} + R_1\langle x\rangle^{1} - 10\langle x-3\rangle^2 + 10\langle x-7\rangle^2 - 240\langle x-10\rangle^{0} \qquad (3)$$

The reactions are found by making x slightly larger than 10 in, where both V and M are zero in this region. Equation (2) will then give

$$-M_1(0) + R_1(1) - 20(10-3) + 20(10-7) - 240(0) = 0$$

Answer

which yields $R_1 = 80$ lbf.

From Eq. (3) we get

$$-M_1(1) + 80(10) - 10(10-3)^2 + 10(10-7)^2 - 240(1) = 0$$

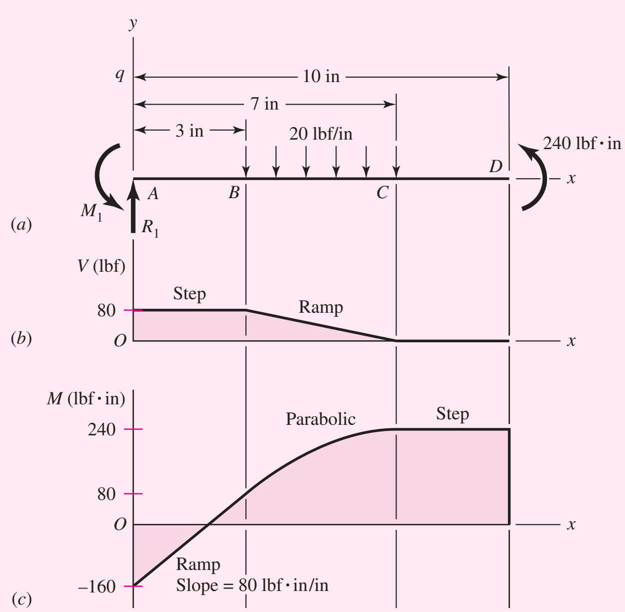
Answer

which yields $M_1 = 160$ lbf · in.

Figures 3–6*b* and *c* show the shear-force and bending-moment diagrams. Note that the impulse terms in Eq. (2), $-M_1\langle x\rangle^{-1}$ and $-240\langle x-10\rangle^{-1}$, are physically not forces

Figure 3–6

(*a*) Loading diagram for a beam cantilevered at *A*.
(*b*) Shear-force diagram.
(*c*) Bending-moment diagram.



and are not shown in the V diagram. Also note that both the M_1 and 240 lbf · in moments are counterclockwise and negative singularity functions; however, by the convention shown in Fig. 3–2 the M_1 and 240 lbf · in are negative and positive bending moments, respectively, which is reflected in Fig. 3–6*c*.

3–4 Stress

When an internal surface is isolated as in Fig. 3–2*b*, the net force and moment acting on the surface manifest themselves as force distributions across the entire area. The force distribution acting at a point on the surface is unique and will have components in the normal and tangential directions called *normal stress* and *tangential shear stress*, respectively. Normal and shear stresses are labeled by the Greek symbols σ and τ, respectively. If the direction of σ is outward from the surface it is considered to be a *tensile stress* and is a positive normal stress. If σ is into the surface it is a *compressive stress* and commonly considered to be a negative quantity. The units of stress in U.S. Customary units are pounds per square inch (psi). For SI units, stress is in newtons per square meter (N/m^2); 1 N/m^2 = 1 pascal (Pa).

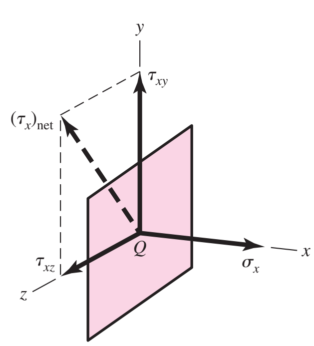
3–5 Cartesian Stress Components

The Cartesian stress components are established by defining three mutually orthogonal surfaces at a point within the body. The normals to each surface will establish the x, y, z Cartesian axes. In general, each surface will have a normal and shear stress. The shear stress may have components along two Cartesian axes. For example, Fig. 3–7 shows an infinitesimal surface area isolation at a point Q within a body where the surface normal is the x direction. The normal stress is labeled σ_x. The symbol σ indicates a normal stress and the subscript x indicates the direction of the surface normal. The net shear stress acting on the surface is $(\tau_x)_{\text{net}}$ which can be resolved into components in the y and z directions, labeled as τ_{xy} and τ_{xz}, respectively (see Fig. 3–7). Note that double subscripts are necessary for the shear. The first subscript indicates the direction of the surface normal whereas the second subscript is the direction of the shear stress.

The state of stress at a point described by three mutually perpendicular surfaces is shown in Fig. 3–8a. It can be shown through coordinate transformation that this is sufficient to determine the state of stress on *any* surface intersecting the point. As the

Figure 3–7

Stress components on surface normal to x direction.



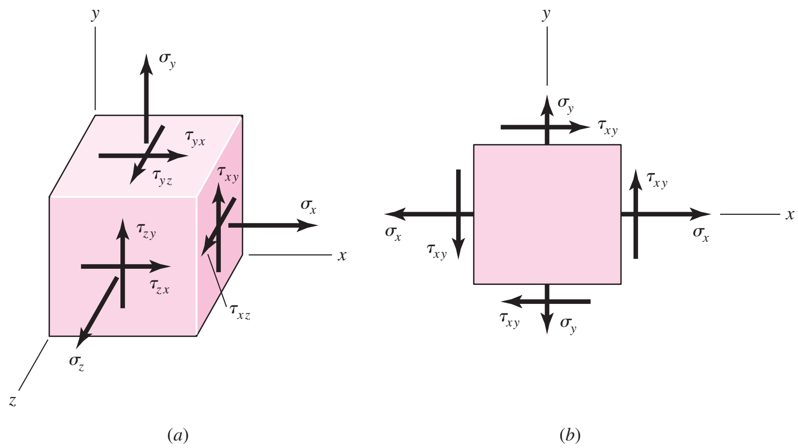


Figure 3–8

(*a*) General three-dimensional stress. (*b*) Plane stress with "cross-shears" equal.

dimensions of the cube in Fig. 3–8*a* approach zero, the stresses on the hidden faces become equal and opposite to those on the opposing visible faces. Thus, in general, a complete state of stress is defined by nine stress components, σ_x, σ_y, σ_z, τ_{xy}, τ_{xz}, τ_{yx}, τ_{yz}, τ_{zx}, and τ_{zy}.

For equilibrium, in most cases, "cross-shears" are equal, hence

$$\tau_{yx} = \tau_{xy} \qquad \tau_{zy} = \tau_{yz} \qquad \tau_{xz} = \tau_{zx} \tag{3–7}$$

This reduces the number of stress components for most three-dimensional states of stress from nine to six quantities, σ_x, σ_y, σ_z, τ_{xy}, τ_{yz}, and τ_{zx}.

A very common state of stress occurs when the stresses on one surface are zero. When this occurs the state of stress is called *plane stress*. Figure 3–8*b* shows a state of plane stress, arbitrarily assuming that the normal for the stress-free surface is the z direction such that $\sigma_z = \tau_{zx} = \tau_{zy} = 0$. It is important to note that the element in Fig. 3–8*b* is still a three-dimensional cube. Also, here it is assumed that the cross-shears are equal such that $\tau_{yx} = \tau_{xy}$, and $\tau_{yz} = \tau_{zy} = \tau_{xz} = \tau_{zx} = 0$.

3–6 Mohr's Circle for Plane Stress

Suppose the $dx\,dy\,dz$ element of Fig. 3–8*b* is cut by an oblique plane with a normal n at an arbitrary angle ϕ counterclockwise from the x axis as shown in Fig. 3–9. This section is concerned with the stresses σ and τ that act upon this oblique plane. By summing the forces caused by all the stress components to zero, the stresses σ and τ are found to be

$$\sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\phi + \tau_{xy}\sin 2\phi \tag{3–8}$$

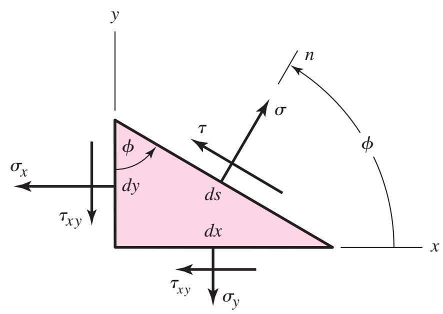
$$\tau = -\frac{\sigma_x - \sigma_y}{2}\sin 2\phi + \tau_{xy}\cos 2\phi \tag{3–9}$$

Equations (3–8) and (3–9) are called the *plane-stress transformation equations*.

Differentiating Eq. (3–8) with respect to ϕ and setting the result equal to zero gives

$$\tan 2\phi_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \tag{3–10}$$

Figure 3–9



Equation (3–10) defines two particular values for the angle $2\phi_p$, one of which defines the maximum normal stress σ_1 and the other, the minimum normal stress σ_2. These two stresses are called the *principal stresses,* and their corresponding directions, the *principal directions.* The angle between the principal directions is 90°. It is important to note that Eq. (3–10) can be written in the form

$$\frac{\sigma_x - \sigma_y}{2} \sin 2\phi_p - \tau_{xy} \cos 2\phi_p = 0 \tag{a}$$

Comparing this with Eq. (3–9), we see that $\tau = 0$, meaning that the *surfaces containing principal stresses have zero shear stresses*.

In a similar manner, we differentiate Eq. (3–9), set the result equal to zero, and obtain

$$\tan 2\phi_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} \tag{3–11}$$

Equation (3–11) defines the two values of $2\phi_s$ at which the shear stress τ reaches an extreme value. The angle between the surfaces containing the maximum shear stresses is 90°. Equation (3–11) can also be written as

$$\frac{\sigma_x - \sigma_y}{2} \cos 2\phi_p + \tau_{xy} \sin 2\phi_p = 0 \tag{b}$$

Substituting this into Eq. (3–8) yields

$$\sigma = \frac{\sigma_x + \sigma_y}{2} \tag{3–12}$$

Equation (3–12) tells us that the two surfaces containing the maximum shear stresses also contain equal normal stresses of $(\sigma_x + \sigma_y)/2$.

Comparing Eqs. (3–10) and (3–11), we see that tan $2\phi_s$ is the negative reciprocal of tan $2\phi_p$. This means that $2\phi_s$ and $2\phi_p$ are angles 90° apart, and thus the angles between the surfaces containing the maximum shear stresses and the surfaces containing the principal stresses are ±45°.

Formulas for the two principal stresses can be obtained by substituting the angle $2\phi_p$ from Eq. (3–10) in Eq. (3–8). The result is

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \tag{3–13}$$

In a similar manner the two extreme-value shear stresses are found to be

$$\tau_1, \tau_2 = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \tag{3–14}$$

Your particular attention is called to the fact that an extreme value of the shear stress *may not be the same as the actual maximum value.* See Sec. 3–7.

It is important to note that the equations given to this point are quite sufficient for performing any plane stress transformation. However, extreme care must be exercised when applying them. For example, say you are attempting to determine the principal state of stress for a problem where $\sigma_x = 14$ MPa, $\sigma_y = -10$ MPa, and $\tau_{xy} = -16$ MPa. Equation (3–10) yields $\phi_p = -26.57°$ and $63.43°$ to locate the principal stress surfaces, whereas, Eq. (3–13) gives $\sigma_1 = 22$ MPa and $\sigma_2 = -18$ MPa for the principal stresses. If all we wanted was the principal stresses, we would be finished. However, what if we wanted to draw the element containing the principal stresses properly oriented relative to the x, y axes? Well, we have two values of ϕ_p and two values for the principal stresses. How do we know which value of ϕ_p corresponds to which value of the principal stress? To clear this up we would need to substitute one of the values of ϕ_p into Eq. (3–8) to determine the normal stress corresponding to that angle.

A graphical method for expressing the relations developed in this section, called *Mohr's circle diagram,* is a very effective means of visualizing the stress state at a point and keeping track of the directions of the various components associated with plane stress. Equations (3–8) and (3–9) can be shown to be a set of parametric equations for σ and τ, where the parameter is 2ϕ. The relationship between σ and τ is that of a circle plotted in the σ, τ plane, where the center of the circle is located at $C = (\sigma, \tau) = [(\sigma_x + \sigma_y)/2, 0]$ and has a radius of $R = \sqrt{[(\sigma_x - \sigma_y)/2]^2 + \tau_{xy}^2}$. A problem arises in the sign of the shear stress. The transformation equations are based on a positive ϕ being counterclockwise, as shown in Fig. 3–9. If a positive τ were plotted above the σ axis, points would rotate clockwise on the circle 2ϕ in the opposite direction of rotation on the element. It would be convenient if the rotations were in the same direction. One could solve the problem easily by plotting positive τ below the axis. However, the classical approach to Mohr's circle uses a different convention for the shear stress.

Mohr's Circle Shear Convention

This convention is followed in drawing Mohr's circle:

- Shear stresses tending to rotate the element clockwise (cw) are plotted *above* the σ axis.
- Shear stresses tending to rotate the element counterclockwise (ccw) are plotted *below* the σ axis.

For example, consider the right face of the element in Fig. 3–8*b*. By Mohr's circle convention the shear stress shown is plotted *below* the σ axis because it tends to rotate the element counterclockwise. The shear stress on the top face of the element is plotted *above* the σ axis because it tends to rotate the element clockwise.

In Fig. 3–10 we create a coordinate system with normal stresses plotted along the abscissa and shear stresses plotted as the ordinates. On the abscissa, tensile (positive) normal stresses are plotted to the right of the origin O and compressive (negative) normal stresses to the left. On the ordinate, clockwise (cw) shear stresses are plotted up; counterclockwise (ccw) shear stresses are plotted down.

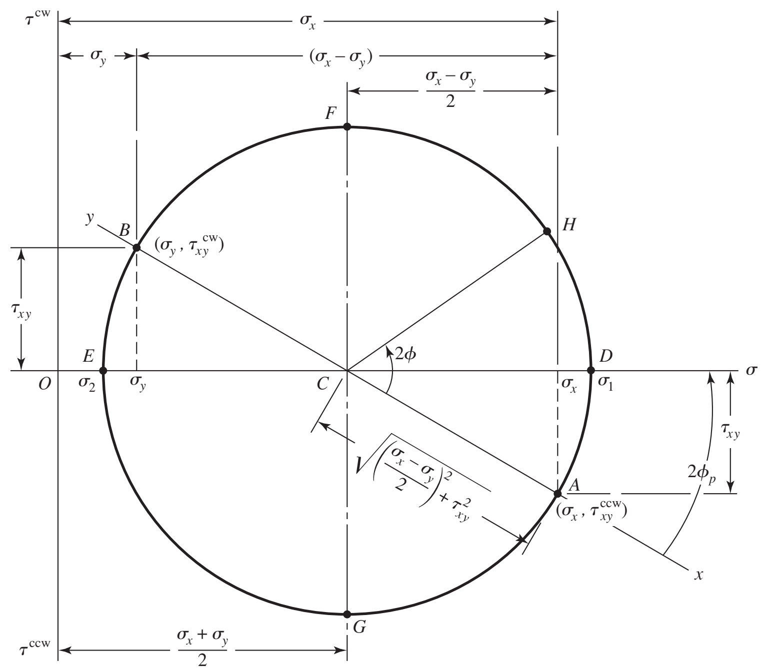


Figure 3–10

Mohr's circle diagram.

Using the stress state of Fig. 3–8*b*, we plot Mohr's circle, Fig. 3–10, by first looking at the right surface of the element containing σ_x to establish the sign of σ_x and the cw or ccw direction of the shear stress. The right face is called the *x face* where $\phi = 0°$. If σ_x is positive and the shear stress τ_{xy} is ccw as shown in Fig. 3–8*b*, we can establish point *A* with coordinates $(\sigma_x, \tau_{xy}^{ccw})$ in Fig. 3–10. Next, we look at the top *y face,* where $\phi = 90°$, which contains σ_y, and repeat the process to obtain point *B* with coordinates $(\sigma_y, \tau_{xy}^{cw})$ as shown in Fig. 3–10. The two states of stress for the element are $\Delta\phi = 90°$ from each other on the element so they will be $2\Delta\phi = 180°$ from each other on Mohr's circle. Points *A* and *B* are the same vertical distance from the σ axis. Thus, AB must be on the diameter of the circle, and the center of the circle *C* is where AB intersects the σ axis. With points *A* and *B* on the circle, and center *C*, the complete circle can then be drawn. Note that the extended ends of line AB are labeled *x* and *y* as references to the normals to the surfaces for which points *A* and *B* represent the stresses.

The entire Mohr's circle represents the state of stress at a *single* point in a structure. Each point on the circle represents the stress state for a *specific* surface intersecting the point in the structure. Each pair of points on the circle 180° apart represent the state of stress on an element whose surfaces are 90° apart. Once the circle is drawn, the states of stress can be visualized for various surfaces intersecting the point being analyzed. For example, the principal stresses σ_1 and σ_2 are points *D* and *E*, respectively, and their values obviously agree with Eq. (3–13). We also see that the shear stresses are zero on the surfaces containing σ_1 and σ_2. The two extreme-value shear stresses, one clockwise and one counterclockwise, occur at *F* and *G* with magnitudes equal to the radius of the circle. The surfaces at *F* and *G* each also contain normal stresses of $(\sigma_x + \sigma_y)/2$ as noted earlier in Eq. (3–12). Finally, the state of stress on an arbitrary surface located at an angle ϕ counterclockwise from the *x* face is point *H*.

At one time, Mohr's circle was used graphically where it was drawn to scale very accurately and values were measured by using a scale and protractor. Here, we are strictly using Mohr's circle as a visualization aid and will use a semigraphical approach, calculating values from the properties of the circle. This is illustrated by the following example.

EXAMPLE 3–4

A stress element has $\sigma_x = 80$ MPa and $\tau_{xy} = 50$ MPa cw, as shown in Fig. 3–11*a*.

(*a*) Using Mohr's circle, find the principal stresses and directions, and show these on a stress element correctly aligned with respect to the xy coordinates. Draw another stress element to show τ_1 and τ_2, find the corresponding normal stresses, and label the drawing completely.

(*b*) Repeat part *a* using the transformation equations only.

Solution

(*a*) In the semigraphical approach used here, we first make an approximate freehand sketch of Mohr's circle and then use the geometry of the figure to obtain the desired information.

Draw the σ and τ axes first (Fig. 3–11*b*) and from the x face locate $\sigma_x = 80$ MPa along the σ axis. On the x face of the element, we see that the shear stress is 50 MPa in the cw direction. Thus, for the x face, this establishes point A ($80, 50^{\text{cw}}$) MPa. Corresponding to the y face, the stress is $\sigma = 0$ and $\tau = 50$ MPa in the ccw direction. This locates point B ($0, 50^{\text{ccw}}$) MPa. The line AB forms the diameter of the required circle, which can now be drawn. The intersection of the circle with the σ axis defines σ_1 and σ_2 as shown. Now, noting the triangle ACD, indicate on the sketch the length of the legs AD and CD as 50 and 40 MPa, respectively. The length of the hypotenuse AC is

Answer
$$\tau_1 = \sqrt{(50)^2 + (40)^2} = 64.0 \text{ MPa}$$

and this should be labeled on the sketch too. Since intersection C is 40 MPa from the origin, the principal stresses are now found to be

Answer
$$\sigma_1 = 40 + 64 = 104 \text{ MPa} \qquad \text{and} \qquad \sigma_2 = 40 - 64 = -24 \text{ MPa}$$

The angle 2ϕ from the x axis *cw* to σ_1 is

Answer
$$2\phi_p = \tan^{-1} \tfrac{50}{40} = 51.3^\circ$$

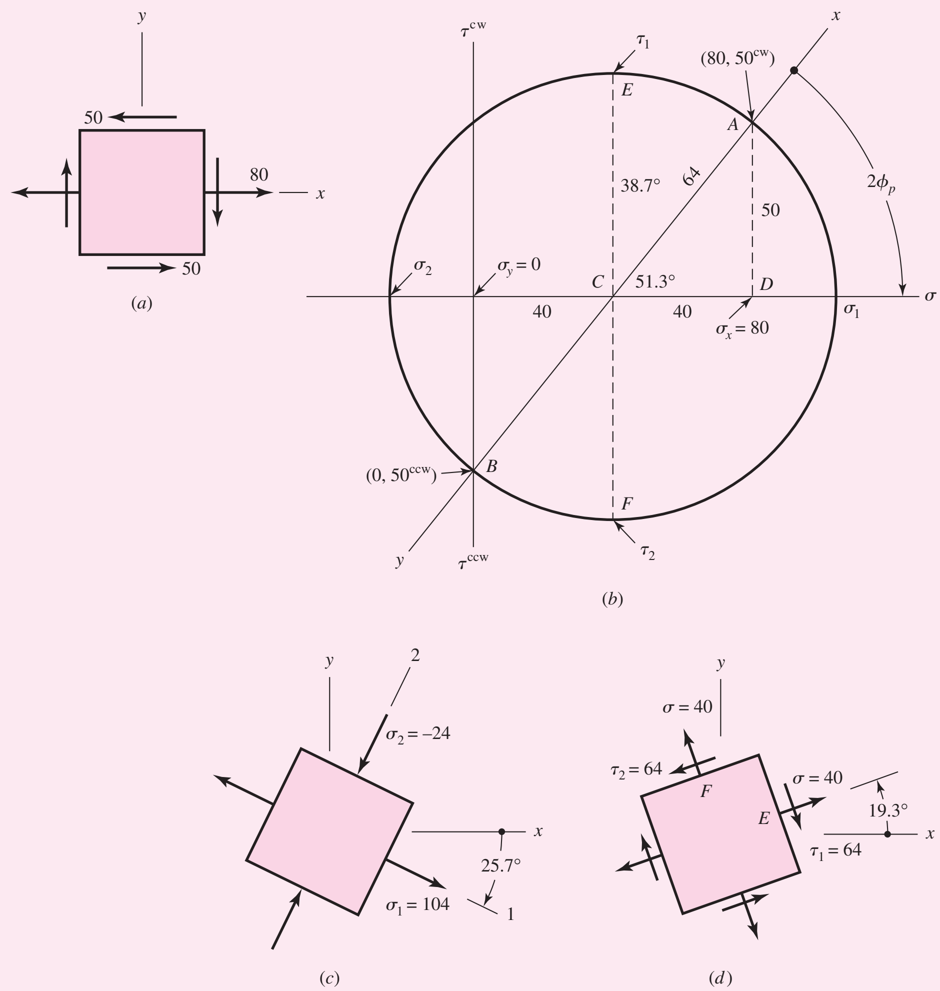
To draw the principal stress element (Fig. 3–11*c*), sketch the x and y axes parallel to the original axes. The angle ϕ_p on the stress element must be measured in the *same* direction as is the angle $2\phi_p$ on the Mohr circle. Thus, from x measure 25.7° (half of 51.3°) clockwise to locate the σ_1 axis. The σ_2 axis is 90° from the σ_1 axis and the stress element can now be completed and labeled as shown. Note that there are *no* shear stresses on this element.

The two maximum shear stresses occur at points E and F in Fig. 3–11*b*. The two normal stresses corresponding to these shear stresses are each 40 MPa, as indicated. Point E is 38.7° ccw from point A on Mohr's circle. Therefore, in Fig. 3–11*d*, draw a stress element oriented 19.3° (half of 38.7°) ccw from x. The element should then be labeled with magnitudes and directions as shown.

In constructing these stress elements it is important to indicate the x and y directions of the original reference system. This completes the link between the original machine element and the orientation of its principal stresses.

Figure 3–11

All stresses in MPa.



Answer

(*b*) The transformation equations are programmable. From Eq. (3–10),

$$\phi_p = \frac{1}{2}\tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) = \frac{1}{2}\tan^{-1}\left(\frac{2(-50)}{80}\right) = -25.7^\circ, 64.3^\circ$$

From Eq. (3–8), for the first angle $\phi_p = -25.7^\circ$,

$$\sigma = \frac{80+0}{2} + \frac{80-0}{2}\cos[2(-25.7)] + (-50)\sin[2(-25.7)] = 104.03 \text{ MPa}$$

The shear on this surface is obtained from Eq. (3–9) as

$$\tau = -\frac{80-0}{2}\sin[2(-25.7)] + (-50)\cos[2(-25.7)] = 0 \text{ MPa}$$

which confirms that 104.03 MPa is a principal stress. From Eq. (3–8), for $\phi_p = 64.3^\circ$,

$$\sigma = \frac{80+0}{2} + \frac{80-0}{2}\cos[2(64.3)] + (-50)\sin[2(64.3)] = -24.03 \text{ MPa}$$

Answer

Substituting $\phi_p = 64.3°$ into Eq. (3–9) again yields $\tau = 0$, indicating that -24.03 MPa is also a principal stress. Once the principal stresses are calculated they can be ordered such that $\sigma_1 \geq \sigma_2$. Thus, $\sigma_1 = 104.03$ MPa and $\sigma_2 = -24.03$ MPa.

Since for $\sigma_1 = 104.03$ MPa, $\phi_p = -25.7°$, and since ϕ is defined positive ccw in the transformation equations, we rotate *clockwise* 25.7° for the surface containing σ_1. We see in Fig. 3–11*c* that this totally agrees with the semigraphical method.

To determine τ_1 and τ_2, we first use Eq. (3–11) to calculate ϕ_s:

$$\phi_s = \frac{1}{2}\tan^{-1}\left(-\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right) = \frac{1}{2}\tan^{-1}\left(-\frac{80}{2(-50)}\right) = 19.3°, 109.3°$$

For $\phi_s = 19.3°$, Eqs. (3–8) and (3–9) yield

Answer

$$\sigma = \frac{80+0}{2} + \frac{80-0}{2}\cos[2(19.3)] + (-50)\sin[2(19.3)] = 40.0 \text{ MPa}$$

$$\tau = -\frac{80-0}{2}\sin[2(19.3)] + (-50)\cos[2(19.3)] = -64.0 \text{ MPa}$$

Remember that Eqs. (3–8) and (3–9) are *coordinate* transformation equations. Imagine that we are rotating the x, y axes 19.3° counterclockwise and y will now point up and to the left. So a negative shear stress on the rotated x face will point down and to the right as shown in Fig. 3–11*d*. Thus again, results agree with the semigraphical method.

For $\phi_s = 109.3°$, Eqs. (3–8) and (3–9) give $\sigma = 40.0$ MPa and $\tau = +64.0$ MPa. Using the same logic for the coordinate transformation we find that results again agree with Fig. 3–11*d*.

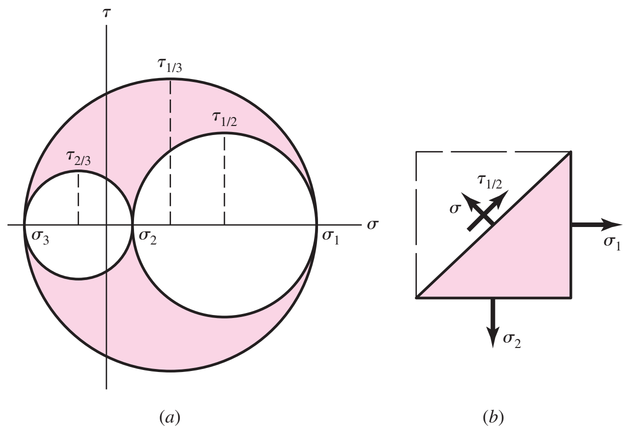
3–7 General Three-Dimensional Stress

As in the case of plane stress, a particular orientation of a stress element occurs in space for which all shear-stress components are zero. When an element has this particular orientation, the normals to the faces are mutually orthogonal and correspond to the principal directions, and the normal stresses associated with these faces are the principal stresses. Since there are three faces, there are three principal directions and three principal stresses σ_1, σ_2, and σ_3. For plane stress, the stress-free surface contains the third principal stress which is zero.

In our studies of plane stress we were able to specify any stress state σ_x, σ_y, and τ_{xy} and find the principal stresses and principal directions. But six components of stress are required to specify a general state of stress in three dimensions, and the problem of determining the principal stresses and directions is more difficult. In design, three-dimensional transformations are rarely performed since most maximum stress states occur under plane stress conditions. One notable exception is contact stress, which is not a case of plane stress, where the three principal stresses are given in Sec. 3–19. In fact, *all* states of stress are truly three-dimensional, where they might be described one- or two-dimensionally with respect to *specific* coordinate axes. Here it is most important to understand the relationship amongst the *three* principal stresses. The process in finding the three principal stresses from the six

Figure 3–12

Mohr's circles for three-dimensional stress.



stress components σ_x, σ_y, σ_z, τ_{xy}, τ_{yz}, and τ_{zx}, involves finding the roots of the cubic equation[1]

$$\sigma^3 - (\sigma_x + \sigma_y + \sigma_z)\sigma^2 + \left(\sigma_x\sigma_y + \sigma_x\sigma_z + \sigma_y\sigma_z - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2\right)\sigma$$
$$- \left(\sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{yz}\tau_{zx} - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{zx}^2 - \sigma_z\tau_{xy}^2\right) = 0 \quad (3\text{–}15)$$

In plotting Mohr's circles for three-dimensional stress, the principal normal stresses are ordered so that $\sigma_1 \geq \sigma_2 \geq \sigma_3$. Then the result appears as in Fig. 3–12*a*. The stress coordinates σ, τ for any arbitrarily located plane will always lie on the boundaries or within the shaded area.

Figure 3–12*a* also shows the three *principal shear stresses* $\tau_{1/2}$, $\tau_{2/3}$, and $\tau_{1/3}$.[2] Each of these occurs on the two planes, one of which is shown in Fig. 3–12*b*. The figure shows that the principal shear stresses are given by the equations

$$\tau_{1/2} = \frac{\sigma_1 - \sigma_2}{2} \qquad \tau_{2/3} = \frac{\sigma_2 - \sigma_3}{2} \qquad \tau_{1/3} = \frac{\sigma_1 - \sigma_3}{2} \quad (3\text{–}16)$$

Of course, $\tau_{max} = \tau_{1/3}$ when the normal principal stresses are ordered ($\sigma_1 > \sigma_2 > \sigma_3$), so always order your principal stresses. Do this in any computer code you generate and you'll always generate τ_{max}.

3–8 Elastic Strain

Normal strain ϵ is defined and discussed in Sec. 2-1 for the tensile specimen and is given by Eq. (2–2) as $\epsilon = \delta/l$, where δ is the total elongation of the bar within the length l. Hooke's law for the tensile specimen is given by Eq. (2–3) as

$$\sigma = E\epsilon \quad (3\text{–}17)$$

where the constant E is called *Young's modulus* or the *modulus of elasticity*.

[1]For development of this equation and further elaboration of three-dimensional stress transformations see: Richard G. Budynas, *Advanced Strength and Applied Stress Analysis,* 2nd ed., McGraw-Hill, New York, 1999, pp. 46–78.

[2]Note the difference between this notation and that for a shear stress, say, τ_{xy}. The use of the shilling mark is not accepted practice, but it is used here to emphasize the distinction.

When a material is placed in tension, there exists not only an axial strain, but also negative strain (contraction) perpendicular to the axial strain. Assuming a linear, homogeneous, isotropic material, this lateral strain is proportional to the axial strain. If the axial direction is x, then the lateral strains are $\epsilon_y = \epsilon_z = -\nu\epsilon_x$. The constant of proportionality ν is called *Poisson's ratio*, which is about 0.3 for most structural metals. See Table A–5 for values of ν for common materials.

If the axial stress is in the x direction, then from Eq. (3–17)

$$\epsilon_x = \frac{\sigma_x}{E} \qquad \epsilon_y = \epsilon_z = -\nu\frac{\sigma_x}{E} \tag{3–18}$$

For a stress element undergoing σ_x, σ_y, and σ_z simultaneously, the normal strains are given by

$$\begin{aligned} \epsilon_x &= \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right] \\ \epsilon_y &= \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right] \\ \epsilon_z &= \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right] \end{aligned} \tag{3–19}$$

Shear strain γ is the change in a right angle of a stress element when subjected to pure shear stress, and Hooke's law for shear is given by

$$\tau = G\gamma \tag{3–20}$$

where the constant G is the *shear modulus of elasticity* or *modulus of rigidity*.

It can be shown for a linear, isotropic, homogeneous material, the three elastic constants are related to each other by

$$E = 2G(1 + \nu) \tag{3–21}$$

3–9 Uniformly Distributed Stresses

The assumption of a uniform distribution of stress is frequently made in design. The result is then often called *pure tension, pure compression,* or *pure shear,* depending upon how the external load is applied to the body under study. The word *simple* is sometimes used instead of *pure* to indicate that there are no other complicating effects. The tension rod is typical. Here a tension load F is applied through pins at the ends of the bar. The assumption of uniform stress means that if we cut the bar at a section remote from the ends and remove one piece, we can replace its effect by applying a uniformly distributed force of magnitude σA to the cut end. So the stress σ is said to be uniformly distributed. It is calculated from the equation

$$\sigma = \frac{F}{A} \tag{3–22}$$

This assumption of uniform stress distribution requires that:

- The bar be straight and of a homogeneous material
- The line of action of the force contains the centroid of the section
- The section be taken remote from the ends and from any discontinuity or abrupt change in cross section

For simple compression, Eq. (3–22) is applicable with F normally being considered a negative quantity. Also, a slender bar in compression may fail by buckling, and this possibility must be eliminated from consideration before Eq. (3–22) is used.[3]

Use of the equation

$$\tau = \frac{F}{A} \tag{3–23}$$

for a body, say, a bolt, in shear assumes a uniform stress distribution too. It is very difficult in practice to obtain a uniform distribution of shear stress. The equation is included because occasions do arise in which this assumption is utilized.

3–10 Normal Stresses for Beams in Bending

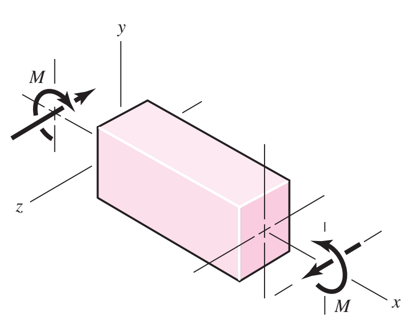
The equations for the normal bending stresses in straight beams are based on the following assumptions:

1. The beam is subjected to pure bending. This means that the shear force is zero, and that no torsion or axial loads are present.
2. The material is isotropic and homogeneous.
3. The material obeys Hooke's law.
4. The beam is initially straight with a cross section that is constant throughout the beam length.
5. The beam has an axis of symmetry in the plane of bending.
6. The proportions of the beam are such that it would fail by bending rather than by crushing, wrinkling, or sidewise buckling.
7. Plane cross sections of the beam remain plane during bending.

In Fig. 3–13 we visualize a portion of a straight beam acted upon by a positive bending moment M shown by the curved arrow showing the physical action of the moment together with a straight arrow indicating the moment vector. The x axis is coincident with the *neutral axis* of the section, and the xz plane, which contains the neutral axes of all cross sections, is called the *neutral plane*. Elements of the beam coincident with this plane have zero stress. The location of the neutral axis with respect to the cross section is coincident with the *centroidal axis* of the cross section.

Figure 3–13

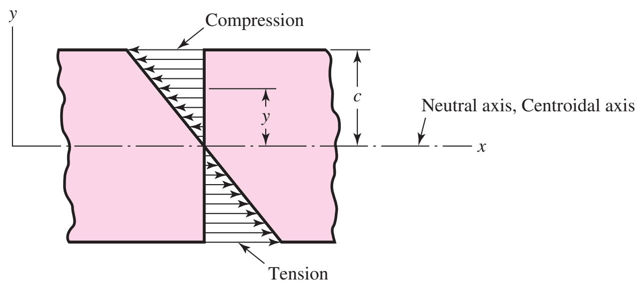
Straight beam in positive bending.



[3]See Sec. 4–11.

Figure 3–14

Bending stresses according to Eq. (3–24).



The bending stress varies linearly with the distance from the neutral axis, y, and is given by

$$\sigma_x = -\frac{My}{I} \tag{3–24}$$

where I is the second *moment of area* about the z axis. That is

$$I = \int y^2 dA \tag{3–25}$$

The stress distribution given by Eq. (3–24) is shown in Fig. 3–14. The maximum magnitude of the bending stress will occur where y has the greatest magnitude. Designating σ_{max} as the maximum *magnitude* of the bending stress, and c as the maximum *magnitude* of y

$$\sigma_{max} = \frac{Mc}{I} \tag{3–26a}$$

Equation (3–24) can still be used to ascertain as to whether σ_{max} is tensile or compressive.

Equation (3–26*a*) is often written as

$$\sigma_{max} = \frac{M}{Z} \tag{3–26b}$$

where $Z = I/c$ is called the *section modulus*.

EXAMPLE 3–5

A beam having a T section with the dimensions shown in Fig. 3–15 is subjected to a bending moment of 1600 N · m that causes tension at the top surface. Locate the neutral axis and find the maximum tensile and compressive bending stresses.

Solution

The area of the composite section is $A = 1956\ \text{mm}^2$. Now divide the T section into two rectangles, numbered 1 and 2, and sum the moments of these areas about the top edge. We then have

$$1956c_1 = 12(75)(6) + 12(88)(56)$$

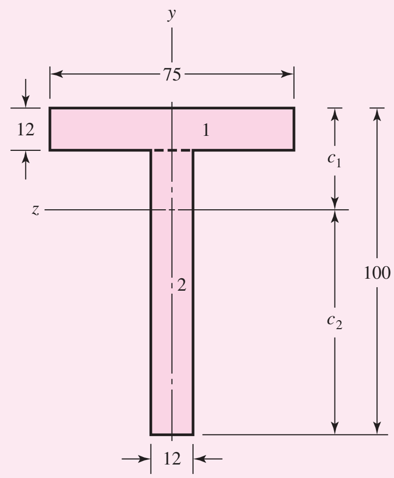
and hence $c_1 = 32.99$ mm. Therefore $c_2 = 100 - 32.99 = 67.01$ mm.

Next we calculate the second moment of area of each rectangle about its own centroidal axis. Using Table A-18, we find for the top rectangle

$$I_1 = \frac{1}{12}bh^3 = \frac{1}{12}(75)12^3 = 1.080 \times 10^4\ \text{mm}^4$$

Figure 3–15

Dimensions in millimeters.



For the bottom rectangle, we have

$$I_2 = \frac{1}{12}(12)88^3 = 6.815 \times 10^5 \text{ mm}^4$$

We now employ the *parallel-axis theorem* to obtain the second moment of area of the composite figure about its own centroidal axis. This theorem states

$$I_z = I_{cg} + Ad^2$$

where I_{cg} is the second moment of area about its own centroidal axis and I_z is the second moment of area about any parallel axis a distance d removed. For the top rectangle, the distance is

$$d_1 = 32.99 - 6 = 26.99 \text{ mm}$$

and for the bottom rectangle,

$$d_2 = 67.01 - 44 = 23.01 \text{ mm}$$

Using the parallel-axis theorem for both rectangles, we now find that

$$\begin{aligned} I &= [1.080 \times 10^4 + 12(75)26.99^2] + [6.815 \times 10^5 + 12(88)23.01^2] \\ &= 1.907 \times 10^6 \text{ mm}^4 \end{aligned}$$

Finally, the maximum tensile stress, which occurs at the top surface, is found to be

Answer
$$\sigma = \frac{Mc_1}{I} = \frac{1600(32.99)10^{-3}}{1.907(10^{-6})} = 27.68(10^6) \text{ Pa} = 27.68 \text{ MPa}$$

Similarly, the maximum compressive stress at the lower surface is found to be

Answer
$$\sigma = -\frac{Mc_2}{I} = -\frac{1600(67.01)10^{-3}}{1.907(10^{-6})} = -56.22(10^6) \text{ Pa} = -56.22 \text{ MPa}$$

Two-Plane Bending

Quite often, in mechanical design, bending occurs in both xy and xz planes. Considering cross sections with one or two planes of symmetry only, the bending stresses are given by

$$\sigma_x = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y} \tag{3–27}$$

where the first term on the right side of the equation is identical to Eq. (3–24), M_y is the bending moment in the xz plane (moment vector in y direction), z is the distance from the neutral y axis, and I_y is the second area moment about the y axis.

For *noncircular* cross sections, Eq. (3–27) is the superposition of stresses caused by the two bending moment components. The maximum tensile and compressive bending stresses occur where the summation gives the greatest positive and negative stresses, respectively. For solid *circular* cross sections, all lateral axes are the same and the plane containing the moment corresponding to the vector sum of M_z and M_y contains the maximum bending stresses. For a beam of diameter d the maximum distance from the neutral axis is $d/2$, and from Table A–18, $I = \pi d^4/64$. The maximum bending stress for a solid circular cross section is then

$$\sigma_m = \frac{Mc}{I} = \frac{(M_y^2 + M_z^2)^{1/2}(d/2)}{\pi d^4/64} = \frac{32}{\pi d^3}(M_y^2 + M_z^2)^{1/2} \tag{3–28}$$

EXAMPLE 3–6 As shown in Fig. 3–16a, beam OC is loaded in the xy plane by a uniform load of 50 lbf/in, and in the xz plane by a concentrated force of 100 lbf at end C. The beam is 8 in long.

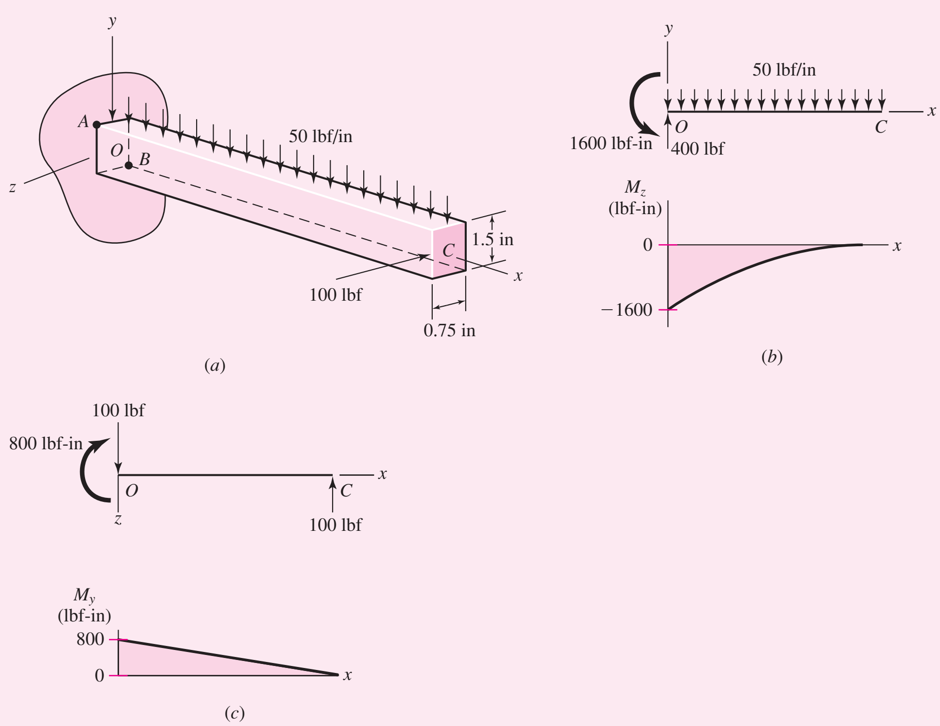


Figure 3–16

(a) Beam loaded in two planes; (b) loading and bending-moment diagrams in xy plane; (c) loading and bending-moment diagrams in xz plane.

(*a*) For the cross section shown determine the maximum tensile and compressive bending stresses and where they act.

(*b*) If the cross section was a solid circular rod of diameter, $d = 1.25$ in, determine the magnitude of the maximum bending stress.

Solution (*a*) The reactions at O and the bending-moment diagrams in the xy and xz planes are shown in Figs. 3–16*b* and *c*, respectively. The maximum moments in both planes occur at O where

$$(M_z)_O = -\frac{1}{2}(50)8^2 = -1600\,\text{lbf-in} \qquad (M_y)_O = 100(8) = 800\,\text{lbf-in}$$

The second moments of area in both planes are

$$I_z = \frac{1}{12}(0.75)1.5^3 = 0.2109\,\text{in}^4 \qquad I_y = \frac{1}{12}(1.5)0.75^3 = 0.05273\,\text{in}^4$$

The maximum tensile stress occurs at point A, shown in Fig. 3–16*a*, where the maximum tensile stress is due to both moments. At A, $y_A = 0.75$ in and $z_A = 0.375$ in. Thus, from Eq. (3–27)

Answer
$$(\sigma_x)_A = -\frac{-1600(0.75)}{0.2109} + \frac{800(0.375)}{0.05273} = 11\,380\,\text{psi} = 11.38\,\text{kpsi}$$

The maximum compressive bending stress occurs at point B where, $y_B = -0.75$ in and $z_B = -0.375$ in. Thus

Answer
$$(\sigma_x)_B = -\frac{-1600(-0.75)}{0.2109} + \frac{800(-0.375)}{0.05273} = -11\,380\,\text{psi} = -11.38\,\text{kpsi}$$

(*b*) For a solid circular cross section of diameter, $d = 1.25$ in, the maximum bending stress at end O is given by Eq. (3–28) as

Answer
$$\sigma_m = \frac{32}{\pi(1.25)^3}\left[800^2 + (-1600)^2\right]^{1/2} = 9326\,\text{psi} = 9.329\,\text{kpsi}$$

Beams with Asymmetrical Sections

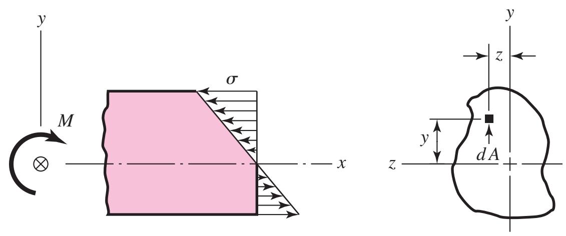
The relations developed earlier in this section can also be applied to beams having asymmetrical sections, provided that the plane of bending coincides with one of the two principal axes of the section. We have found that the stress at a distance y from the neutral axis is

$$\sigma = -\frac{My}{I} \tag{a}$$

Therefore, the force on the element of area dA in Fig. 3–17 is

$$dF = \sigma\,dA = -\frac{My}{I}\,dA$$

Figure 3–17



Taking moments of this force about the y axis and integrating across the section gives

$$M_y = \int z\,dF = \int \sigma z\,dA = -\frac{M}{I}\int yz\,dA \qquad (b)$$

We recognize that the last integral in Eq. (b) is the product of inertia I_{yz}. If the bending moment on the beam is in the plane of one of the principal axes, say the xy plane, then

$$I_{yz} = \int yz\,dA = 0 \qquad (c)$$

With this restriction, the relations developed in Sec. 3–10 hold for any cross-sectional shape. Of course, this means that the designer has a special responsibility to ensure that the bending loads do, in fact, come onto the beam in a principal plane!

3–11 Shear Stresses for Beams in Bending

Most beams have both shear forces and bending moments present. It is only occasionally that we encounter beams subjected to pure bending, that is to say, beams having zero shear force. The flexure formula is developed on the assumption of pure bending. This is done, however, to eliminate the complicating effects of shear force in the development. For engineering purposes, the flexure formula is valid no matter whether a shear force is present or not. For this reason, we shall utilize the same normal bending-stress distribution [Eqs. (3–24) and (3–26)] when shear forces are also present.

In Fig. 3–18a we show a beam segment of constant cross section subjected to a shear force V and a bending moment M at x. Because of external loading and V, the shear force and bending moment change with respect to x. At $x + dx$ the shear force and bending moment are $V + dV$ and $M + dM$, respectively. Considering forces in the x direction only, Fig. 3–18b shows the stress distribution σ_x due to the bending moments. If dM is positive, with the bending moment increasing, the stresses on the right face, for a given value of y, are larger in magnitude than the stresses on the left face. If we further isolate the element by making a slice at $y = y_1$ (see Fig. 3–18b), the net *force* in the x direction will be directed to the left with a value of

$$\int_{y_1}^{c} \frac{(dM)y}{I}\,dA$$

as shown in the rotated view of Fig. 3–18c. For equilibrium, a shear force on the bottom face, directed to the right, is required. This shear force gives rise to a shear stress τ, where, if assumed uniform, the force is $\tau b\,dx$. Thus

$$\tau b\,dx = \int_{y_1}^{c} \frac{(dM)y}{I}\,dA \qquad (a)$$

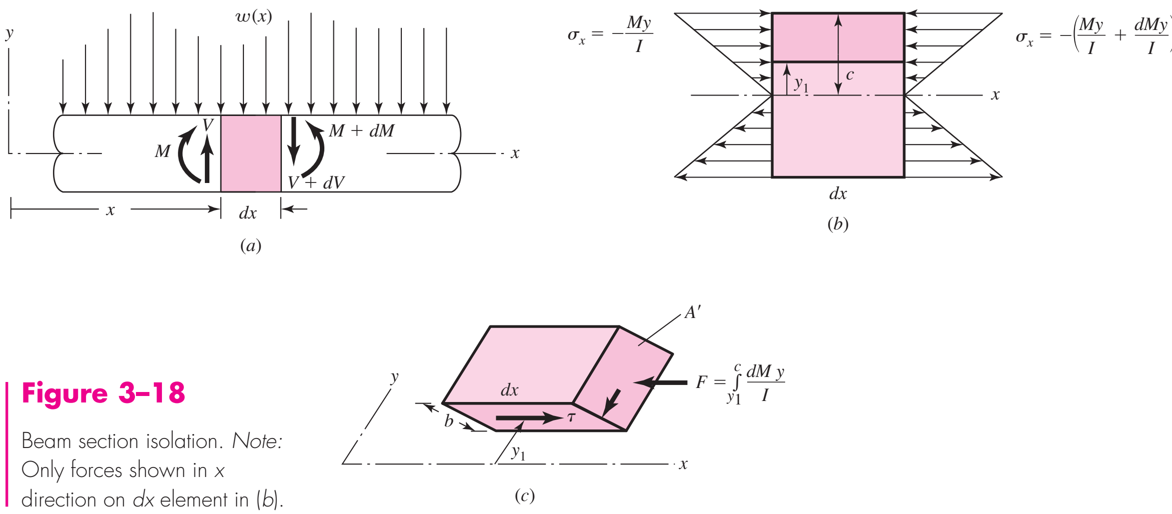


Figure 3–18

Beam section isolation. *Note:* Only forces shown in *x* direction on *dx* element in (*b*).

The term *dM/I* can be removed from within the integral and *b dx* placed on the right side of the equation; then, from Eq. (3–3) with $V = dM/dx$, Eq. (*a*) becomes

$$\tau = \frac{V}{Ib}\int_{y_1}^{c} y\,dA \tag{3–29}$$

In this equation, the integral is the first moment of the area A' with respect to the neutral axis (see Fig. 3–18c). This integral is usually designated as Q. Thus

$$Q = \int_{y_1}^{c} y\,dA = \bar{y}'A' \tag{3–30}$$

where, for the isolated area y_1 to c, $\bar{y}'$ is the distance in the y direction from the neutral plane to the centroid of the area A'. With this, Eq. (3–29) can be written as

$$\tau = \frac{VQ}{Ib} \tag{3–31}$$

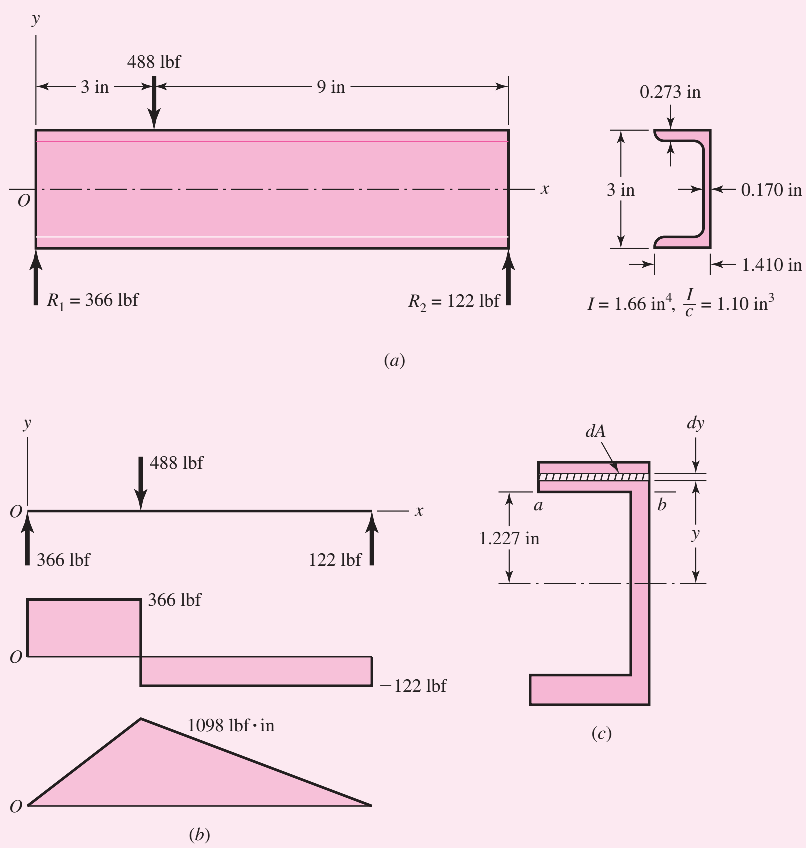
In using this equation, note that b is the width of the section at $y = y_1$. Also, I is the second moment of area of the entire section about the neutral axis.

Because cross shears are equal, and area A' is *finite*, the shear stress τ given by Eq. (3–31) and shown on area A' in Fig. 3–18*c* occurs only at $y = y_1$. The shear stress on the lateral area varies with y (normally maximum at the neutral axis where $y = 0$, and zero at the outer fibers of the beam where $Q = A' = 0$).

EXAMPLE 3–7

A beam 12 in long is to support a load of 488 lbf acting 3 in from the left support, as shown in Fig. 3–19*a*. Basing the design only on bending stress, a designer has selected a 3-in aluminum channel with the cross-sectional dimensions shown. If the direct shear is neglected, the stress in the beam may be actually higher than the designer thinks. Determine the principal stresses considering bending and direct shear and compare them with that considering bending only.

Figure 3–19



Solution The loading, shear-force, and bending-moment diagrams are shown in Fig. 3–19*b*. If the direct shear force is included in the analysis, the maximum stresses at the top and bottom of the beam will be the same as if only bending were considered. The maximum bending stresses are

$$\sigma = \pm\frac{Mc}{I} = \pm\frac{1098(1.5)}{1.66} = \pm 992 \text{ psi}$$

However, the maximum stress due to the combined bending and direct shear stresses may be maximum at the point $(3^-, 1.227)$ that is just to the left of the applied load, where the web joins the flange. To simplify the calculations we assume a cross section with square corners (Fig. 3–19*c*). The normal stress at section *ab*, with $x = 3$ in, is

$$\sigma = -\frac{My}{I} = -\frac{1098(1.227)}{1.66} = -812 \text{ psi}$$

For the shear stress at section *ab*, considering the area above *ab* and using Eq. (3–30) gives

$$Q = \bar{y}'A' = \left(1.227 + \frac{0.273}{2}\right)(1.410)(0.273) = 0.525 \text{ in}^3$$

Using Eq. (3–31) with $V = 366$ lbf, $I = 1.66\ \text{in}^4$, $Q = 0.525\ \text{in}^3$, and $b = 0.170$ in yields

$$\tau_{xy} = -\frac{VQ}{Ib} = -\frac{366(0.525)}{1.66(0.170)} = -681\ \text{psi}$$

The negative sign comes from recognizing that the shear stress is down on an x face of a $dx\ dy$ element at the location being considered.

The principal stresses at the point can now be determined. Using Eq. (3–13), we find that at $x = 3^-$ in, $y = 1.227$ in,

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-812 + 0}{2} \pm \sqrt{\left(\frac{-812 - 0}{2}\right)^2 + (-681)^2} = 387, -1200\ \text{psi}$$

For a point at $x = 3^-$ in, $y = -1.227$ in, the principal stresses are $\sigma_1, \sigma_2 = 1200$, -387 psi. Thus we see that the maximum principal stresses are ± 1200 psi, 21 percent higher than thought by the designer.

Shear Stresses in Standard-Section Beams

The shear stress distribution in a beam depends on how Q/b varies as a function of y_1. Here we will show how to determine the shear stress distribution for a beam with a rectangular cross section and provide results of maximum values of shear stress for other standard cross sections. Figure 3–20 shows a portion of a beam with a rectangular cross section, subjected to a shear force V and a bending moment M. As a result of the bending moment, a normal stress σ is developed on a cross section such as *A-A*, which is in compression above the neutral axis and in tension below. To investigate the shear stress at a distance y_1 above the neutral axis, we select an element of area dA at a distance y above the neutral axis. Then, $dA = b\,dy$, and so Eq. (3–30) becomes

$$Q = \int_{y_1}^{c} y\,dA = b\int_{y_1}^{c} y\,dy = \left.\frac{by^2}{2}\right|_{y_1}^{c} = \frac{b}{2}\left(c^2 - y_1^2\right) \tag{a}$$

Substituting this value for Q into Eq. (3–31) gives

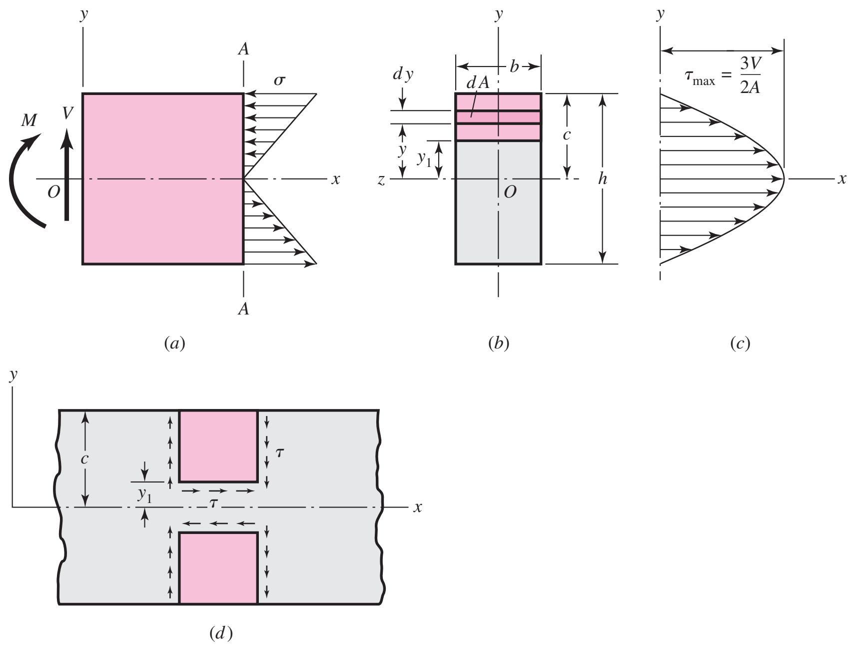
$$\tau = \frac{V}{2I}\left(c^2 - y_1^2\right) \tag{3–32}$$

This is the general equation for shear stress in a rectangular beam. To learn something about it, let us make some substitutions. From Table A–18, the second moment of area for a rectangular section is $I = bh^3/12$; substituting $h = 2c$ and $A = bh = 2bc$ gives

$$I = \frac{Ac^2}{3} \tag{b}$$

Figure 3–20

Shear stresses in a rectangular beam.



If we now use this value of I for Eq. (3–32) and rearrange, we get

$$\tau = \frac{3V}{2A}\left(1 - \frac{y_1^2}{c^2}\right) \tag{3–33}$$

We note that the maximum shear stress exists when $y_1 = 0$, which is at the bending neutral axis. Thus

$$\tau_{\max} = \frac{3V}{2A} \tag{3–34}$$

for a rectangular section. As we move away from the neutral axis, the shear stress decreases parabolically until it is zero at the outer surfaces where $y_1 = \pm c$, as shown in Fig. 3–20*c*. It is particularly interesting and significant here to observe that the shear stress is maximum at the bending neutral axis, where the normal stress due to bending is zero, and that the shear stress is zero at the outer surfaces, where the bending stress is a maximum. Horizontal shear stress is always accompanied by vertical shear stress of the same magnitude, and so the distribution can be diagrammed as shown in Fig. 3–20*d*. Figure 3–20*c* shows that the shear τ on the vertical surfaces varies with y. We are almost always interested in the horizontal shear, τ in Fig. 3–20*d*, which is nearly uniform with constant y. The maximum horizontal shear occurs where the vertical shear is largest. This is usually at the neutral axis but may not be if the width b is smaller somewhere else. Furthermore, if the section is such that b can be minimized on a plane not horizontal, then the horizontal shear stress occurs on an inclined plane. For example, with tubing, the horizontal shear stress occurs on a radial plane and the corresponding "vertical shear" is not vertical, but tangential.

Formulas for the maximum flexural shear stress for the most commonly used shapes are listed in Table 3–2.

Table 3–2

Formulas for Maximum Shear Stress Due to Bending

Beam Shape	Formula	Beam Shape	Formula
Rectangular	$\tau_{\max} = \frac{3V}{2A}$	Hollow, thin-walled round	$\tau_{\max} = \frac{2V}{A}$
Circular	$\tau_{\max} = \frac{4V}{3A}$	Structural I beam (thin-walled) (Web)	$\tau_{\max} = \frac{V}{A_{\text{web}}}$

3–12 Torsion

Any moment vector that is collinear with an axis of a mechanical element is called a *torque vector,* because the moment causes the element to be twisted about that axis. A bar subjected to such a moment is also said to be in *torsion*.

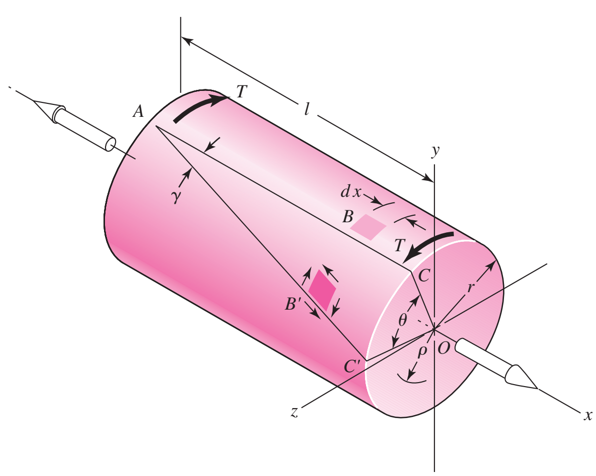
As shown in Fig. 3–21, the torque T applied to a bar can be designated by drawing arrows on the surface of the bar to indicate direction or by drawing torque-vector arrows along the axes of twist of the bar. Torque vectors are the hollow arrows shown on the x axis in Fig. 3–21. Note that they conform to the right-hand rule for vectors.

The *angle of twist,* in radians, for a solid round bar is

$$\theta = \frac{Tl}{GJ} \tag{3–35}$$

where T = torque
l = length
G = modulus of rigidity
J = polar second moment of area

Figure 3–21



Shear stresses develop throughout the cross section. For a round bar in torsion, these stresses are proportional to the radius ρ and are given by

$$\tau = \frac{T\rho}{J} \tag{3–36}$$

Designating r as the radius to the outer surface, we have

$$\tau_{\max} = \frac{Tr}{J} \tag{3–37}$$

The assumptions used in the analysis are:

- The bar is acted upon by a pure torque, and the sections under consideration are remote from the point of application of the load and from a change in diameter.
- Adjacent cross sections originally plane and parallel remain plane and parallel after twisting, and any radial line remains straight.
- The material obeys Hooke's law.

Equation (3–37) applies *only* to circular sections. For a solid round section,

$$J = \frac{\pi d^4}{32} \tag{3–38}$$

where d is the diameter of the bar. For a hollow round section,

$$J = \frac{\pi}{32}\left(d_o^4 - d_i^4\right) \tag{3–39}$$

where the subscripts o and i refer to the outside and inside diameters, respectively.

In using Eq. (3–37) it is often necessary to obtain the torque T from a consideration of the power and speed of a rotating shaft. For convenience when U. S. Customary units are used, three forms of this relation are

$$H = \frac{FV}{33\,000} = \frac{2\pi Tn}{33\,000(12)} = \frac{Tn}{63\,025} \tag{3–40}$$

where H = power, hp
T = torque, lbf · in
n = shaft speed, rev/min
F = force, lbf
V = velocity, ft/min

When SI units are used, the equation is

$$H = T\omega \tag{3–41}$$

where H = power, W
T = torque, N · m
ω = angular velocity, rad/s

The torque T corresponding to the power in watts is given approximately by

$$T = 9.55\frac{H}{n} \tag{3–42}$$

where n is in revolutions per minute.

There are some applications in machinery for noncircular-cross-section members and shafts where a regular polygonal cross section is useful in transmitting torque to a gear or pulley that can have an axial change in position. Because no key or keyway is needed, the possibility of a lost key is avoided. Saint Venant (1855) showed that the maximum shearing stress in a rectangular $b \times c$ section bar occurs in the middle of the *longest* side b and is of the magnitude

$$\tau_{\max} = \frac{T}{\alpha bc^2} \doteq \frac{T}{bc^2}\left(3 + \frac{1.8}{b/c}\right) \tag{3–43}$$

where b is the longer side, c the shorter side, and α a factor that is a function of the ratio b/c as shown in the following table.[4] The angle of twist is given by

$$\theta = \frac{Tl}{\beta bc^3 G} \tag{3–44}$$

where β is a function of b/c, as shown in the table.

b/c	1.00	1.50	1.75	2.00	2.50	3.00	4.00	6.00	8.00	10	∞
α	0.208	0.231	0.239	0.246	0.258	0.267	0.282	0.299	0.307	0.313	0.333
β	0.141	0.196	0.214	0.228	0.249	0.263	0.281	0.299	0.307	0.313	0.333

In Eqs. (3–43) and (3–44) b and c are the width (long side) and thickness (short side) of the bar, respectively. They cannot be interchanged. Equation (3–43) is also approximately valid for equal-sided angles; these can be considered as two rectangles, each of which is capable of carrying half the torque.[5]

[4] S. Timoshenko, *Strength of Materials*, Part I, 3rd ed., D. Van Nostrand Company, New York, 1955, p. 290.

[5] For other sections see W. C. Young and R. G. Budynas, *Roark's Formulas for Stress and Strain,* 7th ed., McGraw-Hill, New York, 2002.

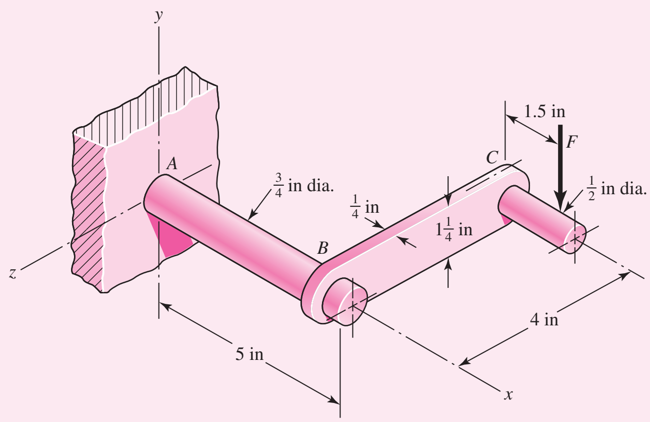
EXAMPLE 3–8

Figure 3–22 shows a crank loaded by a force $F = 300$ lbf that causes twisting and bending of a $\frac{3}{4}$-in-diameter shaft fixed to a support at the origin of the reference system. In actuality, the support may be an inertia that we wish to rotate, but for the purposes of a stress analysis we can consider this a statics problem.

(*a*) Draw separate free-body diagrams of the shaft AB and the arm BC, and compute the values of all forces, moments, and torques that act. Label the directions of the coordinate axes on these diagrams.

(*b*) Compute the maxima of the torsional stress and the bending stress in the arm BC and indicate where these act.

Figure 3–22



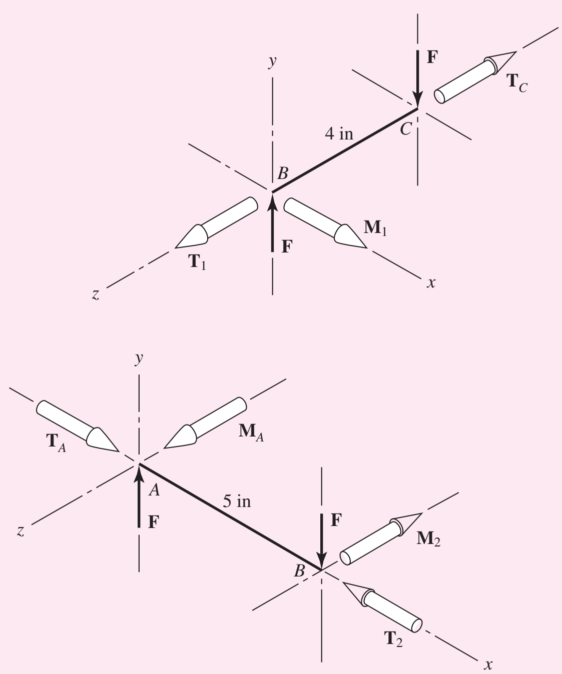
(*c*) Locate a stress element on the top surface of the shaft at *A*, and calculate all the stress components that act upon this element.

(*d*) Determine the maximum normal and shear stresses at *A*.

Solution (*a*) The two free-body diagrams are shown in Fig. 3–23. The results are

At end *C* of arm *BC*:	$\mathbf{F} = -300\mathbf{j}$ lbf, $\mathbf{T}_C = -450\mathbf{k}$ lbf · in
At end *B* of arm *BC*:	$\mathbf{F} = 300\mathbf{j}$ lbf, $\mathbf{M}_1 = 1200\mathbf{i}$ lbf · in, $\mathbf{T}_1 = 450\mathbf{k}$ lbf · in
At end *B* of shaft *AB*:	$\mathbf{F} = -300\mathbf{j}$ lbf, $\mathbf{T}_2 = -1200\mathbf{i}$ lbf · in, $\mathbf{M}_2 = -450\mathbf{k}$ lbf · in
At end *A* of shaft *AB*:	$\mathbf{F} = 300\mathbf{j}$ lbf, $\mathbf{M}_A = 1950\mathbf{k}$ lbf · in, $\mathbf{T}_A = 1200\mathbf{i}$ lbf · in

Figure 3–23



(*b*) For arm BC, the bending moment will reach a maximum near the shaft at B. If we assume this is 1200 lbf · in, then the bending stress for a rectangular section will be

Answer
$$\sigma = \frac{M}{I/c} = \frac{6M}{bh^2} = \frac{6(1200)}{0.25(1.25)^2} = 18\,400 \text{ psi}$$

Of course, this is not exactly correct, because at B the moment is actually being transferred into the shaft, probably through a weldment.

For the torsional stress, use Eq. (3–43). Thus

Answer
$$\tau_{\max} = \frac{T}{bc^2}\left(3 + \frac{1.8}{b/c}\right) = \frac{450}{1.25(0.25^2)}\left(3 + \frac{1.8}{1.25/0.25}\right) = 19\,400 \text{ psi}$$

This stress occurs at the middle of the $1\frac{1}{4}$-in side.

(*c*) For a stress element at A, the bending stress is tensile and is

Answer
$$\sigma_x = \frac{M}{I/c} = \frac{32M}{\pi d^3} = \frac{32(1950)}{\pi(0.75)^3} = 47\,100 \text{ psi}$$

The torsional stress is

Answer
$$\tau_{xz} = \frac{-T}{J/c} = \frac{-16T}{\pi d^3} = \frac{-16(1200)}{\pi(0.75)^3} = -14\,500 \text{ psi}$$

where the reader should verify that the negative sign accounts for the direction of τ_{xz}.

(*d*) Point A is in a state of plane stress where the stresses are in the xz plane. Thus the principal stresses are given by Eq. (3–13) with subscripts corresponding to the x, z axes.

Answer

The maximum normal stress is then given by

$$\sigma_1 = \frac{\sigma_x + \sigma_z}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2}$$

$$= \frac{47.1 + 0}{2} + \sqrt{\left(\frac{47.1 - 0}{2}\right)^2 + (-14.5)^2} = 51.2 \text{ kpsi}$$

Answer

The maximum shear stress at A occurs on surfaces different than the surfaces containing the principal stresses or the surfaces containing the bending and torsional shear stresses. The maximum shear stress is given by Eq. (3–14), again with modified subscripts, and is given by

$$\tau_1 = \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2} = \sqrt{\left(\frac{47.1 - 0}{2}\right)^2 + (-14.5)^2} = 27.7 \text{ kpsi}$$

EXAMPLE 3–9 The 1.5-in-diameter solid steel shaft shown in Fig. 3–24*a* is simply supported at the ends. Two pulleys are keyed to the shaft where pulley B is of diameter 4.0 in and pulley C is of diameter 8.0 in. Considering bending and torsional stresses only, determine the locations and magnitudes of the greatest tensile, compressive, and shear stresses in the shaft.

Solution Figure 3–24*b* shows the net forces, reactions, and torsional moments on the shaft. Although this is a three-dimensional problem and vectors might seem appropriate, we will look at the components of the moment vector by performing a two-plane analysis. Figure 3–24*c* shows the loading in the xy plane, as viewed down the z axis, where bending moments are actually vectors in the z direction. Thus we label the moment diagram as M_z versus x. For the xz plane, we look down the y axis, and the moment diagram is M_y versus x as shown in Fig. 3–24*d*.

The net moment on a section is the vector sum of the components. That is,

$$M = \sqrt{M_y^2 + M_z^2} \tag{1}$$

At point B,

$$M_B = \sqrt{2000^2 + 8000^2} = 8246 \text{ lbf} \cdot \text{in}$$

At point C,

$$M_C = \sqrt{4000^2 + 4000^2} = 5657 \text{ lbf} \cdot \text{in}$$

Thus the maximum bending moment is 8246 lbf · in and the maximum bending stress at pulley B is

$$\sigma = \frac{M\,d/2}{\pi d^4/64} = \frac{32M}{\pi d^3} = \frac{32(8246)}{\pi(1.5^3)} = 24\ 890 \text{ psi}$$

The maximum torsional shear stress occurs between B and C and is

$$\tau = \frac{T\,d/2}{\pi d^4/32} = \frac{16T}{\pi d^3} = \frac{16(1600)}{\pi(1.5^3)} = 2414 \text{ psi}$$

The maximum bending and torsional shear stresses occur just to the right of pulley B at points E and F as shown in Fig. 3–24*e*. At point E, the maximum tensile stress will be σ_1 given by

Answer

$$\sigma_1 = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{24\ 890}{2} + \sqrt{\left(\frac{24\ 890}{2}\right)^2 + 2414^2} = 25\ 120 \text{ psi}$$

At point F, the maximum compressive stress will be σ_2 given by

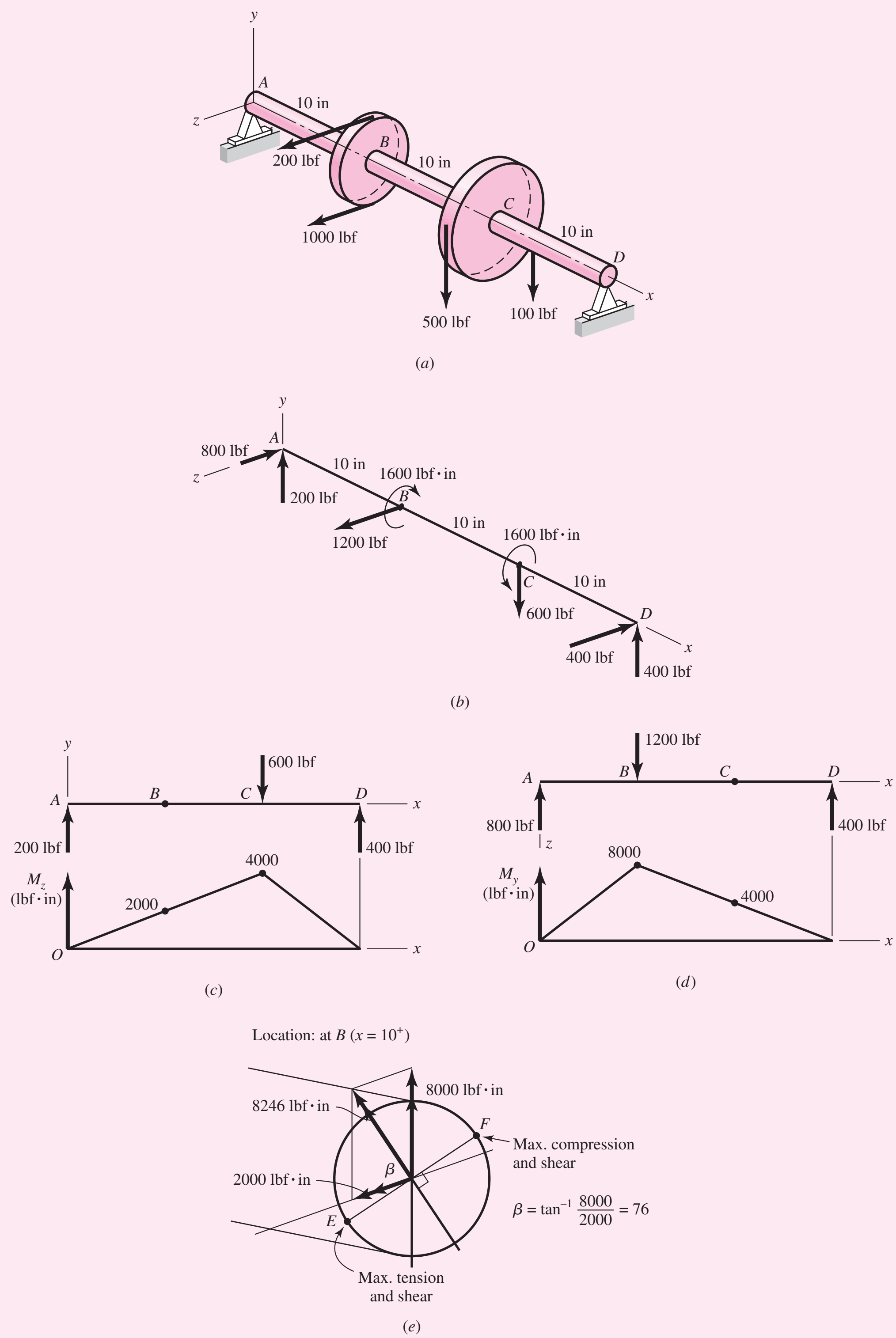
Answer

$$\sigma_2 = \frac{-\sigma}{2} - \sqrt{\left(\frac{-\sigma}{2}\right)^2 + \tau^2} = \frac{-24\ 890}{2} - \sqrt{\left(\frac{-24\ 890}{2}\right)^2 + 2414^2} = -25\ 120 \text{ psi}$$

The extreme shear stress also occurs at E and F and is

Answer

$$\tau_1 = \sqrt{\left(\frac{\pm\sigma}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{\pm 24\ 890}{2}\right)^2 + 2414^2} = 12\ 680 \text{ psi}$$



y
A
z
10 in
B
200 lbf
10 in
1000 lbf
C
10 in
D
x
500 lbf
100 lbf
(a)
y
800 lbf
A
z
10 in
1600 lbf · in
200 lbf
B
10 in
1200 lbf
1600 lbf · in
C
10 in
600 lbf
D
x
400 lbf
400 lbf
(b)
y
600 lbf
A
B
C
D
x
200 lbf
400 lbf
4000
M_z
(lbf · in)
2000
O
x
(c)
1200 lbf
A
B
C
D
x
800 lbf
z
400 lbf
8000
M_y
(lbf · in)
4000
O
x
(d)
Location: at B ($x = 10^+$)
8000 lbf · in
8246 lbf · in
F
Max. compression
and shear
β
2000 lbf · in
$\beta = \tan^{-1}\frac{8000}{2000} = 76$
E
Max. tension
and shear
(e)

Figure 3–24

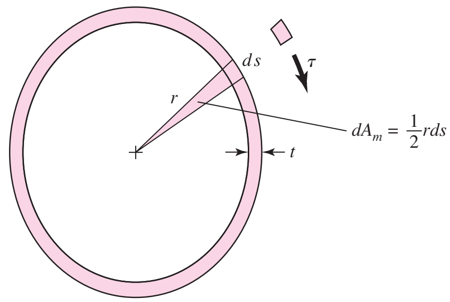


Figure 3–25

The depicted cross section is elliptical, but the section need not be symmetrical nor of constant thickness.

Closed Thin-Walled Tubes ($t \ll r$)[6]

In closed thin-walled tubes, it can be shown that the product of shear stress times thickness of the wall τt is constant, meaning that the shear stress τ is inversely proportional to the wall thickness t. The total torque T on a tube such as depicted in Fig. 3–25 is given by

$$T = \int \tau t r \, ds = (\tau t) \int r \, ds = \tau t (2A_m) = 2A_m t \tau$$

where A_m is the *area enclosed by the section median line*. Solving for τ gives

$$\tau = \frac{T}{2A_m t} \tag{3–45}$$

For constant wall thickness t, the angular twist (radians) per unit of length of the tube θ_1 is given by

$$\theta_1 = \frac{T L_m}{4 G A_m^2 t} \tag{3–46}$$

where L_m is the *perimeter of the section median line*. These equations presume the buckling of the tube is prevented by ribs, stiffeners, bulkheads, and so on, and that the stresses are below the proportional limit.

[6]See Sec. 3–13, F. P. Beer, E. R. Johnston, and J. T. De Wolf, *Mechanics of Materials,* 4th ed., McGraw-Hill, New York, 2006.

EXAMPLE 3–10

A welded steel tube is 40 in long, has a $\frac{1}{8}$-in wall thickness, and a 2.5-in by 3.6-in rectangular cross section as shown in Fig. 3–26. Assume an allowable shear stress of 11 500 psi and a shear modulus of $11.5(10^6)$ psi.

(*a*) Estimate the allowable torque T.

(*b*) Estimate the angle of twist due to the torque.

Solution

(*a*) Within the section median line, the area enclosed is

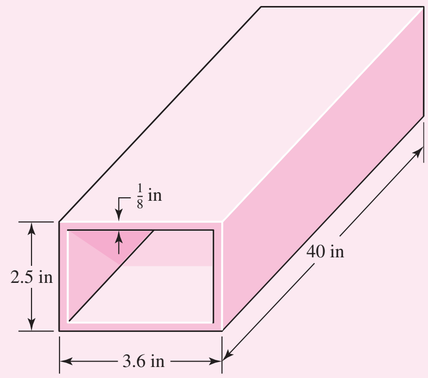
$$A_m = (2.5 - 0.125)(3.6 - 0.125) = 8.253 \text{ in}^2$$

and the length of the median perimeter is

$$L_m = 2[(2.5 - 0.125) + (3.6 - 0.125)] = 11.70 \text{ in}$$

Figure 3–26

A rectangular steel tube produced by welding.



Answer From Eq. (3–45) the torque T is

$$T = 2A_m t\tau = 2(8.253)0.125(11\,500) = 23\,730 \text{ lbf}\cdot\text{in}$$

Answer (*b*) The angle of twist θ from Eq. (3–46) is

$$\theta = \theta_1 l = \frac{TL_m}{4GA_m^2 t} l = \frac{23\,730(11.70)}{4(11.5\times 10^6)(8.253^2)(0.125)}(40) = 0.0284 \text{ rad} = 1.62^\circ$$

EXAMPLE 3–11 Compare the shear stress on a circular cylindrical tube with an outside diameter of 1 in and an inside diameter of 0.9 in, predicted by Eq. (3–37), to that estimated by Eq. (3–45).

Solution From Eq. (3–37),

$$\tau_{\max} = \frac{Tr}{J} = \frac{Tr}{(\pi/32)\left(d_o^4 - d_i^4\right)} = \frac{T(0.5)}{(\pi/32)(1^4 - 0.9^4)} = 14.809T$$

From Eq. (3–45),

$$\tau = \frac{T}{2A_m t} = \frac{T}{2(\pi 0.95^2/4)0.05} = 14.108T$$

Taking Eq. (3–37) as correct, the error in the thin-wall estimate is −4.7 percent.

Open Thin-Walled Sections

When the median wall line is not closed, it is said to be *open*. Figure 3–27 presents some examples. Open sections in torsion, where the wall is thin, have relations derived from the membrane analogy theory[7] resulting in:

$$\tau = G\theta_1 c = \frac{3T}{Lc^2} \tag{3–47}$$

[7]See S. P. Timoshenko and J. N. Goodier, *Theory of Elasticity,* 3rd ed., McGraw-Hill, New York, 1970, Sec.109.

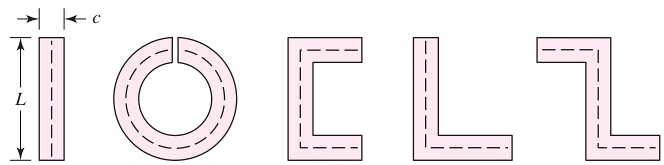


Figure 3–27

Some open thin-wall sections.

where τ is the shear stress, G is the shear modulus, θ_1 is the angle of twist per unit length, T is torque, and L is the length of the median line. The wall thickness is designated c (rather than t) to remind you that you are in open sections. By studying the table that follows Eq. (3–44) you will discover that membrane theory presumes $b/c \to \infty$. Note that open thin-walled sections in torsion should be avoided in design. As indicated in Eq. (3–47), the shear stress and the angle of twist are inversely proportional to c^2 and c^3, respectively. Thus, for small wall thickness, stress and twist can become quite large. For example, consider the thin round tube with a slit in Fig. 3–27. For a ratio of wall thickness of outside diameter of $c/d_o = 0.1$, the open section has greater magnitudes of stress and angle of twist by factors of 12.3 and 61.5, respectively, compared to a closed section of the same dimensions.

EXAMPLE 3–12

A 12-in-long strip of steel is $\frac{1}{8}$ in thick and 1 in wide, as shown in Fig. 3–28. If the allowable shear stress is 11 500 psi and the shear modulus is $11.5(10^6)$ psi, find the torque corresponding to the allowable shear stress and the angle of twist, in degrees, (*a*) using Eq. (3–47) and (*b*) using Eqs. (3–43) and (3–44).

Solution

(*a*) The length of the median line is 1 in. From Eq. (3–47),

$$T = \frac{Lc^2\tau}{3} = \frac{(1)(1/8)^2 11\,500}{3} = 59.90 \text{ lbf} \cdot \text{in}$$

$$\theta = \theta_1 l = \frac{\tau l}{Gc} = \frac{11500(12)}{11.5(10^6)(1/8)} = 0.0960 \text{ rad} = 5.5°$$

A torsional spring rate k_t can be expressed as T/θ:

$$k_t = 59.90/0.0960 = 624 \text{ lbf} \cdot \text{in/rad}$$

(*b*) From Eq. (3–43),

$$T = \frac{\tau_{\max} bc^2}{3 + 1.8/(b/c)} = \frac{11\,500(1)(0.125)^2}{3 + 1.8/(1/0.125)} = 55.72 \text{ lbf} \cdot \text{in}$$

From Eq. (3–44), with $b/c = 1/0.125 = 8$,

$$\theta = \frac{Tl}{\beta bc^3 G} = \frac{55.72(12)}{0.307(1)0.125^3(11.5)10^6} = 0.0970 \text{ rad} = 5.6°$$

$$k_t = 55.72/0.0970 = 574 \text{ lbf} \cdot \text{in/rad}$$

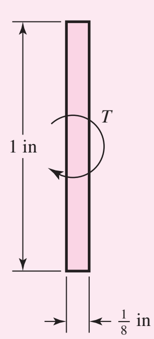


Figure 3–28

The cross-section of a thin strip of steel subjected to a torsional moment T.

3–13 Stress Concentration

In the development of the basic stress equations for tension, compression, bending, and torsion, it was assumed that no geometric irregularities occurred in the member under consideration. But it is quite difficult to design a machine without permitting some changes in the cross sections of the members. Rotating shafts must have shoulders designed on them so that the bearings can be properly seated and so that they will take thrust loads; and the shafts must have key slots machined into them for securing pulleys and gears. A bolt has a head on one end and screw threads on the other end, both of which account for abrupt changes in the cross section. Other parts require holes, oil grooves, and notches of various kinds. Any discontinuity in a machine part alters the stress distribution in the neighborhood of the discontinuity so that the elementary stress equations no longer describe the state of stress in the part at these locations. Such discontinuities are called *stress raisers,* and the regions in which they occur are called areas of *stress concentration*.

The distribution of elastic stress across a section of a member may be uniform as in a bar in tension, linear as a beam in bending, or even rapid and curvaceous as in a sharply curved beam. Stress concentrations can arise from some irregularity not inherent in the member, such as tool marks, holes, notches, grooves, or threads. The *nominal stress* is said to exist if the member is free of the stress raiser. This definition is not always honored, so check the definition on the stress-concentration chart or table you are using.

A *theoretical,* or *geometric, stress-concentration factor* K_t or K_{ts} is used to relate the actual maximum stress at the discontinuity to the nominal stress. The factors are defined by the equations

$$K_t = \frac{\sigma_{\max}}{\sigma_0} \qquad K_{ts} = \frac{\tau_{\max}}{\tau_0} \tag{3–48}$$

where K_t is used for normal stresses and K_{ts} for shear stresses. The nominal stress σ_0 or τ_0 is more difficult to define. Generally, it is the stress calculated by using the elementary stress equations and the net area, or net cross section. But sometimes the gross cross section is used instead, and so it is always wise to double check your source of K_t or K_{ts} before calculating the maximum stress.

The subscript t in K_t means that this stress-concentration factor depends for its value only on the *geometry* of the part. That is, the particular material used has no effect on the value of K_t. This is why it is called a *theoretical* stress-concentration factor.

The analysis of geometric shapes to determine stress-concentration factors is a difficult problem, and not many solutions can be found. Most stress-concentration factors are found by using experimental techniques.[8] Though the finite-element method has been used, the fact that the elements are indeed finite prevents finding the true maximum stress. Experimental approaches generally used include photoelasticity, grid methods, brittle-coating methods, and electrical strain-gauge methods. Of course, the grid and strain-gauge methods both suffer from the same drawback as the finite-element method.

Stress-concentration factors for a variety of geometries may be found in Tables A–15 and A–16.

[8]The best source book is W. D. Pilkey, *Peterson's Stress Concentration Factors,* 2nd ed., John Wiley & Sons, New York, 1997.

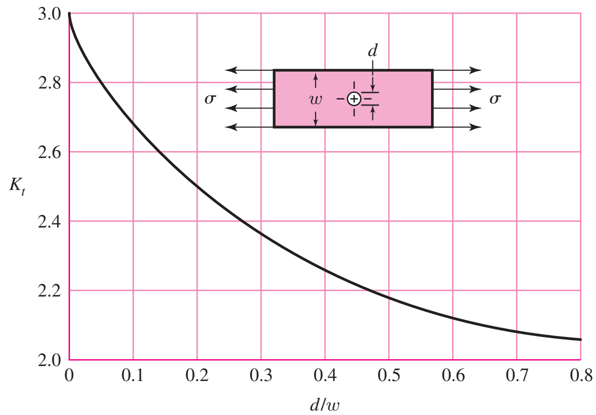


Figure 3–29

Thin plate in tension or simple compression with a transverse central hole. The net tensile force is $F = \sigma wt$, where t is the thickness of the plate. The nominal stress is given by

$$\sigma_0 = \frac{F}{(w-d)t} = \frac{w}{(w-d)}\sigma$$

An example is shown in Fig. 3–29, that of a thin plate loaded in tension where the plate contains a centrally located hole.

In *static loading,* stress-concentration factors are applied as follows. In ductile ($\epsilon_f \geq 0.05$) materials, the stress-concentration factor *is not* usually applied to predict the critical stress, because plastic strain in the region of the stress is localized and has a strengthening effect. In *brittle materials* ($\epsilon_f < 0.05$), the geometric stress-concentration factor K_t is applied to the nominal stress before comparing it with strength. Gray cast iron has so many inherent stress raisers that the stress raisers introduced by the designer have only a modest (but additive) effect.

EXAMPLE 3–13 Be Alert to Viewpoint

On a "spade" rod end (or lug) a load is transferred through a pin to a rectangular-cross-section rod or strap. The theoretical or geometric stress-concentration factor for this geometry is known as follows, on the basis of the net area $A = (w - d)t$ as shown in Fig. 3–30.

d/w	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
K_t	7.4	5.4	4.6	3.7	3.2	2.8	2.6	2.45

As presented in the table, K_t is a decreasing monotone. This rod end is similar to the square-ended lug depicted in Fig. A–15-12 of appendix A.

$$\sigma_{\max} = K_t \sigma_0 \tag{a}$$

$$\sigma_{\max} = \frac{K_t F}{A} = K_t \frac{F}{(w-d)t} \tag{b}$$

It is insightful to base the stress concentration factor on the *unnotched* area, wt. Let

$$\sigma_{\max} = K_t' \frac{F}{wt} \tag{c}$$

By equating Eqs. (b) and (c) and solving for K_t' we obtain

$$K_t' = \frac{wt}{F} K_t \frac{F}{(w-d)t} = \frac{K_t}{1 - d/w} \tag{d}$$

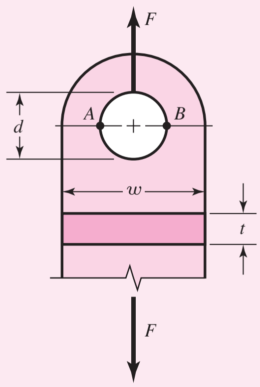


Figure 3–30

A round-ended lug end to a rectangular cross-section rod. The maximum tensile stress in the lug occurs at locations A and B. The net area $A = (w - d)t$ is used in the definition of K_t, but there is an advantage to using the total area wt.

A power regression curve-fit for the data in the above table in the form $K_t = a(d/w)^b$ gives the result $a = \exp(0.204\,521\,2) = 1.227$, $b = -0.935$, and $r^2 = 0.9947$. Thus

$$K_t = 1.227\left(\frac{d}{w}\right)^{-0.935} \tag{e}$$

which is a decreasing monotone (and unexciting). However, from Eq. (d),

$$K'_t = \frac{1.227}{1 - d/w}\left(\frac{d}{w}\right)^{-0.935} \tag{f}$$

Form another table from Eq. (f):

d/w	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60
K'_t	8.507	6.907	5.980	5.403	5.038	4.817	4.707	4.692	4.769	4.946

which shows a stationary-point minimum for K'_t. This can be found by differentiating Eq. (f) with respect to d/w and setting it equal to zero:

$$\frac{dK'_t}{d(d/w)} = \frac{(1 - d/w)ab(d/w)^{b-1} + a(d/w)^b}{[1 - (d/w)]^2} = 0$$

where $b = -0.935$, from which

$$\left(\frac{d}{w}\right)^* = \frac{b}{b-1} = \frac{-0.935}{-0.935 - 1} = 0.483$$

with a corresponding K'_t of 4.687. Knowing the section $w \times t$ lets the designer specify the strongest lug immediately by specifying a pin diameter of $0.483w$ (or, as a rule of thumb, of half the width). The theoretical K_t data in the original form, or a plot based on the data using net area, would not suggest this. The right viewpoint can suggest valuable insights.

3–14 Stresses in Pressurized Cylinders

Cylindrical pressure vessels, hydraulic cylinders, gun barrels, and pipes carrying fluids at high pressures develop both radial and tangential stresses with values that depend upon the radius of the element under consideration. In determining the radial stress σ_r and the tangential stress σ_t, we make use of the assumption that the longitudinal elongation is constant around the circumference of the cylinder. In other words, a right section of the cylinder remains plane after stressing.

Referring to Fig. 3–31, we designate the inside radius of the cylinder by r_i, the outside radius by r_o, the internal pressure by p_i, and the external pressure by p_o. Then it can be shown that tangential and radial stresses exist whose magnitudes are[9]

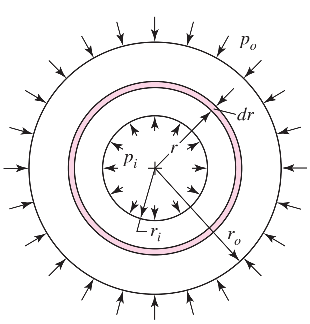


Figure 3–31

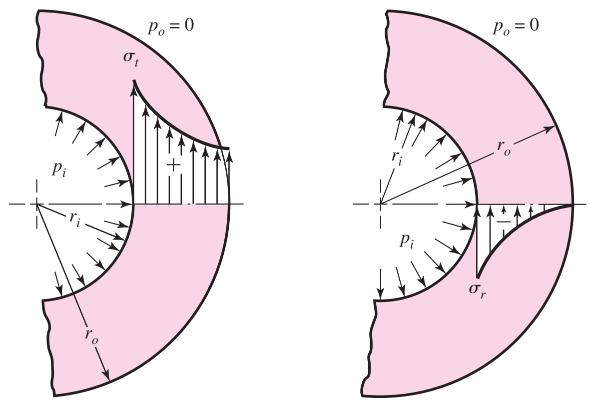
A cylinder subjected to both internal and external pressure.

$$\sigma_t = \frac{p_i r_i^2 - p_o r_o^2 - r_i^2 r_o^2 (p_o - p_i)/r^2}{r_o^2 - r_i^2}$$
$$\sigma_r = \frac{p_i r_i^2 - p_o r_o^2 + r_i^2 r_o^2 (p_o - p_i)/r^2}{r_o^2 - r_i^2} \tag{3–49}$$

[9] See Richard G. Budynas, *Advanced Strength and Applied Stress Analysis,* 2nd ed., McGraw-Hill, New York, 1999, pp. 348–352.

Figure 3–32

Distribution of stresses in a thick-walled cylinder subjected to internal pressure.



(*a*) Tangential stress distribution

(*b*) Radial stress distribution

As usual, positive values indicate tension and negative values, compression.

The special case of $p_o = 0$ gives

$$\sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r^2}\right)$$
$$\sigma_r = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r^2}\right) \tag{3–50}$$

The equations of set (3–50) are plotted in Fig. 3–32 to show the distribution of stresses over the wall thickness. It should be realized that longitudinal stresses exist when the end reactions to the internal pressure are taken by the pressure vessel itself. This stress is found to be

$$\sigma_l = \frac{p_i r_i^2}{r_o^2 - r_i^2} \tag{3–51}$$

We further note that Eqs. (3–49), (3–50), and (3–51) apply only to sections taken a significant distance from the ends and away from any areas of stress concentration.

Thin-Walled Vessels

When the wall thickness of a cylindrical pressure vessel is about one-twentieth, or less, of its radius, the radial stress that results from pressurizing the vessel is quite small compared with the tangential stress. Under these conditions the tangential stress can be obtained as follows: Let an internal pressure p be exerted on the wall of a cylinder of thickness t and inside diameter d_i. The force tending to separate two halves of a unit length of the cylinder is pd_i. This force is resisted by the tangential stress, also called the *hoop stress,* acting uniformly over the stressed area. We then have $pd_i = 2t\sigma_t$, or

$$(\sigma_t)_{\text{av}} = \frac{pd_i}{2t} \tag{3–52}$$

This equation gives the *average* tangential stress and is valid regardless of the wall thickness. For a thin-walled vessel an approximation to the maximum tangential stress is

$$(\sigma_t)_{\max} = \frac{p(d_i + t)}{2t} \tag{3–53}$$

where $d_i + t$ is the average diameter.

In a closed cylinder, the longitudinal stress σ_l exists because of the pressure upon the ends of the vessel. If we assume this stress is also distributed uniformly over the wall thickness, we can easily find it to be

$$\sigma_l = \frac{pd_i}{4t} \tag{3–54}$$

EXAMPLE 3–14

An aluminum-alloy pressure vessel is made of tubing having an outside diameter of 8 in and a wall thickness of $\frac{1}{4}$ in.

(*a*) What pressure can the cylinder carry if the permissible tangential stress is 12 kpsi and the theory for thin-walled vessels is assumed to apply?

(*b*) On the basis of the pressure found in part (*a*), compute all of the stress components using the theory for thick-walled cylinders.

Solution

(*a*) Here $d_i = 8 - 2(0.25) = 7.5$ in, $r_i = 7.5/2 = 3.75$ in, and $r_o = 8/2 = 4$ in. Then $t/r_i = 0.25/3.75 = 0.067$. Since this ratio is greater than $\frac{1}{20}$, the theory for thin-walled vessels may not yield safe results.

We first solve Eq. (3–53) to obtain the allowable pressure. This gives

Answer

$$p = \frac{2t(\sigma_t)_{\max}}{d_i + t} = \frac{2(0.25)(12)(10)^3}{7.5 + 0.25} = 774 \text{ psi}$$

Then, from Eq. (3–54), we find the average longitudinal stress to be

$$\sigma_l = \frac{pd_i}{4t} = \frac{774(7.5)}{4(0.25)} = 5810 \text{ psi}$$

(*b*) The maximum tangential stress will occur at the inside radius, and so we use $r = r_i$ in the first equation of Eq. (3–50). This gives

Answer

$$(\sigma_t)_{\max} = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r_i^2}\right) = p_i \frac{r_o^2 + r_i^2}{r_o^2 - r_i^2} = 774 \frac{4^2 + 3.75^2}{4^2 - 3.75^2} = 12\,000 \text{ psi}$$

Similarly, the maximum radial stress is found, from the second equation of Eq. (3–50) to be

Answer

$$\sigma_r = -p_i = -774 \text{ psi}$$

Equation (3–51) gives the longitudinal stress as

Answer

$$\sigma_l = \frac{p_i r_i^2}{r_o^2 - r_i^2} = \frac{774(3.75)^2}{4^2 - 3.75^2} = 5620 \text{ psi}$$

These three stresses, σ_t, σ_r, and σ_l, are principal stresses, since there is no shear on these surfaces. Note that there is no significant difference in the tangential stresses in parts (*a*) and (*b*), and so the thin-wall theory can be considered satisfactory.

3–15 Stresses in Rotating Rings

Many rotating elements, such as flywheels and blowers, can be simplified to a rotating ring to determine the stresses. When this is done it is found that the same tangential and radial stresses exist as in the theory for thick-walled cylinders except that they are caused by inertial forces acting on all the particles of the ring. The tangential and radial stresses so found are subject to the following restrictions:

- The outside radius of the ring, or disk, is large compared with the thickness $r_o \geq 10t$.
- The thickness of the ring or disk is constant.
- The stresses are constant over the thickness.

The stresses are[10]

$$\begin{aligned}\sigma_t &= \rho\omega^2 \left(\frac{3+\nu}{8}\right)\left(r_i^2 + r_o^2 + \frac{r_i^2 r_o^2}{r^2} - \frac{1+3\nu}{3+\nu} r^2\right)\\ \sigma_r &= \rho\omega^2 \left(\frac{3+\nu}{8}\right)\left(r_i^2 + r_o^2 - \frac{r_i^2 r_o^2}{r^2} - r^2\right)\end{aligned} \tag{3–55}$$

where r is the radius to the stress element under consideration, ρ is the mass density, and ω is the angular velocity of the ring in radians per second. For a rotating disk, use $r_i = 0$ in these equations.

3–16 Press and Shrink Fits

When two cylindrical parts are assembled by shrinking or press fitting one part upon another, a contact pressure is created between the two parts. The stresses resulting from this pressure may easily be determined with the equations of the preceding sections.

Figure 3–33 shows two cylindrical members that have been assembled with a shrink fit. Prior to assembly, the outer radius of the inner member was larger than the inner radius of the outer member by the *radial interference* δ. After assembly, an interference contact pressure p develops between the members at the nominal radius R, causing radial stresses $\sigma_r = -p$ in each member at the contacting surfaces. This pressure is given by[11]

$$p = \frac{\delta}{R\left[\dfrac{1}{E_o}\left(\dfrac{r_o^2 + R^2}{r_o^2 - R^2} + \nu_o\right) + \dfrac{1}{E_i}\left(\dfrac{R^2 + r_i^2}{R^2 - r_i^2} - \nu_i\right)\right]} \tag{3–56}$$

where the subscripts o and i on the material properties correspond to the outer and inner members, respectively. If the two members are of the same material with $E_o = E_i = E$, $\nu_o = \upsilon_i$, the relation simplifies to

$$p = \frac{E\delta}{2R^3}\left[\frac{(r_o^2 - R^2)(R^2 - r_i^2)}{r_o^2 - r_i^2}\right] \tag{3–57}$$

For Eqs. (3–56) or (3–57), diameters can be used in place of R, r_i, and r_o, provided δ is the diametral interference (twice the radial interference).

[10] Ibid, pp. 348–357.

[11] Ibid, pp. 348–354.

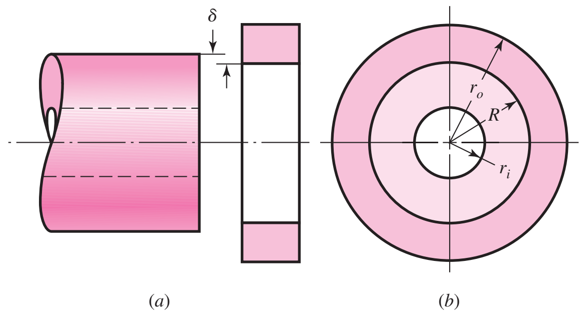


Figure 3–33

Notation for press and shrink fits. (*a*) Unassembled parts; (*b*) after assembly.

With p, Eq. (3–49) can be used to determine the radial and tangential stresses in each member. For the inner member, $p_o = p$ and $p_i = 0$, For the outer member, $p_o = 0$ and $p_i = p$. For example, the magnitudes of the tangential stresses at the transition radius R are maximum for both members. For the inner member

$$(\sigma_t)_i\Big|_{r=R} = -p\frac{R^2 + r_i^2}{R^2 - r_i^2} \tag{3–58}$$

and, for the outer member

$$(\sigma_t)_o\Big|_{r=R} = p\frac{r_o^2 + R^2}{r_o^2 - R^2} \tag{3–59}$$

Assumptions

It is assumed that both members have the same length. In the case of a hub that has been press-fitted onto a shaft, this assumption would not be true, and there would be an increased pressure at each end of the hub. It is customary to allow for this condition by employing a stress-concentration factor. The value of this factor depends upon the contact pressure and the design of the female member, but its theoretical value is seldom greater than 2.

3–17 Temperature Effects

When the temperature of an unrestrained body is uniformly increased, the body expands, and the normal strain is

$$\epsilon_x = \epsilon_y = \epsilon_z = \alpha(\Delta T) \tag{3–60}$$

where α is the coefficient of thermal expansion and ΔT is the temperature change, in degrees. In this action the body experiences a simple volume increase with the components of shear strain all zero.

If a straight bar is restrained at the ends so as to prevent lengthwise expansion and then is subjected to a uniform increase in temperature, a compressive stress will develop because of the axial constraint. The stress is

$$\sigma = -\epsilon E = -\alpha(\Delta T)E \tag{3–61}$$

In a similar manner, if a uniform flat plate is restrained at the edges and also subjected to a uniform temperature rise, the compressive stress developed is given by the equation

$$\sigma = -\frac{\alpha(\Delta T)E}{1 - \nu} \tag{3–62}$$

Table 3–3

Coefficients of Thermal Expansion (Linear Mean Coefficients for the Temperature Range 0–100°C)

Material	Celsius Scale (°C^{-1})	Fahrenheit Scale (°F^{-1})
Aluminum	$23.9(10)^{-6}$	$13.3(10)^{-6}$
Brass, cast	$18.7(10)^{-6}$	$10.4(10)^{-6}$
Carbon steel	$10.8(10)^{-6}$	$6.0(10)^{-6}$
Cast iron	$10.6(10)^{-6}$	$5.9(10)^{-6}$
Magnesium	$25.2(10)^{-6}$	$14.0(10)^{-6}$
Nickel steel	$13.1(10)^{-6}$	$7.3(10)^{-6}$
Stainless steel	$17.3(10)^{-6}$	$9.6(10)^{-6}$
Tungsten	$4.3(10)^{-6}$	$2.4(10)^{-6}$

The stresses expressed by Eqs. (3–61) and (3–62) are called *thermal stresses*. They arise because of a temperature change in a clamped or restrained member. Such stresses, for example, occur during welding, since parts to be welded must be clamped before welding. Table 3–3 lists approximate values of the coefficients of thermal expansion.

3–18 Curved Beams in Bending

The distribution of stress in a curved flexural member is determined by using the following assumptions:

- The cross section has an axis of symmetry in a plane along the length of the beam.
- Plane cross sections remain plane after bending.
- The modulus of elasticity is the same in tension as in compression.

We shall find that the neutral axis and the centroidal axis of a curved beam, unlike the axes of a straight beam, are not coincident and also that the stress does not vary linearly from the neutral axis. The notation shown in Fig. 3–34 is defined as follows:

$$r_o = \text{radius of outer fiber}$$
$$r_i = \text{radius of inner fiber}$$

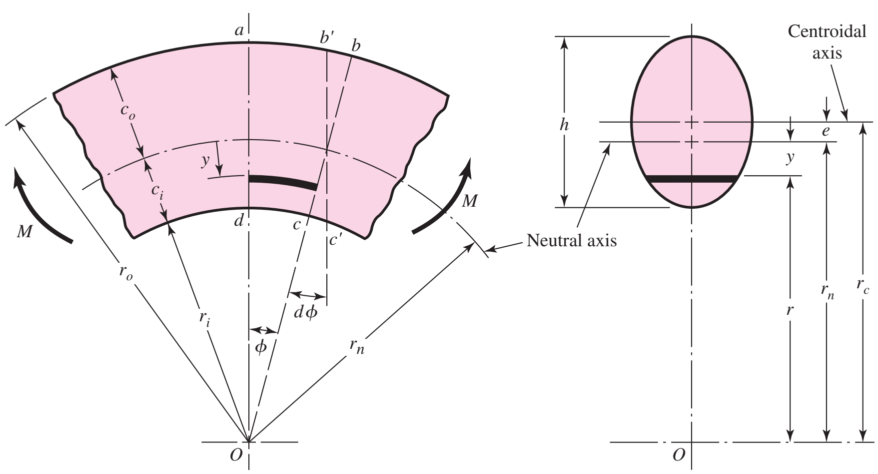


Figure 3–34

Note that y is positive in the direction toward the center of curvature, point O.

h = depth of section
c_o = distance from neutral axis to outer fiber
c_i = distance from neutral axis to inner fiber
r_n = radius of neutral axis
r_c = radius of centroidal axis
e = distance from centroidal axis to neutral axis
M = bending moment; positive M decreases curvature

Figure 3–34 shows that the neutral and centroidal axes are not coincident.[12] It turns out that the location of the neutral axis with respect to the center of curvature O is given by the equation

$$r_n = \frac{A}{\displaystyle\int \frac{dA}{r}} \tag{3–63}$$

The stress distribution can be found by balancing the external applied moment against the internal resisting moment. The result is found to be

$$\sigma = \frac{My}{Ae(r_n - y)} \tag{3–64}$$

where M is positive in the direction shown in Fig. 3–34. Equation (3–63) shows that the stress distribution is hyperbolic. The critical stresses occur at the inner and outer surfaces where $y = c_i$ and $y = -c_o$, respectively, and are

$$\sigma_i = \frac{Mc_i}{Aer_i} \qquad \sigma_o = -\frac{Mc_o}{Aer_o} \tag{3–65}$$

These equations are valid for pure bending. In the usual and more general case, such as a crane hook, the U frame of a press, or the frame of a clamp, the bending moment is due to forces acting to one side of the cross section under consideration. In this case the bending moment is computed about the *centroidal axis,* not the neutral axis. Also, an additional axial tensile or compressive stress must be added to the bending stresses given by Eqs. (3–64) and (3–65) to obtain the resultant stresses acting on the section.

[12]For a complete development of the relations in this section, see Richard G. Budynas, *Advanced Strength and Applied Stress Analysis,* 2nd ed., Mcgraw-Hill, New York, 1999, pp. 309–317.

EXAMPLE 3–15

Plot the distribution of stresses across section *A-A* of the crane hook shown in Fig.3–35*a*. The cross section is rectangular, with $b = 0.75$ in and $h = 4$ in, and the load is $F = 5000$ lbf.

Solution

Since $A = bh$, we have $dA = b\,dr$ and, from Eq. (3–63),

$$r_n = \frac{A}{\displaystyle\int \frac{dA}{r}} = \frac{bh}{\displaystyle\int_{r_i}^{r_o} \frac{b}{r}\,dr} = \frac{h}{\ln \dfrac{r_o}{r_i}} \tag{1}$$

From Fig. 3–35*b*, we see that $r_i = 2$ in, $r_o = 6$ in, $r_c = 4$ in, and $A = 3$ in^2. Thus, from Eq. (1),

$$r_n = \frac{h}{\ln(r_o/r_i)} = \frac{4}{\ln\frac{6}{2}} = 3.641 \text{ in}$$

and so the eccentricity is $e = r_c - r_n = 4 - 3.641 = 0.359$ in. The moment M is positive and is $M = Fr_c = 5000(4) = 20\,000$ lbf · in. Adding the axial component of stress to Eq. (3–64) gives

$$\sigma = \frac{F}{A} + \frac{My}{Ae(r_n - y)} = \frac{5000}{3} + \frac{(20\,000)(3.641 - r)}{3(0.359)r} \tag{2}$$

Substituting values of r from 2 to 6 in results in the stress distribution shown in Fig. 3–35*c*. The stresses at the inner and outer radii are found to be 16.9 and −5.63 kpsi, respectively, as shown.

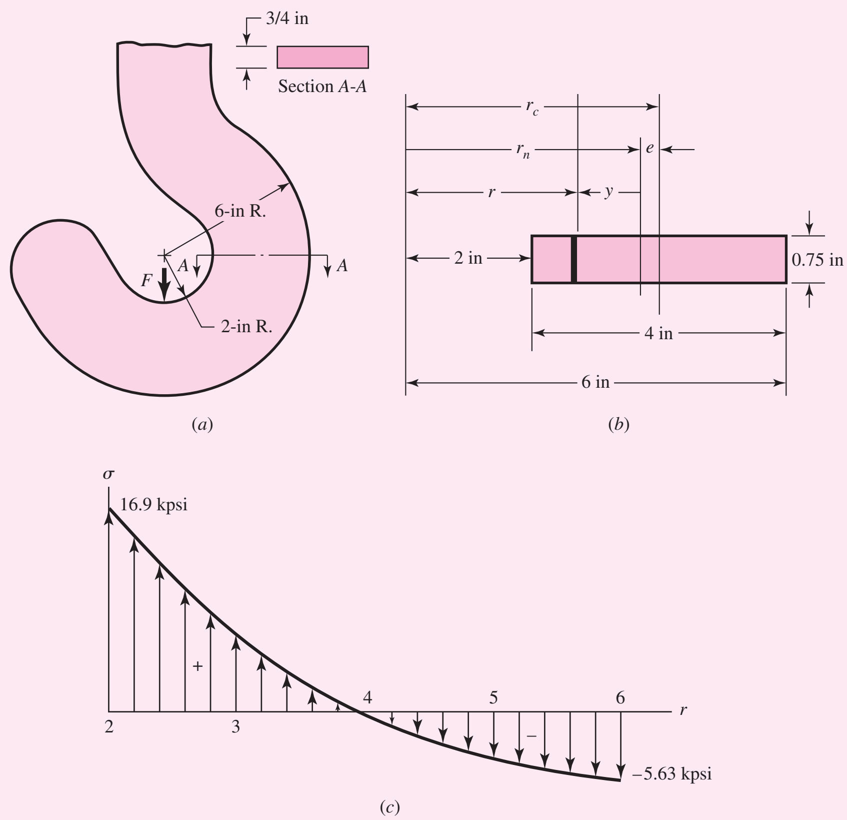


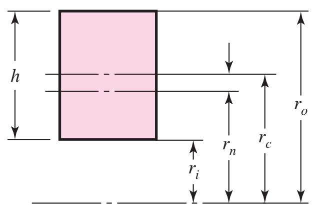
Figure 3–35

(*a*) Plan view of crane hook; (*b*) cross section and notation; (*c*) resulting stress distribution. There is no stress concentration.

Note in the hook example, the symmetrical rectangular cross section causes the maximum tensile stress to be 3 times greater than the maximum compressive stress. If we wanted to design the hook to use material more effectively we would use more material at the inner radius and less material at the outer radius. For this reason, trapezoidal, T, or unsymmetric I, cross sections are commonly used. Sections most frequently encountered in the stress analysis of curved beams are shown in Table 3–4.

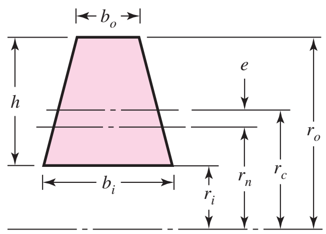
Table 3–4

Formulas for Sections of Curved Beams



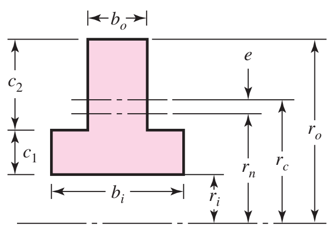
$$r_c = r_i + \frac{h}{2}$$

$$r_n = \frac{h}{\ln (r_o/r_i)}$$



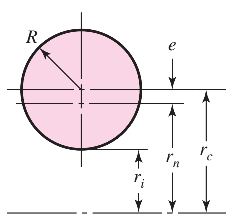
$$r_c = r_i + \frac{h}{3}\,\frac{b_i + 2b_o}{b_i + b_o}$$

$$r_n = \frac{A}{b_o - b_i + [(b_i r_o - b_o r_i)/h] \ln (r_o/r_i)}$$



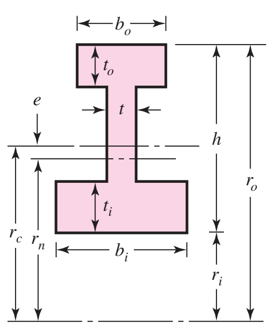
$$r_c = r_i + \frac{b_i c_1^2 + 2b_o c_1 c_2 + b_o c_2^2}{2(b_o c_2 + b_i c_1)}$$

$$r_n = \frac{b_i c_1 + b_o c_2}{b_i \ln [(r_i + c_1)/r_i] + b_o \ln [r_o/(r_i + c_1)]}$$



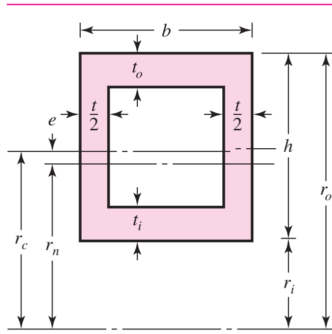
$$r_c = r_i + R$$

$$r_n = \frac{R^2}{2\left(r_c - \sqrt{r_c^2 - R^2}\right)}$$



$$r_c = r_i + \frac{\frac{1}{2}h^2 t + \frac{1}{2}t_i^2(b_i - t) + t_o(b_o - t)(h - t_o/2)}{t_i(b_i - t) + t_o(b_o - t) + ht}$$

$$r_n = \frac{t_i(b_i - t) + t_o(b_o - t) + ht_o}{b_i \ln \dfrac{r_i + t}{r_i} + t \ln \dfrac{r_o - t_o}{r_i + t_i} + b_o \ln \dfrac{r_o}{r_o - t_o}}$$



$$r_c = r_i + \frac{\frac{1}{2}h^2 t + \frac{1}{2}t_i^2(b - t) + t_o(b - t)(h - t_o/2)}{ht + (b - t)(t_i + t_o)}$$

$$r_n = \frac{(b - t)(t_i + t_o) + ht}{b\left(\ln \dfrac{r_i + t_i}{r_i} + \ln \dfrac{r_o}{r_o - t_o}\right) + t \ln \dfrac{r_o - t_o}{r_i + t_i}}$$

Alternative Calculations for e

Calculating r_n and r_c mathematically and subtracting the difference can lead to large errors if not done carefully, since r_n and r_c are typically large values compared to e. Since e is in the denominator of Eqs. (3–64) and (3–65), a large error in e can lead to an inaccurate stress calculation. Furthermore, if you have a complex cross section that the tables do not handle, alternative methods for determining e are needed. For a quick and simple approximation of e, it can be shown that[13]

$$e \doteq \frac{I}{r_c A} \tag{3–66}$$

This approximation is good for a large curvature where e is small with $r_n \doteq r_c$. Substituting Eq. (3–66) into Eq. (3–64), with $r_n - y = r$, gives

$$\sigma \doteq \frac{My}{I}\frac{r_c}{r} \tag{3–67}$$

If $r_n \doteq r_c$, which it should be to use Eq. (3–67), then it is only necessary to calculate r_c, and to measure y from this axis. Determining r_c for a complex cross section can be done easily by most CAD programs or numerically as shown in the before mentioned reference. Observe that as the curvature increases, $r \to r_c$, and Eq. (3–67) becomes the straight-beam formulation, Eq. (3–24). Note that the negative sign is missing because y in Fig. 3–34 is vertically downward, opposite that for the straight-beam equation.

[13] Ibid., pp 317–321. Also presents a numerical method.

EXAMPLE 3–16

Consider the circular section in Table 3–4 with $r_c = 3$ in and $R = 1$ in. Determine e by using the formula from the table and approximately by using Eq. (3–66). Compare the results of the two solutions.

Solution

Using the formula from Table 3–4 gives

$$r_n = \frac{R^2}{2\left(r_c - \sqrt{r_c^2 - R^2}\right)} = \frac{1^2}{2\left(3 - \sqrt{3^2 - 1}\right)} = 2.91421 \text{ in}$$

This gives an eccentricity of

Answer

$$e = r_c - r_n = 3 - 2.91421 = 0.08579 \text{ in}$$

The approximate method, using Eq. (3–66), yields

Answer

$$e \doteq \frac{I}{r_c A} = \frac{\pi R^4/4}{r_c(\pi R^2)} = \frac{R^2}{4r_c} = \frac{1^2}{4(3)} = 0.08333 \text{ in}$$

This differs from the exact solution by −2.9 percent.

3–19 Contact Stresses

When two bodies having curved surfaces are pressed together, point or line contact changes to area contact, and the stresses developed in the two bodies are three-dimensional. Contact-stress problems arise in the contact of a wheel and a rail, in automotive valve cams and tappets, in mating gear teeth, and in the action of rolling bearings. Typical failures are seen as cracks, pits, or flaking in the surface material.

The most general case of contact stress occurs when each contacting body has a double radius of curvature; that is, when the radius in the plane of rolling is different from the radius in a perpendicular plane, both planes taken through the axis of the contacting force. Here we shall consider only the two special cases of contacting spheres and contacting cylinders.[14] The results presented here are due to Hertz and so are frequently known as *Hertzian stresses*.

Spherical Contact

When two solid spheres of diameters d_1 and d_2 are pressed together with a force F, a circular area of contact of radius a is obtained. Specifying E_1, ν_1 and E_2, ν_2 as the respective elastic constants of the two spheres, the radius a is given by the equation

$$a = \sqrt[3]{\frac{3F}{8}\frac{\left(1-\nu_1^2\right)/E_1 + \left(1-\nu_2^2\right)/E_2}{1/d_1 + 1/d_2}} \tag{3–68}$$

The pressure distribution within the contact area of each sphere is hemispherical, as shown in Fig. 3–36*b*. The maximum pressure occurs at the center of the contact area and is

$$p_{\max} = \frac{3F}{2\pi a^2} \tag{3–69}$$

Equations (3–68) and (3–69) are perfectly general and also apply to the contact of a sphere and a plane surface or of a sphere and an internal spherical surface. For a plane surface, use $d = \infty$. For an internal surface, the diameter is expressed as a negative quantity.

The maximum stresses occur on the z axis, and these are principal stresses. Their values are

$$\sigma_1 = \sigma_2 = \sigma_x = \sigma_y = -p_{\max}\left[\left(1 - \left|\frac{z}{a}\right|\tan^{-1}\frac{1}{|z/a|}\right)(1+\nu) - \frac{1}{2\left(1+\dfrac{z^2}{a^2}\right)}\right] \tag{3–70}$$

$$\sigma_3 = \sigma_z = \frac{-p_{\max}}{1+\dfrac{z^2}{a^2}} \tag{3–71}$$

[14] A more comprehensive presentation of contact stresses may be found in Arthur P. Boresi and Richard J. Schmidt, *Advanced Mechanics of Materials,* 6th ed., Wiley, New York, 2003 pp. 589–623.

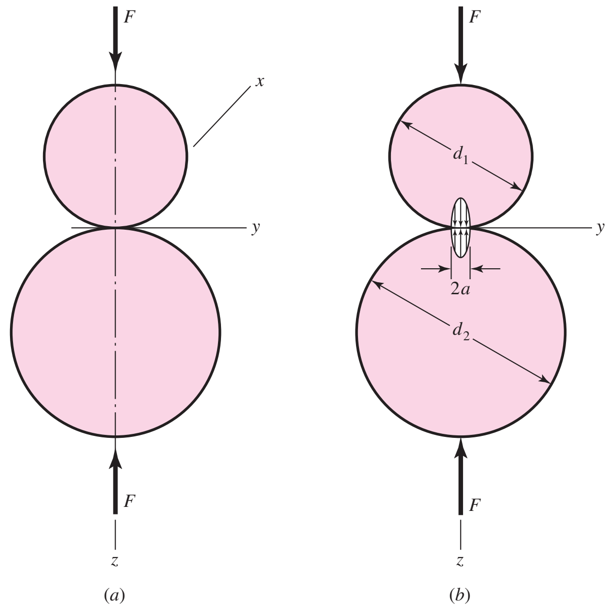


Figure 3–36

(*a*) Two spheres held in contact by force *F*; (*b*) contact stress has a hemispherical distribution across contact zone diameter 2*a*.

These equations are valid for either sphere, but the value used for Poisson's ratio must correspond with the sphere under consideration. The equations are even more complicated when stress states off the z axis are to be determined, because here the x and y coordinates must also be included. But these are not required for design purposes, because the maxima occur on the z axis.

Mohr's circles for the stress state described by Eqs. (3–70) and (3–71) are a point and two coincident circles. Since $\sigma_1 = \sigma_2$, we have $\tau_{1/2} = 0$ and

$$\tau_{\max} = \tau_{1/3} = \tau_{2/3} = \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_2 - \sigma_3}{2} \tag{3–72}$$

Figure 3–37 is a plot of Eqs. (3–70), (3–71), and (3–72) for a distance to $3a$ below the surface. Note that the shear stress reaches a maximum value slightly below the surface. It is the opinion of many authorities that this maximum shear stress is responsible for the surface fatigue failure of contacting elements. The explanation is that a crack originates at the point of maximum shear stress below the surface and progresses to the surface and that the pressure of the lubricant wedges the chip loose.

Cylindrical Contact

Figure 3–38 illustrates a similar situation in which the contacting elements are two cylinders of length l and diameters d_1 and d_2. As shown in Fig. 3–38*b*, the area of contact is a narrow rectangle of width $2b$ and length l, and the pressure distribution is elliptical. The half-width b is given by the equation

$$b = \sqrt{\frac{2F}{\pi l} \frac{\left(1 - \nu_1^2\right)/E_1 + \left(1 - \nu_2^2\right)/E_2}{1/d_1 + 1/d_2}} \tag{3–73}$$

Figure 3–37

Magnitude of the stress components below the surface as a function of the maximum pressure of contacting spheres. Note that the maximum shear stress is slightly below the surface at $z = 0.48a$ and is approximately $0.3p_{max}$. The chart is based on a Poisson ratio of 0.30. Note that the normal stresses are all compressive stresses.

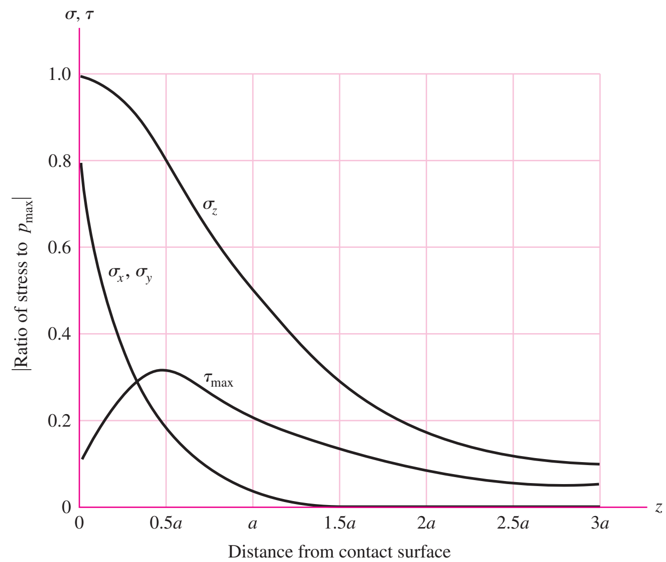
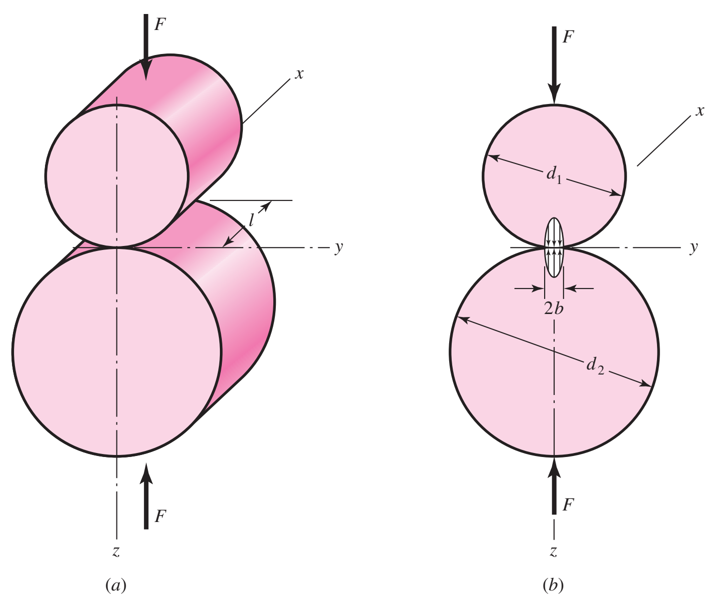


Figure 3–38

(*a*) Two right circular cylinders held in contact by forces F uniformly distributed along cylinder length l. (*b*) Contact stress has an elliptical distribution across the contact zone width $2b$.



The maximum pressure is

$$p_{max} = \frac{2F}{\pi bl} \tag{3–74}$$

Equations (3–73) and (3–74) apply to a cylinder and a plane surface, such as a rail, by making $d = \infty$ for the plane surface. The equations also apply to the contact of a cylinder and an internal cylindrical surface; in this case d is made negative for the internal surface.

The stress state along the z axis is given by the equations

$$\sigma_x = -2\nu p_{\max}\left(\sqrt{1+\frac{z^2}{b^2}} - \left|\frac{z}{b}\right|\right) \tag{3–75}$$

$$\sigma_y = -p_{\max}\left(\frac{1+2\dfrac{z^2}{b^2}}{\sqrt{1+\dfrac{z^2}{b^2}}} - 2\left|\frac{z}{b}\right|\right) \tag{3–76}$$

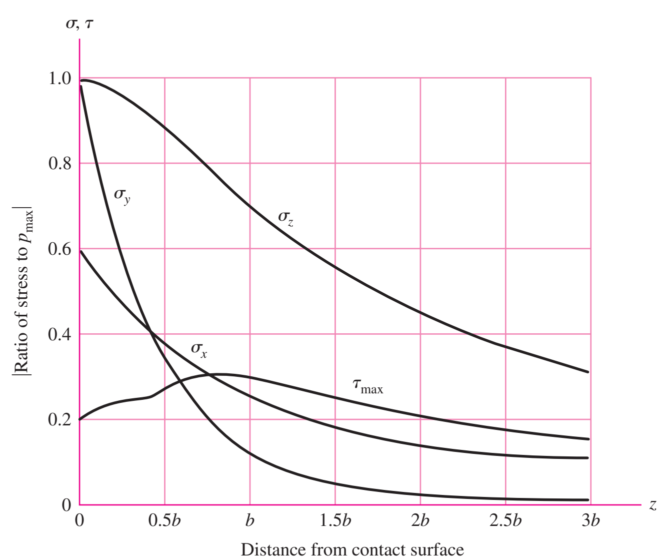
$$\sigma_3 = \sigma_z = \frac{-p_{\max}}{\sqrt{1+z^2/b^2}} \tag{3–77}$$

These three equations are plotted in Fig. 3–39 up to a distance of $3b$ below the surface. For $0 \le z \le 0.436b$, $\sigma_1 = \sigma_x$, and $\tau_{\max} = (\sigma_1 - \sigma_3)/2 = (\sigma_x - \sigma_z)/2$. For $z \ge 0.436b$, $\sigma_1 = \sigma_y$, and $\tau_{\max} = (\sigma_y - \sigma_z)/2$. A plot of $\tau_{\max}$ is also included in Fig. 3–39, where the greatest value occurs at $z/b = 0.786$ with a value of $0.300\ p_{\max}$.

Hertz (1881) provided the preceding mathematical models of the stress field when the contact zone is free of shear stress. Another important contact stress case is *line of contact* with friction providing the shearing stress on the contact zone. Such shearing stresses are small with cams and rollers, but in cams with flatfaced followers, wheel-rail contact, and gear teeth, the stresses are elevated above the Hertzian field. Investigations of the effect on the stress field due to normal and shear stresses in the contact zone were begun theoretically by Lundberg (1939), and continued by Mindlin (1949), Smith-Liu (1949), and Poritsky (1949) independently. For further detail, see the reference cited in Footnote 14.

Figure 3–39

Magnitude of the stress components below the surface as a function of the maximum pressure for contacting cylinders. The largest value of $\tau_{\max}$ occurs at $z/b = 0.786$. Its maximum value is $0.30p_{\max}$. The chart is based on a Poisson ratio of 0.30. Note that all normal stresses are compressive stresses.



3–20 Summary

The ability to quantify the stress condition at a critical location in a machine element is an important skill of the engineer. Why? Whether the member fails or not is assessed by comparing the (damaging) stress at a critical location with the corresponding material strength at this location. This chapter has addressed the description of stress.

Stresses can be estimated with great precision where the geometry is sufficiently simple that theory easily provides the necessary quantitative relationships. In other cases, approximations are used. There are numerical approximations such as finite element analysis (FEA, see Chap. 19), whose results tend to converge on the true values. There are experimental measurements, strain gauging, for example, allowing *inference* of stresses from the measured strain conditions. Whatever the method(s), the goal is a robust description of the stress condition at a critical location.

The nature of research results and understanding in any field is that the longer we work on it, the more involved things seem to be, and new approaches are sought to help with the complications. As newer schemes are introduced, engineers, hungry for the improvement the new approach *promises,* begin to use the approach. Optimism usually recedes, as further experience adds concerns. Tasks that promised to extend the capabilities of the nonexpert eventually show that expertise is not optional.

In stress analysis, the computer can be helpful if the necessary equations are available. Spreadsheet analysis can quickly reduce complicated calculations for parametric studies, easily handling "what if" questions relating trade-offs (e.g., less of a costly material or more of a cheaper material). It can even give insight into optimization opportunities.

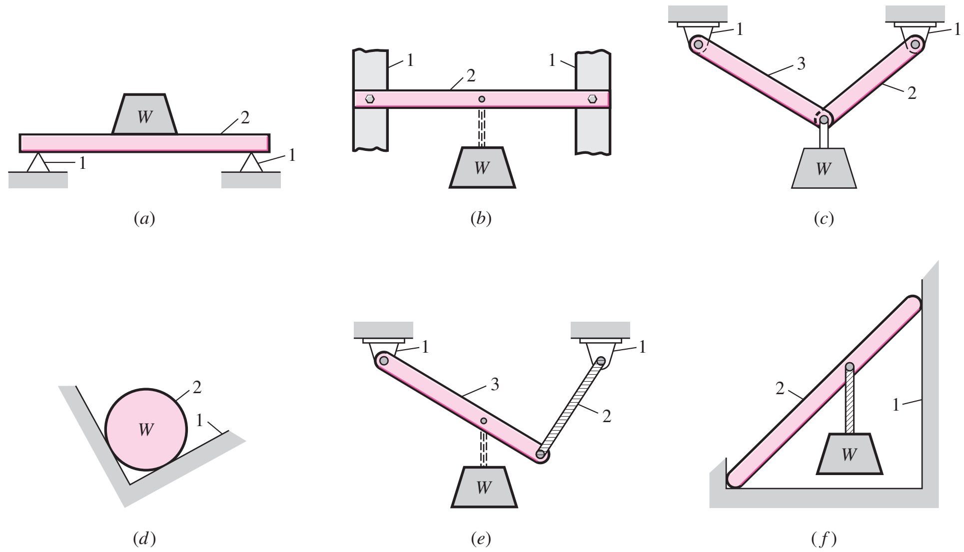
When the necessary equations are not available, then methods such as FEA are attractive, but cautions are in order. Even when you have access to a powerful FEA code, you should be near an expert while you are learning. There are nagging questions of convergence at discontinuities. Elastic analysis is much easier than elastic-plastic analysis. The results are no better than the modeling of reality that was used to formulate the problem. Chapter 19 provides an idea of what finite-element analysis is and how it can be used in design. The chapter is by no means comprehensive in finite-element theory and the application of finite elements in practice. Both skill sets require much exposure and experience to be adept.

PROBLEMS

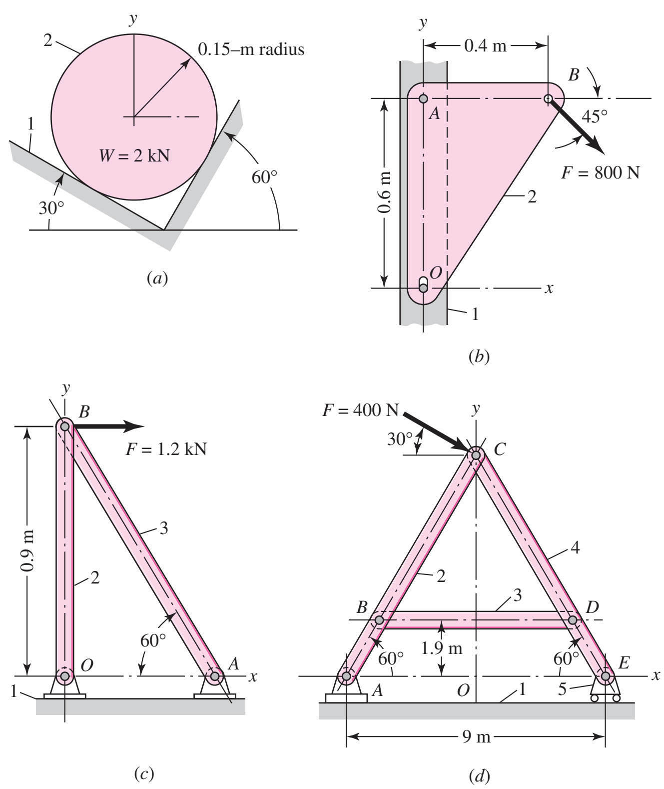
3–1 The symbol W is used in the various figure parts to specify the weight of an element. If not given, assume the parts are weightless. For each figure part, sketch a free-body diagram of each element, including the frame. Try to get the forces in the proper directions, but do not compute magnitudes.

3–2 Using the figure part selected by your instructor, sketch a free-body diagram of each element in the figure. Compute the magnitude and direction of each force using an algebraic or vector method, as specified.

3–3 Find the reactions at the supports and plot the shear-force and bending-moment diagrams for each of the beams shown in the figure on page 123. Label the diagrams properly.



Problem 3–1

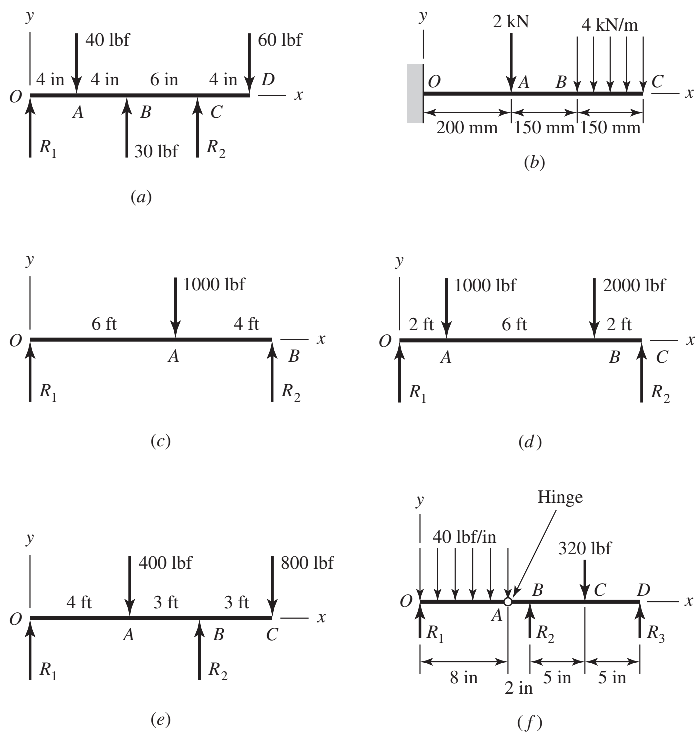


Problem 3–2

3–4 Repeat Prob. 3–3 using singularity functions exclusively (for reactions as well).

3–5 Select a beam from Table A–9 and find general expressions for the loading, shear-force, bending-moment, and support reactions. Use the method specified by your instructor.

Problem 3–3



3–6 A beam carrying a uniform load is simply supported with the supports set back a distance a from the ends as shown in the figure. The bending moment at x can be found from summing moments to zero at section x:

$$\sum M = M + \frac{1}{2}w(a+x)^2 - \frac{1}{2}wlx = 0$$

or

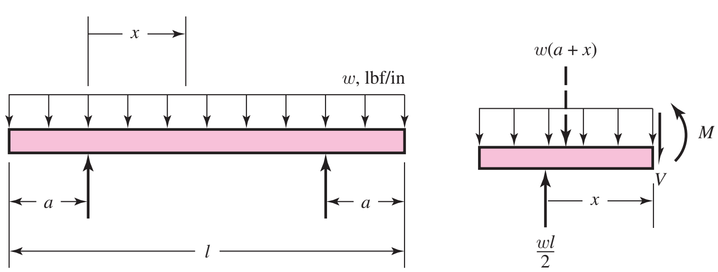
$$M = \frac{w}{2}[lx - (a+x)^2]$$

where w is the loading intensity in lbf/in. The designer wishes to minimize the necessary weight of the supporting beam by choosing a setback resulting in the smallest possible maximum bending stress.

(a) If the beam is configured with $a = 2.25$ in, $l = 10$ in, and $w = 100$ lbf/in, find the magnitude of the severest bending moment in the beam.

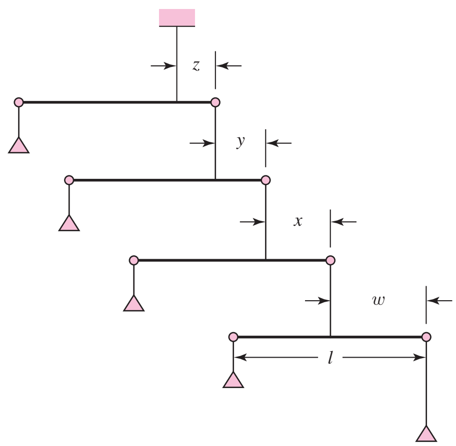
(b) Since the configuration in part (a) is not optimal, find the optimal setback a that will result in the lightest-weight beam.

Problem 3–6



3–7 An artist wishes to construct a mobile using pendants, string, and span wire with eyelets as shown in the figure.
(a) At what positions w, x, y, and z should the suspension strings be attached to the span wires?
(b) Is the mobile stable? If so, justify; if not, suggest a remedy.

Problem 3–7



3–8 For each of the plane stress states listed below, draw a Mohr's circle diagram properly labeled, find the principal normal and shear stresses, and determine the angle from the x axis to σ_1. Draw stress elements as in Fig. 3–11c and d and label all details.
(a) $\sigma_x = 12$, $\sigma_y = 6$, $\tau_{xy} = 4$ cw
(b) $\sigma_x = 16$, $\sigma_y = 9$, $\tau_{xy} = 5$ ccw
(c) $\sigma_x = 10$, $\sigma_y = 24$, $\tau_{xy} = 6$ ccw
(d) $\sigma_x = 9$, $\sigma_y = 19$, $\tau_{xy} = 8$ cw

3–9 Repeat Prob. 3–8 for:
(a) $\sigma_x = -4$, $\sigma_y = 12$, $\tau_{xy} = 7$ ccw
(b) $\sigma_x = 6$, $\sigma_y = -5$, $\tau_{xy} = 8$ ccw
(c) $\sigma_x = -8$, $\sigma_y = 7$, $\tau_{xy} = 6$ cw
(d) $\sigma_x = 9$, $\sigma_y = -6$, $\tau_{xy} = 3$ cw

3–10 Repeat Prob. 3–8 for:
(a) $\sigma_x = 20$, $\sigma_y = -10$, $\tau_{xy} = 8$ cw
(b) $\sigma_x = 30$, $\sigma_y = -10$, $\tau_{xy} = 10$ ccw
(c) $\sigma_x = -10$, $\sigma_y = 18$, $\tau_{xy} = 9$ cw
(d) $\sigma_x = -12$, $\sigma_y = 22$, $\tau_{xy} = 12$ cw

3–11 For each of the stress states listed below, find all three principal normal and shear stresses. Draw a complete Mohr's three-circle diagram and label all points of interest.
(a) $\sigma_x = 10$, $\sigma_y = -4$
(b) $\sigma_x = 10$, $\tau_{xy} = 4$ ccw
(c) $\sigma_x = -2$, $\sigma_y = -8$, $\tau_{xy} = 4$ cw
(d) $\sigma_x = 10$, $\sigma_y = -30$, $\tau_{xy} = 10$ ccw

3–12 Repeat Prob. 3–11 for:
(a) $\sigma_x = -80$, $\sigma_y = -30$, $\tau_{xy} = 20$ cw
(b) $\sigma_x = 30$, $\sigma_y = -60$, $\tau_{xy} = 30$ cw
(c) $\sigma_x = 40$, $\sigma_z = -30$, $\tau_{xy} = 20$ ccw
(d) $\sigma_x = 50$, $\sigma_z = -20$, $\tau_{xy} = 30$ cw

3–13 A $\frac{1}{2}$-in-diameter steel tension rod is 72 in long and carries a load of 2000 lbf. Find the tensile stress, the total deformation, the unit strains, and the change in the rod diameter.

3–14 Twin diagonal aluminum alloy tension rods 15 mm in diameter are used in a rectangular frame to prevent collapse. The rods can safely support a tensile stress of 135 MPa. If the rods are initially 3 m in length, how much must they be stretched to develop this stress?

3–15 Electrical strain gauges were applied to a notched specimen to determine the stresses in the notch. The results were $\epsilon_x = 0.0021$ and $\epsilon_y = -0.00067$. Find σ_x and σ_y if the material is carbon steel.

3–16 An engineer wishes to determine the shearing strength of a certain epoxy cement. The problem is to devise a test specimen such that the joint is subject to pure shear. The joint shown in the figure, in which two bars are offset at an angle θ so as to keep the loading force F centroidal with the straight shanks, seems to accomplish this purpose. Using the contact area A and designating S_{su} as the *ultimate shearing strength,* the engineer obtains

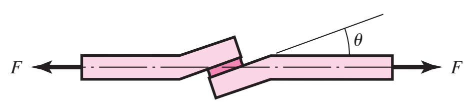
$$S_{su} = \frac{F}{A}\cos\theta$$

The engineer's supervisor, in reviewing the test results, says the expression should be

$$S_{su} = \frac{F}{A}\left(1 + \frac{1}{4}\tan^2\theta\right)^{1/2}\cos\theta$$

Resolve the discrepancy. What is your position?

Problem 3–16



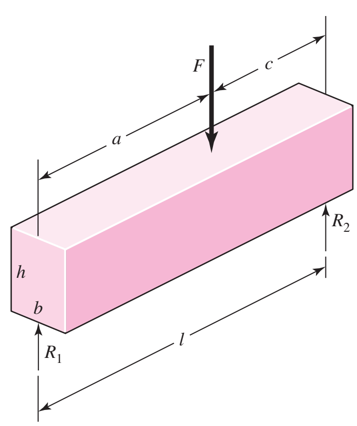
3–17 The state of stress at a point is $\sigma_x = -2$, $\sigma_y = 6$, $\sigma_z = -4$, $\tau_{xy} = 3$, $\tau_{yz} = 2$, and $\tau_{zx} = -5$ kpsi. Determine the principal stresses, draw a complete Mohr's three-circle diagram, labeling all points of interest, and report the maximum shear stress for this case.

3–18 Repeat Prob. 3–17 with $\sigma_x = 10$, $\sigma_y = 0$, $\sigma_z = 10$, $\tau_{xy} = 20$, $\tau_{yz} = -10\sqrt{2}$, and $\tau_{zx} = 0$ MPa.

3–19 Repeat Prob. 3–17 with $\sigma_x = 1$, $\sigma_y = 4$, $\sigma_z = 4$, $\tau_{xy} = 2$, $\tau_{yz} = -4$, and $\tau_{zx} = -2$ kpsi.

3–20 The Roman method for addressing uncertainty in design was to build a copy of a design that was satisfactory and had proven durable. Although the early Romans did not have the intellectual tools to deal with scaling size up or down, you do. Consider a simply supported, rectangular-cross-section beam with a concentrated load F, as depicted in the figure.

Problem 3–20



(*a*) Show that the stress-to-load equation is

$$F = \frac{\sigma b h^2 l}{6ac}$$

(*b*) Subscript every parameter with m (for model) and divide into the above equation. Introduce a scale factor, $s = a_m/a = b_m/b = c_m/c$ etc. Since the Roman method was to not "lean on" the material any more than the proven design, set $\sigma_m/\sigma = 1$. Express F_m in terms of the scale factors and F, and comment on what you have learned.

3–21 Using our experience with concentrated loading on a simple beam, Prob. 3–20, consider a uniformly loaded simple beam (Table A–9–7).

(*a*) Show that the stress-to-load equation for a rectangular-cross-section beam is given by

$$W = \frac{4}{3}\frac{\sigma b h^2}{l}$$

where $W = wl$.

(*b*) Subscript every parameter with m (for model) and divide the model equation into the prototype equation. Introduce the scale factor s as in Prob. 3–20, setting $\sigma_m/\sigma = 1$. Express W_m and w_m in terms of the scale factor, and comment on what you have learned.

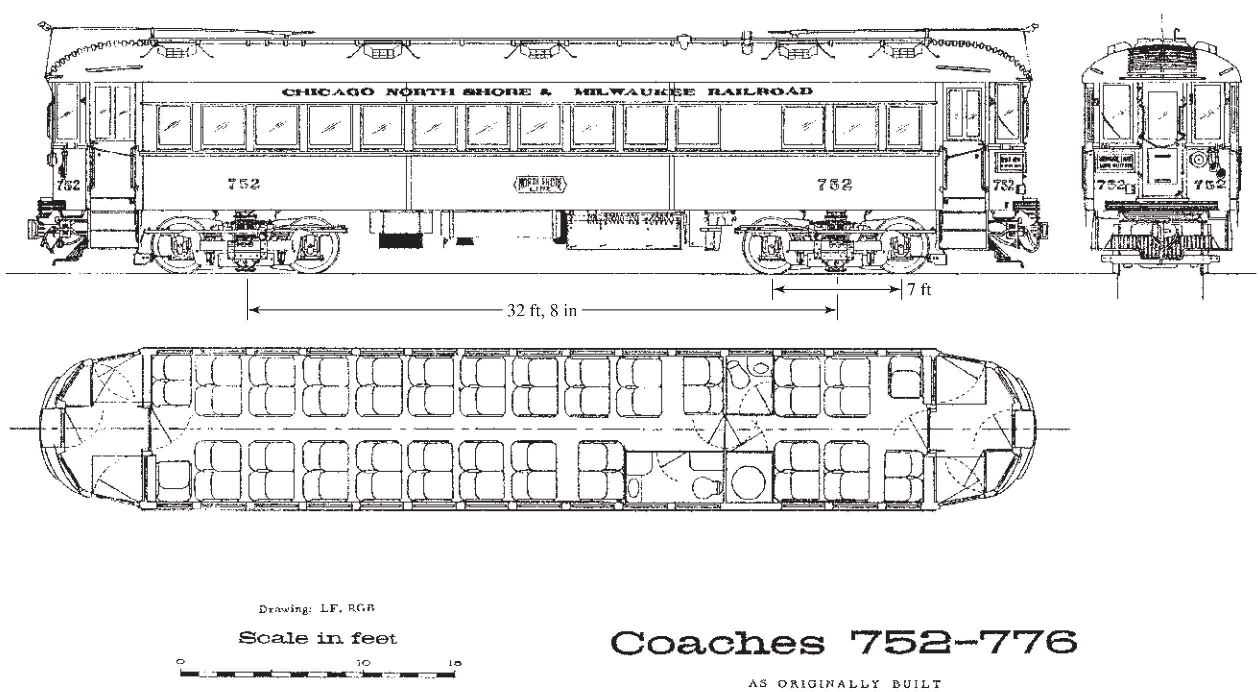
3–22 The Chicago North Shore & Milwaukee Railroad was an electric railway running between the cities in its corporate title. It had passenger cars as shown in the figure, which weighed 104.4 kip, had 32-ft, 8-in truck centers, 7-ft-wheelbase trucks, and a coupled length of 55 ft, $3\frac{1}{4}$ in. Consider the case of a single car on a 100-ft-long, simply supported deck plate girder bridge.

(*a*) What was the largest bending moment in the bridge?

(*b*) Where on the bridge was the moment located?

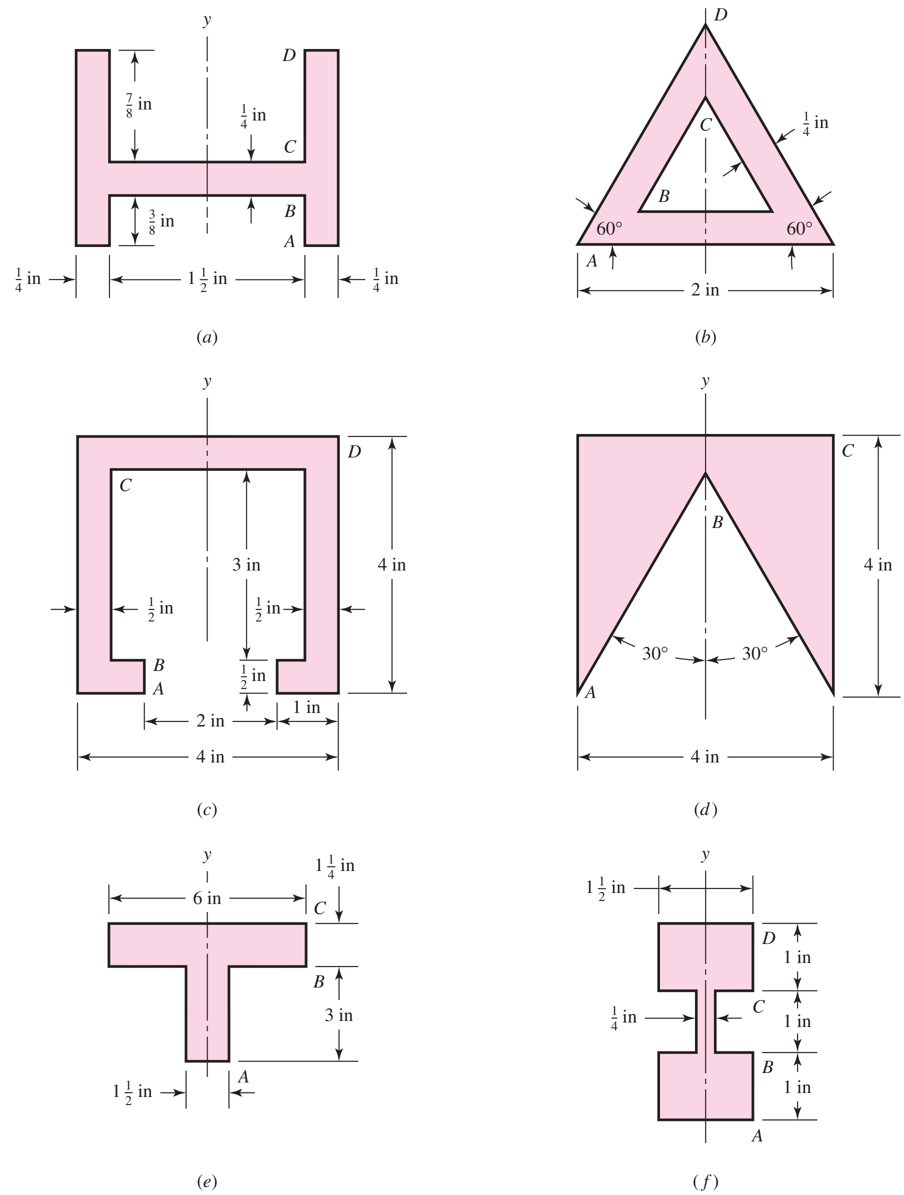
(*c*) What was the position of the car on the bridge?

(*d*) Under which axle is the bending moment?



Problem 3–22

3–23 For each section illustrated, find the second moment of area, the location of the neutral axis, and the distances from the neutral axis to the top and bottom surfaces. Suppose a positive bending moment of 10 kip · in is applied; find the resulting stresses at the top and bottom surfaces and at every abrupt change in cross section.

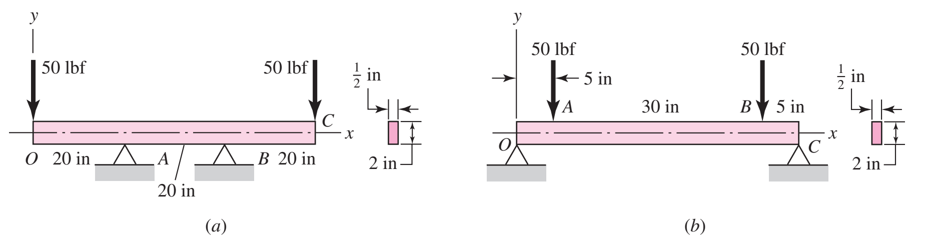


Problem 3–23

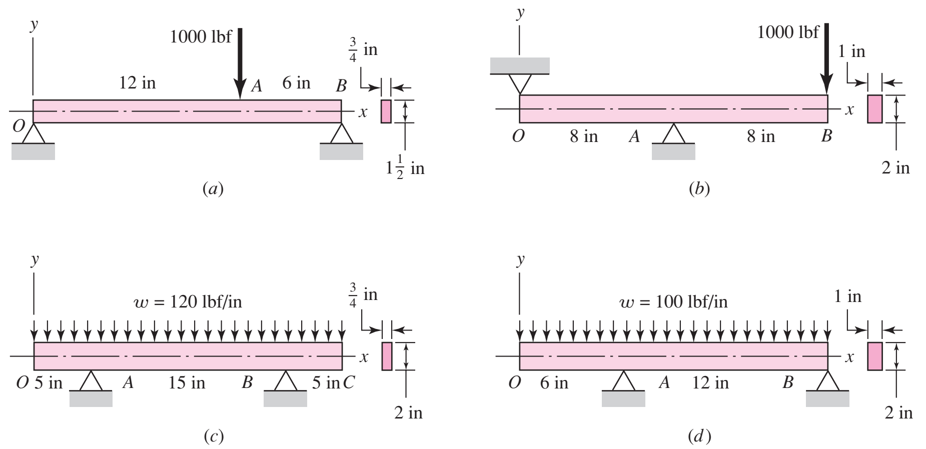
3–24 From basic mechanics of materials, in the derivation of the bending stresses, it is found that the radius of curvature of the neutral axis, ρ, is given by $\rho = EI/M$. Find the x and y coordinates of the center of curvature corresponding to the place where the beam is bent the most, for each beam shown in the figure. The beams are both made of Douglas fir (see Table A–5) and have rectangular sections.

3–25 For each beam illustrated in the figure, find the locations and magnitudes of the maximum tensile bending stress and the maximum shear stress due to V.

Problem 3–24



Problem 3–25



3–26 The figure illustrates a number of beam sections. Use an allowable bending stress of 1.2 kpsi for wood and 12 kpsi for steel and find the maximum safe uniformly distributed load that each beam can carry if the given lengths are between simple supports.

(*a*) Wood joist $1\frac{1}{2}$ by $9\frac{1}{2}$ in and 12 ft long

(*b*) Steel tube, 2 in OD by $\frac{3}{8}$-in wall thickness, 48 in long

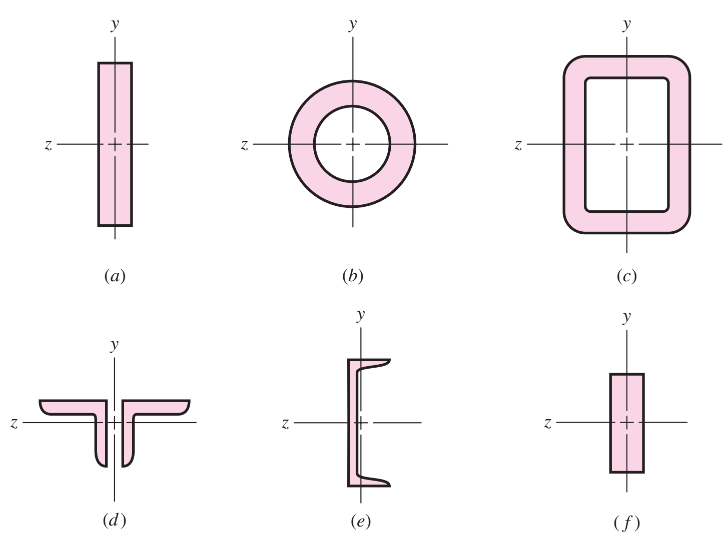
(*c*) Hollow steel tube 3 by 2 in, outside dimensions, formed from $\frac{3}{16}$-in material and welded, 48 in long

(*d*) Steel angles $3 \times 3 \times \frac{1}{4}$ in and 72 in long

(*e*) A 5.4-lb, 4-in steel channel, 72 in long

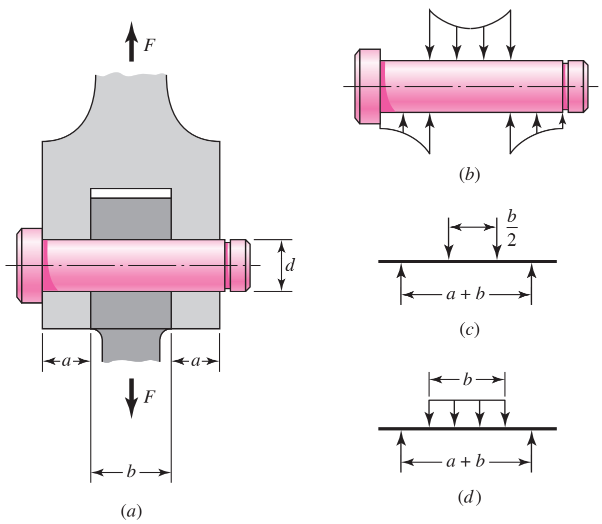
(*f*) A 4-in $\times$ 1-in steel bar, 72 in long

Problem 3–26



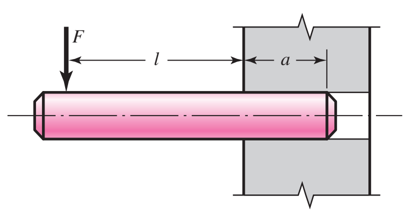
3–27 A pin in a knuckle joint carrying a tensile load F deflects somewhat on account of this loading, making the distribution of reaction and load as shown in part b of the figure. The usual designer's assumption of loading is shown in part c; others sometimes choose the loading shown in part d. If $a = 0.5$ in, $b = 0.75$ in, $d = 0.5$ in, and $F = 1000$ lbf, estimate the maximum bending stress and the maximum shear stress due to V for each approximation.

Problem 3–27



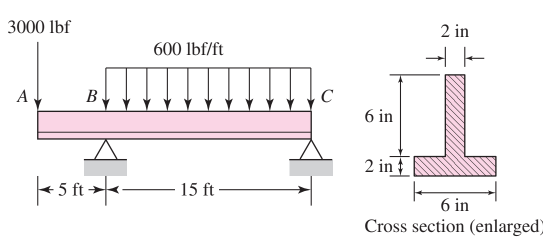
3–28 The figure illustrates a pin tightly fitted into a hole of a substantial member. A usual analysis is one that assumes concentrated reactions R and M at distance l from F. Suppose the reaction is distributed linearly along distance a. Is the resulting moment reaction larger or smaller than the concentrated reaction? What is the loading intensity q? What do you think of using the usual assumption?

Problem 3–28



3–29 For the beam shown, determine (a) the maximum tensile and compressive bending stresses, (b) the maximum shear stress due to V, and (c) the maximum shear stress in the beam.

Problem 3–29



3–30 Consider a simply supported beam of rectangular cross section of constant width b and variable depth h, so proportioned that the maximum stress σ_x at the outer surface due to bending is constant, when subjected to a load F at a distance a from the left support and a distance c from the right support. Show that the depth h at location x is given by

$$h = \sqrt{\frac{6Fcx}{lb\sigma_{\max}}} \qquad 0 \le x \le a$$

3–31 In Prob. 3–30, $h \to 0$ as $x \to 0$, which cannot occur. If the maximum shear stress $\tau_{\max}$ due to direct shear is to be constant in this region, show that the depth h at location x is given by

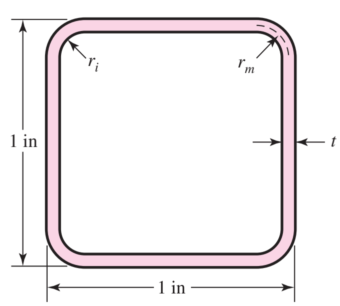
$$h = \frac{3}{2}\frac{Fc}{lb\tau_{\max}} \qquad 0 \le x \le \frac{3}{8}\frac{Fc\sigma_{\max}}{lb\tau_{\max}^2}$$

3–32 Consider a simply supported static beam of circular cross section of diameter d, so proportioned by varying the diameter such that the maximum stress σ_x at the surface due to bending is constant, when subjected to a steady load F located at a distance a from the left support and a distance b from the right support. Show that the diameter d at a location x is given by

$$d = \left(\frac{32Fbx}{\pi l\sigma_{\max}}\right)^{1/3} \qquad 0 \le x \le a$$

3–33 Two steel thin-wall tubes in torsion of equal length are to be compared. The first is of square cross section, side length b, and wall thickness t. The second is a round of diameter b and wall thickness t. The largest allowable shear stress is τ_{all} and is to be the same in both cases. How does the angle of twist per unit length compare in each case?

3–34 Begin with a 1-in-square thin-wall steel tube, wall thickness $t = 0.05$ in, length 40 in, then introduce corner radii of inside radii r_i, with allowable shear stress τ_{all} of 11 500 psi, shear modulus of $11.5(10^6)$ psi; now form a table. Use a column of inside corner radii in the range $0 \le r_i \le 0.45$ in. Useful columns include median line radius r_m, periphery of the median line L_m, area enclosed by median curve, torque T, and the angular twist θ. The cross section will vary from square to circular round. A computer program will reduce the calculation effort. Study the table. What have you learned?



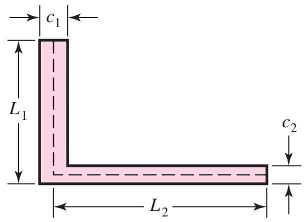
Problem 3–34

3–35 An unequal leg angle shown in the figure carries a torque T. Show that

$$T = \frac{G\theta_1}{3}\sum L_i c_i^3$$

$$\tau_{\max} = G\theta_1 c_{\max}$$

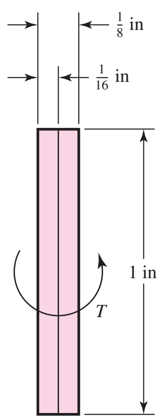
Problem 3–35



3–36 In Prob. 3–35 the angle has one leg thickness $\frac{1}{16}$ in and the other $\frac{1}{8}$ in, with both leg lengths $\frac{5}{8}$ in. The allowable shear stress is $\tau_{\text{all}} = 12\,000$ psi for this steel angle.
(*a*) Find the torque carried by each leg, and the largest shear stress therein.
(*b*) Find the angle of twist per unit length of the section.

3–37 Two 12 in long thin rectangular steel strips are placed together as shown. Using a maximum allowable shear stress of 12 000 psi, determine the maximum torque and angular twist, and the torsional spring rate. Compare these with a single strip of cross section 1 in by $\frac{1}{8}$ in.

Problem 3–37



3–38 Using a maximum allowable shear stress of 60 MPa, find the shaft diameter needed to transmit 35 kw when
(*a*) The shaft speed is 2000 rev/min.
(*b*) The shaft speed is 200 rev/min.

3–39 A 15-mm-diameter steel bar is to be used as a torsion spring. If the torsional stress in the bar is not to exceed 110 MPa when one end is twisted through an angle of 30°, what must be the length of the bar?

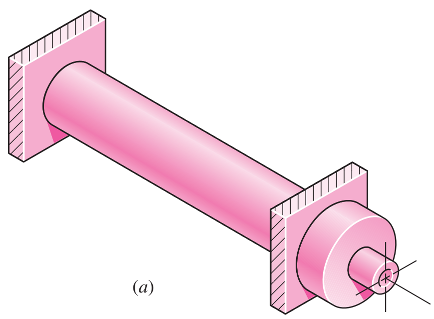
3–40 A 3-in-diameter solid steel shaft, used as a torque transmitter, is replaced with a 3-in hollow shaft having a $\frac{1}{4}$-in wall thickness. If both materials have the same strength, what is the percentage reduction in torque transmission? What is the percentage reduction in shaft weight?

3–41 A hollow steel shaft is to transmit 5400 N · m of torque and is to be sized so that the torsional stress does not exceed 150 MPa.
(*a*) If the inside diameter is three-fourths of the outside diameter, what size shaft should be used? Use preferred sizes.
(*b*) What is the stress on the inside of the shaft when full torque is applied?

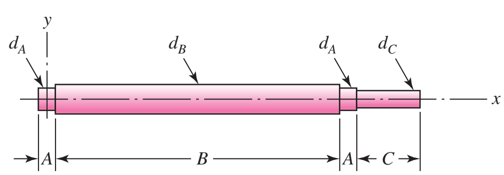
3–42 The figure shows an endless-belt conveyor drive roll. The roll has a diameter of 6 in and is driven at 5 rev/min by a geared-motor source rated at 1 hp. Determine a suitable shaft diameter d_C for an allowable torsional stress of 14 kpsi.

(*a*) What would be the stress in the shaft you have sized if the motor starting torque is twice the running torque?

(*b*) Is bending stress likely to be a problem? What is the effect of different roll lengths B on bending?



(*a*)



(*b*)

Problem 3–42

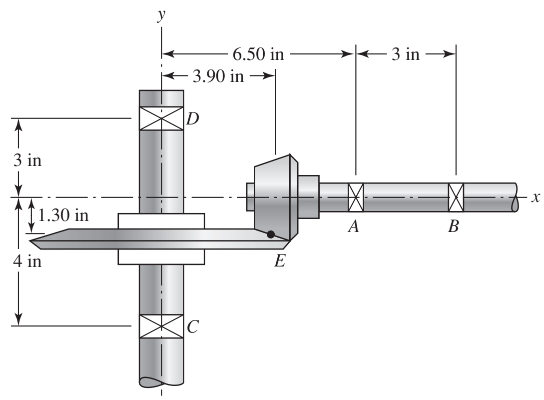
3–43 The conveyer drive roll in the figure for Prob. 3–42 is 150 mm in diameter and is driven at 8 rev/min by a geared-motor source rated at 1 kW. Find a suitable shaft diameter d_C based on an allowable torsional stress of 75 MPa.

3–44 For the same cross-sectional area $A = s^2 = \pi d^2/4$, for a square cross-sectional area shaft and a circular cross-sectional area shaft, in torsion which has the higher maximum shear stress, and by what multiple is it higher?

3–45 For the same cross-sectional area $A = s^2 = \pi d^2/4$, for a square cross-sectional area shaft and a circular cross-sectional area shaft, both of length l, in torsion which has the greater angular twist θ, and by what multiple is it greater?

3–46 In the figure, shaft AB is rotating at 1000 rev/min and transmits 10 hp to shaft CD through a set of bevel gears contacting at point E. The contact force at E on the gear of shaft CD is determined to be $(\mathbf{F}_E)_{CD} = -92.8\mathbf{i} - 362.8\mathbf{j} + 808.0\mathbf{k}$ lbf. For shaft CD: (*a*) draw a free-body diagram and determine the reactions at C and D assuming simple supports (assume also that bearing C is a thrust bearing), (*b*) draw the shear-force and bending-moment diagrams, and (*c*) assuming that the shaft diameter is 1.25 in, determine the maximum tensile and shear stresses in the beam.

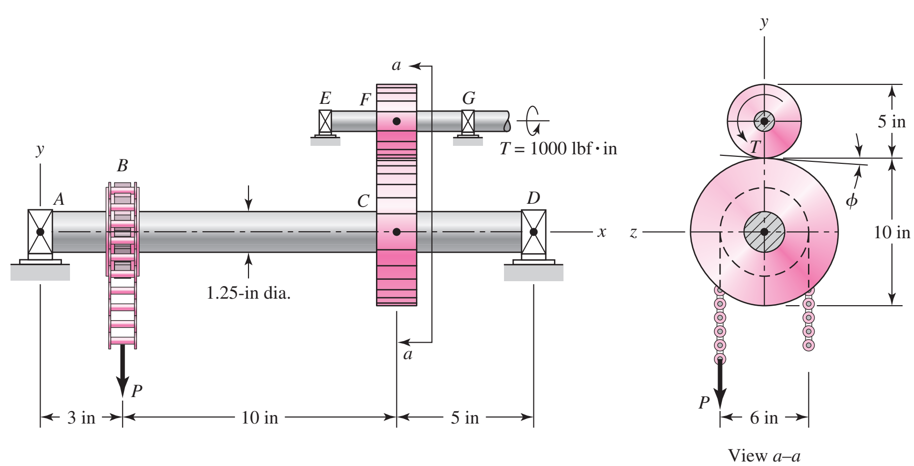
Problem 3–46



3–47 Repeat the analysis of Prob. 3–46 for shaft *AB*. Let the diameter of the shaft be 1.0 in, and assume that bearing *A* is a thrust bearing.

3–48 A torque of $T = 1000$ lbf · in is applied to the shaft *EFG*, which is running at constant speed and contains gear *F*. Gear *F* transmits torque to shaft *ABCD* through gear *C*, which drives the chain sprocket at *B*, transmitting a force *P* as shown. Sprocket *B*, gear *C*, and gear *F* have pitch diameters of 6, 10, and 5 in, respectively. The contact force between the gears is transmitted through the pressure angle $\phi = 20°$. Assuming no frictional losses and considering the bearings at *A*, *D*, *E*, and *G* to be simple supports, locate the point on shaft *ABCD* that contains the maximum tensile bending and maximum torsional shear stresses. From this, determine the maximum tensile and shear stresses in the shaft.

Problem 3–48



3–49 If the tension-loaded plate of Fig. 3–29 is infinitely wide, then the stress state anywhere in the plate can be described in polar coordinates as[15]

$$\sigma_r = \frac{1}{2}\sigma\left[1 - \frac{d^2}{4r^2} + \left(1 - \frac{d^2}{4r^2}\right)\left(1 - \frac{3d^2}{4r^2}\right)\cos 2\theta\right]$$

[15] See R. G. Budynas, *Advanced Strength and Applied Stress Analysis,* 2nd ed. McGraw-Hill, New York, 1999, pp. 235–238.

$$\sigma_\theta = \frac{1}{2}\sigma\left[1 + \frac{d^2}{4r^2} - \left(1 + \frac{3}{16}\frac{d^4}{r^4}\right)\cos 2\theta\right]$$

$$\tau_{r\theta} = -\frac{1}{2}\sigma\left(1 - \frac{1}{4}\frac{d^2}{r^2}\right)\left(1 + \frac{3}{4}\frac{d^2}{r^2}\right)\sin 2\theta$$

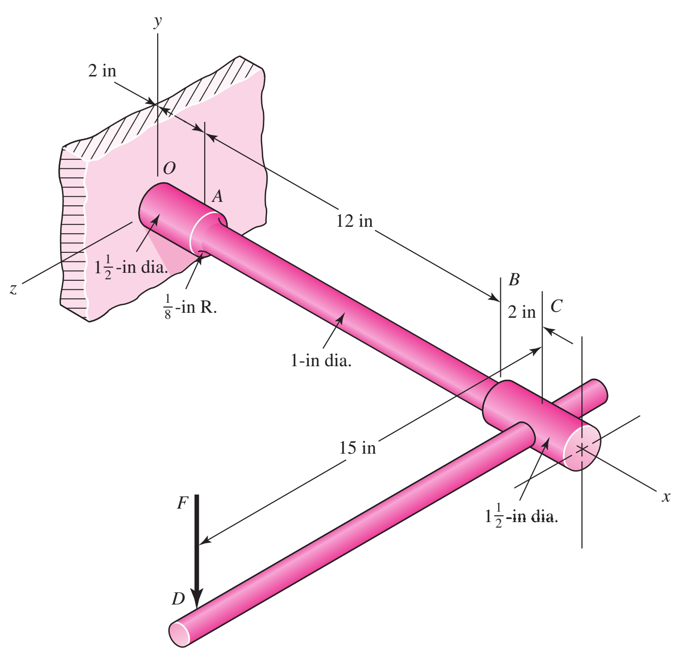
for the radial, tangential, and shear components, respectively. Here r is the distance from the center to the point of interest and θ is measured positive counterclockwise from the horizontal axis.

(*a*) Find the stress components at the top and side of the hole for $r = d/2$.

(*b*) If $d = 10$ mm, plot a graph of the tangential stress distribution σ_θ/σ for $\theta = 90°$ from $r = 5$ mm to 20 mm.

(*c*) Repeat part (*b*) for $\theta = 0°$

3–50 Considering the stress concentration at point A in the figure, determine the maximum normal and shear stresses at A if $F = 200$ lbf.



Problem 3–50

3–51 Develop the formulas for the maximum radial and tangential stresses in a thick-walled cylinder due to internal pressure only.

3–52 Repeat Prob. 3–51 where the cylinder is subject to external pressure only. At what radii do the maximum stresses occur?

3–53 Develop the stress relations for a thin-walled spherical pressure vessel.

3–54 A pressure cylinder has a diameter of 150 mm and has a 6-mm wall thickness. What pressure can this vessel carry if the maximum shear stress is not to exceed 25 Mpa?

3–55 A cylindrical pressure vessel has an outside diameter of 10 in and a wall thickness of $\frac{3}{8}$ in. If the internal pressure is 350 psi, what is the maximum shear stress in the vessel walls?

3–56 An AISI 1020 cold-drawn steel tube has an ID of $1\frac{1}{4}$ in and an OD of $1\frac{3}{4}$ in. What maximum external pressure can this tube take if the largest principal normal stress is not to exceed 80 percent of the minimum yield strength of the material?

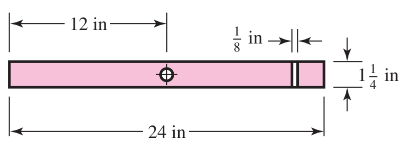
3–57 An AISI 1020 cold-drawn steel tube has an ID of 40 mm and an OD of 50 mm. What maximum internal pressure can this tube take if the largest principal normal stress is not to exceed 80 percent of the minimum yield strength of the material?

3–58 Find the maximum shear stress in a 10-in circular saw if it runs idle at 7200 rev/min. The saw is 14 gauge (0.0747 in) and is used on a $\frac{3}{4}$-in arbor. The thickness is uniform. What is the maximum radial component of stress?

3–59 The maximum recommended speed for a 300-mm-diameter abrasive grinding wheel is 2069 rev/min. Assume that the material is isotropic; use a bore of 25 mm, $\nu = 0.24$, and a mass density of 3320 kg/m^3; and find the maximum tensile stress at this speed.

3–60 An abrasive cutoff wheel has a diameter of 6 in, is $\frac{1}{16}$ in thick, and has a 1-in bore. It weighs 6 oz and is designed to run at 10 000 rev/min. If the material is isotropic and $\nu = 0.20$, find the maximum shear stress at the design speed.

3–61 A rotary lawn-mower blade rotates at 3000 rev/min. The steel blade has a uniform cross section $\frac{1}{8}$ in thick by $1\frac{1}{4}$ in wide, and has a $\frac{1}{2}$-in-diameter hole in the center as shown in the figure. Estimate the nominal tensile stress at the central section due to rotation.



Problem 3–61

3–62 to 3–67 The table lists the maximum and minimum hole and shaft dimensions for a variety of standard press and shrink fits. The materials are both hot-rolled steel. Find the maximum and minimum values of the radial interference and the corresponding interface pressure. Use a collar diameter of 80 mm for the metric sizes and 3 in for those in inch units.

Problem Number	Fit Designation*	Basic Size	Hole D_{max}	Hole D_{min}	Shaft d_{max}	Shaft d_{min}
3–62	40H7/p6	40 mm	40.025	40.000	40.042	40.026
3–63	(1.5 in)H7/p6	1.5 in	1.5010	1.5000	1.5016	1.5010
3–64	40H7/s6	40 mm	40.025	40.000	40.059	40.043
3–65	(1.5 in)H7/s6	1.5 in	1.5010	1.5000	1.5023	1.5017
3–66	40H7/u6	40 mm	40.025	40.000	40.076	40.060
3–67	(1.5 in)H7/u6	1.5 in	1.5010	1.5000	1.5030	1.5024

*Note: See Table 7–9 for description of fits.

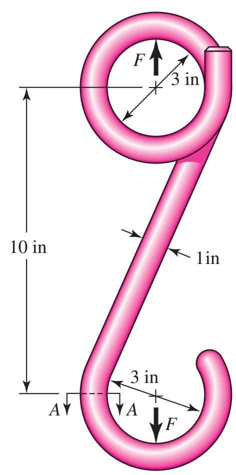
3–68 to 3–71 The table gives data concerning the shrink fit of two cylinders of differing materials and dimensional specification in inches. Elastic constants for different materials may be found in Table A–5. Identify the radial interference δ, then find the interference pressure p, and the tangential normal stress on both sides of the fit surface. If dimensional tolerances are given at fit surfaces, repeat the problem for the highest and lowest stress levels.

Problem Number	Inner Cylinder Material	d_i	d_0	Outer Cylinder Material	D_i	D_0
3–68	Steel	0	1.002	Steel	1.000	2.00
3–69	Steel	0	1.002	Cast iron	1.000	2.00
3–70	Steel	0	1.002/1.003	Steel	1.000/1.001	2.00
3–71	Steel	0	2.005/2.003	Aluminum	2.000/2.002	4.00

3–72 Force fits of a shaft and gear are assembled in an air-operated arbor press. An estimate of assembly force and torque capacity of the fit is needed. Assume the coefficient of friction is f, the fit interface pressure is p, the nominal shaft or hole radius is R, and the axial length of the gear bore is l.
(*a*) Show that the estimate of the axial force is $F_{ax} = 2\pi f Rlp$.
(*b*) Show the estimate of the torque capacity of the fit is $T = 2\pi f R^2 lp$.

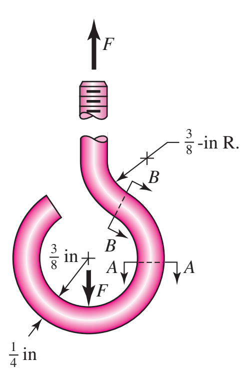
3–73 A utility hook was formed from a 1-in-diameter round rod into the geometry shown in the figure. What are the stresses at the inner and outer surfaces at section A-A if the load F is 1000 lbf?

Problem 3–73



3–74 The steel eyebolt shown in the figure is loaded with a force F of 100 lbf. The bolt is formed of $\frac{1}{4}$-in-diameter wire to a $\frac{3}{8}$-in radius in the eye and at the shank. Estimate the stresses at the inner and outer surfaces at sections A-A and B-B.

Problem 3–74

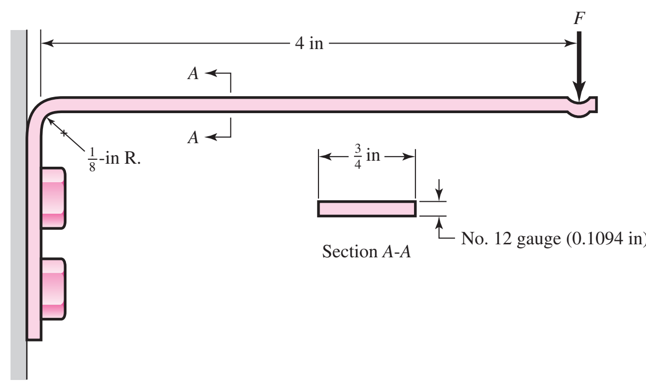


3–75 Shown in the figure is a 12-gauge (0.1094-in) by $\frac{3}{4}$-in latching spring that supports a load of $F = 3$ lbf. The inside radius of the bend is $\frac{1}{8}$ in. Estimate the stresses at the inner and outer surfaces at the critical section.

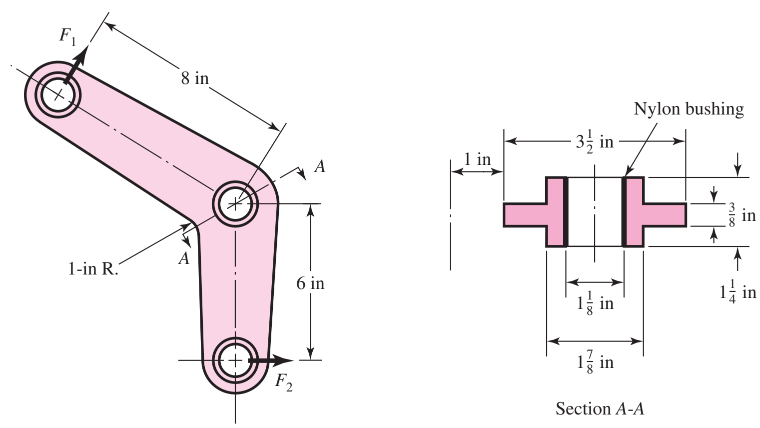
3–76 The cast-iron bell-crank lever depicted in the figure is acted upon by forces F_1 of 250 lbf and F_2 of 333 lbf. The section A-A at the central pivot has a curved inner surface with a radius of $r_i = 1$ in. Estimate the stresses at the inner and outer surfaces of the curved portion of the lever.

3–77 The crane hook depicted in Fig. 3–35 has a 1-in-diameter hole in the center of the critical section. For a load of 5 kip, estimate the bending stresses at the inner and outer surfaces at the critical section.

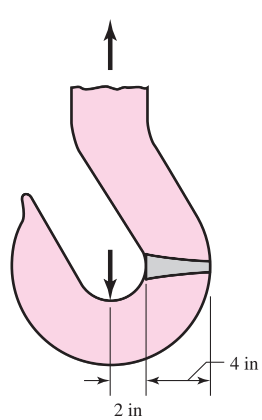
3–78 A 20-kip load is carried by the crane hook shown in the figure. The cross section of the hook uses two concave flanks. The width of the cross section is given by $b = 2/r$, where r is the radius from the center. The inside radius r_i is 2 in, and the outside radius $r_o = 6$ in. Find the stresses at the inner and outer surfaces at the critical section.



Problem 3–75



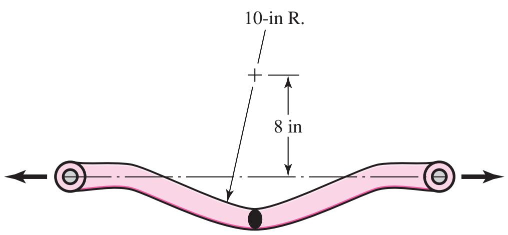
Problem 3–76



Problem 3–78

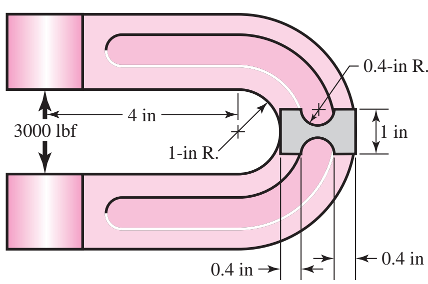
3–79 An offset tensile link is shaped to clear an obstruction with a geometry as shown in the figure. The cross section at the critical location is elliptical, with a major axis of 4 in and a minor axis of 2 in. For a load of 20 kip, estimate the stresses at the inner and outer surfaces of the critical section.

Problem 3–79



3–80 A cast-steel C frame as shown in the figure has a rectangular cross section of 1 in by 1.6 in, with a 0.4-in-radius semicircular notch on both sides that forms midflank fluting as shown. Estimate A, r_c, r_n, and e, and for a load of 3000 lbf, estimate the inner and outer surface stresses at the throat C. *Note:* Table 3–4 can be used to determine r_n for this section. From the table, the integral $\int dA/r$ can be evaluated for a rectangle and a circle by evaluating A/r_n for each shape [see Eq. (3–64)]. Subtracting A/r_n of the circle from that of the rectangle yields $\int dA/r$ for the C frame, and r_n can then be evaluated.

Problem 3–80

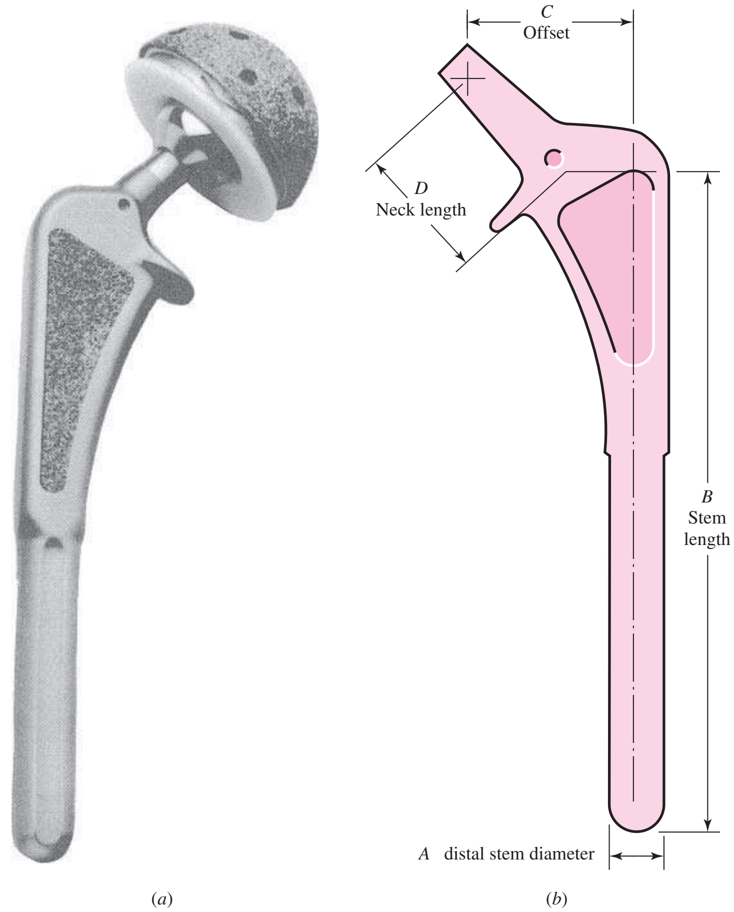


3–81 Two carbon steel balls, each 25 mm in diameter, are pressed together by a force F. In terms of the force F, find the maximum values of the principal stress, and the maximum shear stress, in MPa.

3–82 One of the balls in Prob. 3–81 is replaced by a flat carbon steel plate. If $F = 18$ N, at what depth does the maximum shear stress occur?

3–83 An aluminum alloy roller with diameter 1 in and length 2 in rolls on the inside of a cast-iron ring having an inside radius of 4 in, which is 2 in thick. Find the maximum contact force F that can be used if the shear stress is not to exceed 4000 psi.

3–84 The figure shows a hip prosthesis containing a stem that is cemented into a reamed cavity in the femur. The cup is cemented and fastened to the hip with bone screws. Shown are porous layers of titanium into which bone tissue will grow to form a longer-lasting bond than that afforded by cement alone. The bearing surfaces are a plastic cup and a titanium femoral head. The lip shown in the figures bears against the cutoff end of the femur to transfer the load to the leg from the hip. Walking will induce several million stress fluctuations per year for an average person, so there is danger that the prosthesis will loosen the cement bonds or that metal cracks may occur because of the many repetitions of stress. Prostheses like this are made in many different sizes. Typical



Problem 3–84

Porous hip prosthesis. *(Photograph and drawing courtesy of Zimmer, Inc., Warsaw, Indiana.)*

dimensions are ball diameter 50 mm, stem diameter 15 mm, stem length 155 mm, offset 38 mm, and neck length 39 mm. Develop an outline to follow in making a complete stress analysis of this prosthesis. Describe the material properties needed, the equations required, and how the loading is to be defined.

3–85 Simplify Eqs. (3–70), (3–71), and (3–72) by setting $z = 0$ and finding σ_x/p_{max}, σ_y/p_{max}, σ_z/p_{max}, and $\tau_{2/3}/p_{max}$ and, for cast iron, check the ordinate intercepts of the four loci in Fig. 3–37.

3–86 A 6-in-diameter cast-iron wheel, 2 in wide, rolls on a flat steel surface carrying an 800-lbf load.

(*a*) Find the Hertzian stresses σ_x, σ_y, σ_z, and $\tau_{2/3}$.

(*b*) What happens to the stresses at a point A that is 0.010 in below the wheel rim surface during a revolution?